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Brownian motion in one-dimensional disordered systems

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Résumé. — Le mouvement brownien d'une particule plongée dans un potentiel aléatoire du type Wiener-Lévy est étudié. Les fluctuations thermiques exercent une force aléatoire ou communiquent une énergie aléatoire à la particule qui effectue un mouvement brownien. Ces deux types de bruit conduisent à deux relations d'échelle distinctes de l'espace (L)-temps (t) : $L \approx \ln^2 t$ ou $L \approx t^\nu$. L'application de ce modèle au cas d'une paroi de domaine est discutée et les expressions des susceptibilités ou conductibilités de basse fréquence sont établies en vue d'une comparaison éventuelle aux expériences.

Abstract. — The Brownian motion of a particle in a one-dimensional random potential of Wiener-Lévy type is studied. Thermal fluctuations exert random forces or transfer random energies which set the particle in Brownian motion. These two kinds of noise provide different asymptotic space (L)-time (t) scaling relations : $L \approx \ln^2 t$ or $L \approx t^\nu$. The application of this model to a single domain wall is discussed and the expression of low frequency magnetic susceptibility and electrical conductivity are established and possible comparisons with experiments are discussed.

1. Introduction.

The recent interest in the diffusion problem of a particle in a random one-dimensional lattice deals with models which are a bit far from reality. The basic microscopic features of these models are the symmetric or antisymmetric transition rates between the nearest neighbour sites, taken as independent random variables and obeying a given probability density (see e.g. Ref. [1]). These assumptions are not well established in the physical situations discussed in this context as for instance the one-dimensional conductors. I propose to study here the Brownian motion of a particle or defect like a rigid domain wall in a disordered medium which exerts weak pinning forces. There is a crucial distinction between the energy fluctuation thermal noise and the force fluctuating thermal noise which leads to two different types of law, the first of which is similar to the Sinai regime [2, 3] while the second one produces a new asymptotic power law of diffusion. By a simple scaling method involving the most probable relevant variables, the space-time relations are established as well as the related measurable quantities like the frequency dependent magnetic susceptibility and electrical conductivity. Possible applications to a magnetic domain wall or a charged wall of discommensuration in a one-dimensional solid are discussed.

2. The model.

Let us consider an elementary displacement δx of the particle along the one-dimensional axis Ox and suppose that a random increment δV of the potential energy results from this displacement. $V(x)$ is a random function with an independent random increment which is represented by a random curve of Brownian or Wiener-Levy type. Any extended rigid defect embedded in a disordered medium has the stated property of being able to change its potential energy for an elementary displacement δx if impurities or defects are present in the swept volume. In this sense, the above assumption made for $V(x)$ seems very general with the condition that the defect would be extended enough to explore large values of $V(x)$. This hypothesis was put forward in 1942 by Néel [4] in terms of the polygonal contour energy of a Bloch wall in order to successfully explain the Rayleigh regime of ferromagnetic substances [5]. Additional hypotheses concerning the origin of the noise are necessary for a description of Brownian motion. Two situations are possible. By contact with the thermal reservoir the thermal fluctuations act either as forces (magnetic field) or as energy. Historically, the Brownian motion of colloidal particles belongs to the first category where the random forces originate from elastic collisions with atoms of the fluid within which the particles are immersed. In magnetism, the local fluctuations of the magnetization create fields which act as random forces on the Bloch wall. On the contrary the thermal fluctuations of energy are often due to inelastic collisions with the excitations of the thermal bath (phonons, etc...). In the case of *thermal fluctuations of forces or fields* for displacing the particle the fluctuating force must overcome the pinning force $-dV/dX$ in one of the directions chosen. The shift of the particle stops at the site where the pinning force is stronger than the random force. This mechanism has been applied to wall motion and has provided a good understanding of the magnetic hysteresis and well as thermal magnetic creep [6]. The *thermal fluctuation of energy* must be considered otherwise : in the classical regime the fluctuating energy must overcome as usual the potential barriers of the Wiener-Levy curve $V(x)$ for a shift of the particle. In both cases, the transfer mechanism induces long range displacement which cannot be handled by the standard methods of calculating random walks. Derived from the explicit scaling approach of Alexander *et al.* [1] a scaling method is made use of to set up the space-time relation in the asymptotic regime $t \rightarrow \infty$. There are 3 necessary stages.

A. Consider a well of length L of the Wiener-Levy curve $V(x)$. Calculate the most probable variable $X(L)$ — maximum pinning force or minimum energy — as a function of L .

B. For a long period of time t deduce from the noise the most probable fluctuating variable $Y^*(t)$ — maximum force or energy.

C. Estimate the most probable time of escape of the well by putting $X^*(L) = Y^*(t)$. This equality provides the space-time relation $L(t)$ for the considered random walk.

3. Thermal fluctuations of forces or fields.

In stage A, the well is built up with slopes of the Wiener-Levy potential $S_i = -\partial V/\partial x_i$ on discrete intervals. At $x = 0$ and $x = L$, the terminal slopes $-S_0, +S_L$ are supposed steeper than S (a given slope) while within the well the slopes S_i ($i = 1, 2, \dots, L - 1$) are such that $|S_i| < S$. Let $G(S_i)$ be the Gaussian probability density of variance σ_s

$$G(S_i) = \frac{1}{\sigma_s \sqrt{2\pi}} \exp\left(-\frac{S_i^2}{2\sigma_s^2}\right) \quad (1)$$

and

$$\eta(S) = 2 \int_0^S G(S_i) dS \quad (2)$$

the probability of finding a slope S_i less than S at site i . The probability of occurrence of the well described above with independent random slope S_i is

$$\eta(S)^L(1 - \eta(S)). \tag{3}$$

The most probable value of the slope S^* is given by the maximum of(3) :

$$L = \frac{\eta(S^*)}{1 - \eta(S^*)}. \tag{4}$$

For large L , $\eta(S^*) \lesssim 1$. The first term of the expansion of the error function (2)

$$\eta(S^*) \simeq 1 - \frac{\sigma_s}{\sqrt{\pi} S^*} \exp\left(-\frac{S^{*2}}{2 \sigma_s^2}\right) \tag{5}$$

gives

$$S^{*2} \simeq 2 \sigma_s^2 \left(\ln L + \ln \left(\sqrt{\frac{2}{\pi}} \frac{\sigma_s}{S^*} \right) \right). \tag{6}$$

The last term of(5) can be considered as constant in all of the practical situations. Finally

$$S^{*2} \simeq 2 \sigma_s^2 \ln L + \text{const.} \tag{7}$$

B. Suppose that the fluctuating forces are described by Gaussian white noise in a discrete time interval t , the probability density of which is :

$$G(f) = \frac{1}{\sigma_f \sqrt{2 \pi}} \exp\left(-\frac{f^2}{2 \sigma_f^2}\right). \tag{8}$$

Let $f_m = \max \{ |f_1|, |f_2|, \dots, |f_t| \}$. The probability of finding $|f| \leq f_m \leq |f + df|$ is given by $d\mu^t/df$ where

$$\mu(f) = 2 \int_0^{|f|} G(f') df'. \tag{9}$$

The most probable maximum value $|f_m^*|$ during the time t is given by the maximum of $d\mu^t/df$, one finds for long time t :

$$f_m^{*2} \simeq 2 \sigma_f^2 \left(\ln t + \ln \left(\sqrt{\frac{2}{\pi}} \frac{\sigma_f}{f_m^*} \right) \right) \tag{10}$$

where, as in (5), the last term can be considered as constant for practical purposes

$$f_m^{*2} \simeq 2 \sigma_f^2 \ln t + \text{const.} \tag{11}$$

C. The most probable time of escape from the well is estimated by writing the relation :

$$S^* \simeq f_m^*$$

which gives

$$L \approx t^\nu. \tag{12}$$

Here $\nu = \sigma_f^2 / \sigma_s^2$ describes the ratio of the thermal fluctuations over the variance of the pinning forces. The physical meaning is now clear : the stronger the pinning forces as compared to the thermal fluctuations, the smaller ν and consequently the confinement of the particle is stronger. As Gaussian thermal fluctuations σ_f^2 are proportional to T , therefore $\nu = T/T_0$ where T_0 is related to the variance of the pinning forces σ_s^2 .

Actually the relation (11) which gives the most probable maximum force at time t is reminiscent of the magnetic « reptation » of ferromagnetic substances described by Néel [6]. In both situations it is the Gaussian distribution law which gives the variation of f_m^{*2} with $\ln t$. The situation which is analysed here is that L is large but, this is not actually a strict condition for the asymptotic regime. It is a condition rather for a simplified expression of the most probable pinning forces edging the well. The asymptotic regime is really defined by the limit where t is going to infinity. This condition is fulfilled when the exponent ν is much less than 1 or, equivalently, $\sigma_s \gg \sigma_f$. It is then a regime of strong confinement as expected, since the diffusion constant, proportional to L^2/t or $t^{2\nu-1}$, goes to zero as $t \rightarrow \infty$ because $\nu < 1/2$.

4. Thermal fluctuations of energy.

In stage A, the potential well of the Wiener-Levy curve is such that $V(0) = V(L)$ and $V(x_i) \leq V(0)$ for $x_i \in [1, L - 1]$. The most probable depth ΔE^* of the Wiener-Levy well is equal to : $1/2 \sigma_s \sqrt{aL}$ (a : step size of the space discretization). Consider now, in step B, a sequence of energies extracted from the Boltzmann distribution law and let E_m be the maximum energy of the sequence :

$$E_m = \max \{ E_1, E_2, \dots, E_t \} . \quad (13)$$

It is easy to get the most probable energy of a t -sequence :

$$E_m^* = T \ln t \quad (14)$$

which is the well-known Arrhenius law of activation. Now the most probable time of escape from the well (step C) is obtained by the relation :

$$E_m^* \simeq \Delta E^* \quad (15)$$

which yields

$$L = \frac{4 T^2}{a \sigma_s^2} \ln^2 t . \quad (16)$$

The $\ln^2 t$ time dependence of L is characteristic of a Sinai regime [2, 3] while T^2 indicates the origin of the fluctuation from the thermal bath of the canonical ensemble. Very strong localization is contained in this expression since, as the walk progresses, deeper and deeper wells are explored.

5. Magnetic susceptibility and electrical conductivity.

The asymptotic space-time scaling relations are not directly measurable. But the space variable L^2 stands for the space correlation function $\overline{x(0) x(t)}$.

Let us define the power spectrum of x as : $S_x(\omega)$ by the standard relation

$$S_x(\omega) = \lim_{\theta \rightarrow \infty} \frac{1}{\theta} \left| \int_0^\theta e^{i\omega t} x(t) dt \right|^2 \quad (17)$$

combined with the asymptotic scaling relation

$$x(\tau t) = \lambda(\tau) x(t) \tag{18}$$

where τ and λ are respectively the time and space dilation factors. It is easy to obtain

$$S_x(\omega) \approx (\omega \lambda^2(\omega))^{-1}. \tag{19}$$

Since

$$\lambda(\omega) = L^{-1}(t = 1/\omega) \tag{20}$$

one gets :

$$S_x(\omega) \approx \frac{L^2(t = 1/\omega)}{\omega}. \tag{21}$$

One deduces from the scaling relations (12) and (16)

(force)
$$S_x(\omega) \approx \omega^{-(1+2T/T_0)} \tag{22}$$

(energy)
$$S_x(\omega) \approx T^4 \omega^{-1} \ln^4 \omega. \tag{23}$$

The frequency dependence of (23) has been derived recently by Marinari *et al.* [7] from similar assumptions and is considered as a possible source of $1/f$ noise. In the context of ferromagnetism, $x(t)$, the position of the Bloch wall is related linearly to the total magnetization of the substance. Consequently the magnetic susceptibility $\chi(\omega)$ is equal to :

$$\chi(\omega) = \frac{1}{\Omega_0 T} S_M(\omega) \tag{24}$$

where $S_M(\omega)$ is the spectrum power of the magnetization and Ω_0 the total volume. By assuming the proportionality between $S_M(\omega)$ and $S_x(\omega)$ one gets finally the following low frequency susceptibility :

(force)
$$\chi(\omega) \approx T^{-1} \omega^{-(1+2\frac{T}{T_0})} \tag{25}$$

(energy)
$$\chi(\omega) \approx T^3 \omega^{-1} \ln^4 \omega. \tag{26}$$

The frequency dependence is then different in both cases approaching a $1/\omega$ behaviour at low temperature (the limit $T \rightarrow 0$ is out of validity of this calculation because in each case the space variable must be asymptotically large i.e. : $t^{T/T_0} \gg 1$ or $T^2 \ln^2 t \gg 1$ for the validity of the scaling relations (12) and (16)). The final divergence as $\omega \rightarrow 0$ reflects the ultimate excursion of the wall at infinity (Bloch wall cannot be annihilated anywhere in this model).

In the context of transport let us consider now a charged wall. From the general scaling hypothesis of Alexander *et al.* [1] a simple relation can be obtained between the complex electrical conductivity $\sigma(\omega)$ and the space-time relation :

$$\sigma(\omega) \approx \frac{1}{T} \frac{\omega}{i} L^2 \left(t = \frac{i}{\omega} \right) \tag{27}$$

with relations (10) and (13), one obtains immediately :

(force)
$$\sigma(\omega) \approx \frac{1}{T} \left(\frac{\omega}{i} \right)^{1-2\frac{T}{T_0}} \tag{28}$$

(energy)
$$\sigma(\omega) \approx \frac{\omega}{i} T^3 \ln^4 \left(\frac{\omega}{i} \right). \tag{29}$$

Both kinds of conductivity vanish when $\omega \rightarrow 0$. This is the situation expected in strong localization regime. The expression (29) is partly reminiscent of the AC hopping conductivity for exponentially localized states as deduced from a phenomenological derivation using the Debye theory of dielectric relaxation [8].

Finally the recurrence probability of the initial position at time t is obtained in Fourier variables. Let $P_0(\omega)$ be the frequency development of the initial probability amplitude. The general scaling hypothesis of Alexander *et al.* provides the following relation :

$$P_0(\omega) \approx \omega^{-1} L^{-1}(t = 1/\omega). \quad (30)$$

In table I, the corresponding relations for both types of Brownian motion are reported as well as a résumé of the main formulae of this section. It must be noticed that the frequency dependence of the T_1 — relaxation rate in a classical experiment of magnetic resonance is just proportional to this $P_0(\omega)$ calculated here.

6. Discussion.

The two kinds of Brownian motions pointed out in the previous sections are very different from the standard random walk where L is proportional to $t^{1/2}$. The existence of the random potential of the Wiener-Levy type induced strong confinement of the particle since the exponent ν is always less than 1/2. The action of the thermal bath is different in both cases of source of fluctuations and provides different scaling relations. Then a detailed analysis of the mechanism of noise must be made before deciding what the most probable regime is. These two aspects cannot be reconciled in the limit of infinite time since the thermodynamic limit of a particle immersed in a random potential $V(x)$ of Wiener-Levy type probably does not exist.

The possibility of observation of Brownian motion of this type in ferromagnetic substances requires that several conditions must be fulfilled.

First, the Bloch wall must be postulated to be rigid and capable of displacement in one dimension as a whole. Actually this behaviour is observed as a response to an applied magnetic field and the model of random potential energy $V(x)$ depending only on single variable x is usually accepted (see for example Ref. [5]). However the spontaneous Brownian motion is more questionable since the domain wall can be displaced a part at a time by thermal fluctuations. The dynamics of these

Table I. — Low frequency $\omega \rightarrow 0$ response functions for both types of Brownian motions

	Space-time asymptotic relation	Magnetic susceptibility	Frequency development of the initial probability amplitude	Complex electrical conductivity
Thermal fluctuations of forces or fields ($T \ll T_0$)	$L \approx t^{T/T_0}$	$\chi(\omega) \approx \frac{1}{T\omega^{1+2T/T_0}}$	$P_0(\omega) \approx \frac{1}{\omega^{1-T/T_0}}$	$\sigma(\omega) \approx \frac{1}{T} \left(\frac{\omega}{i}\right)^{1-2T/T_0}$
Thermal fluctuations of energy	$L \approx T^2 \ln^2 t$	$\chi(\omega) \approx T^3 \frac{\ln^4 \omega}{\omega}$	$P_0(\omega) \approx \frac{1}{T^2 \omega \ln^2 \omega}$	$\sigma(\omega) \approx T^3 \frac{\omega}{i} \ln^4 \left(\frac{\omega}{i}\right)$

L : space variable ; t : time variable ; T : temperature of the bath ; T_0 : characteristic temperature related to the pinning forces.

internal modes of flexibility are usually assumed to exist at higher frequencies (spin waves) than the transverse motion as a whole of the domain wall. Then, in the asymptotic regime of very long time which is considered here, only the translational degree of freedom subsists and the type of one-dimensional Brownian motion proposed here could be observable at very low frequency.

Among the two regimes of noise envisaged previously, the thermal fluctuating fields are more probable for Bloch walls. In this case the characteristic exponent $\nu = \sigma_f^2/\sigma_p^2$ can be analysed more quantitatively. It was established by Néel [9] that the mean square of thermal random field σ_f^2 is proportional to T/Ω where Ω is a characteristic volume affected by the thermal fluctuations of the spontaneous magnetization in the domains. In fact both quantities σ_f^2 and σ_p^2 are measurable in classical hysteresis experiments where σ_f^2 is proportional to the after-effect relaxation fields and σ_p^2 to the coercitive fields. At low temperature, standard purity and crystalline perfection of ferromagnetic samples, the ratio $\sigma_f^2/\sigma_p^2 = T/T_0$ is too small and is typically of the order of 10^{-2} or 10^{-3} [10] for any observation of the Brownian motion. It has been suggested [11] that near the critical temperature this ratio increases strongly approaching a sizeable value which could be an important factor of acceleration of this random walk and could make this motion observable.

For one-dimensional systems, large density waves could be describe by this model. Particularly, the ground state of an incommensurate system can be described in terms of walls of discommensuration. When these walls carry an effective charge the electrical conductivity can be of the type indicated in table I. Well below T_c , transport coefficients and particularly their frequency dependence could be measurable and compared to the proposed theoretical formulae.

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