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Enhanced ion-damping in an ion-acoustic instability

Tu Khiet

Laboratoire de Physique des gaz et des plasmas (*), Université Paris-Sud, 91405 Orsay, France

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Résumé. — On établit une équation quasi linéaire renormalisée pour la fonction de distribution des ions et on calcule le taux d'amortissement dû aux ions énergiques. Le spectre d'énergie se comporte comme k^{-2} sans coupure artificielle.

Abstract. — We derive a renormalized quasilinear equation for the ion distribution function and we calculate the modified damping rate from energetic ions. The energy spectrum behaves as k^{-2} without artificial cut-off.

Ion acoustic turbulence is of great interest to many problems in laboratory and space plasma physics. In turbulent heating experiments it acts to slow down the heating current, in the solar wind and laser-produced plasma, it struggles against the instability by reducing the heat flux. As a consequence the plasma resistivity is anomalously enhanced in the first case and the conductivity diminished in the second one. In both cases the transport coefficients are shown to be directly connected to the turbulent energy spectrum which remains ill-known up to date. In this short communication, we examine explicitly the saturation mechanism of an ion-acoustic instability driven by an electron drift-current; the results can be easily extended to an instability-dominated laser plasma. This study is aimed to provide insight on the role of the ion in the saturation. In this optics, induced scattering [1] and modification of ion orbit [2] have been previously studied but they both lead to a too high saturated energy in a system having a large electron-to-ion temperature ratio. We shall examine the production mechanism of energetic ions which are speculated to quench the instability. For that purpose we shall derive an equation for the ion-background function using a Fourier transformation in velocity space in conjunction with a technique of expansion in cumulants in the same manner as for the calculation of a renormalized propagator in strong turbulence theory. Our equation is shown to be a renormalized quasilinear one which presents an advantage that it does not imply a further resolution

of a diffusion equation. Finally we shall calculate the modified ion damping which subsequently gives the saturated state by requiring an exact balance with electron growth rate.

We consider a homogeneous plasma with $T_e \gg T_i$, where T_e (T_i) is the electron (ion) temperature. The electron distribution function is chosen to be a shifted Maxwellian with a drift velocity v_d largely exceeding in magnitude the ion-acoustic speed $c_s = (T_e/M)^{1/2}$, M being the ion mass. In these conditions, ion-acoustic waves are generated with frequency

$$\omega_k = kc_s(1 + k^2 \lambda_e^2)^{-1/2},$$

λ_e being the electron Debye length, and growth rate $\gamma = \gamma_e + \gamma_i$ where

$$\begin{aligned} \gamma_e &= \frac{\omega_{pe}^2}{n_0 k^2} \left(\frac{\partial \varepsilon}{\partial \omega} \right)^{-1} \int d^3 v k \cdot \frac{\partial F_e}{\partial v} \delta(\omega - k \cdot v) \\ \gamma_i &= - \frac{\pi^2 \omega_k^4}{n_0 k^3} F_i \left(\frac{\omega}{k} \right) \end{aligned} \quad (1)$$

where we have only assumed that F_i is isotropic in velocity space. It is also interesting to note that the ion damping rate is essentially determined by ion population in the range of velocity between $c_s/\sqrt{2}$ and c_s . As time progresses, the waves can drive the particles to higher velocity and subsequently modify the ion distribution in the domain of interest. This effect can be taken into account by using the conservation on the number of particles along a particle orbit, i.e.,

$$f(\mathbf{r}, \mathbf{v}, t) = \mathcal{F}(\mathbf{r}(t), \mathbf{v}(t)) \quad (2)$$

(*) Laboratoire associé au C.N.R.S.

where $\mathcal{F}(\mathbf{r}, \mathbf{v}) = f(\mathbf{r}, \mathbf{v}, t = 0)$, $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are the position and velocity of a particle at $t = 0$, knowing that they take the values \mathbf{r} and \mathbf{v} , respectively, at time t . From the fact that \mathcal{F} is a superposition of a homogeneous part, $F_0(\mathbf{v})$, and an inhomogeneous part, δf , with $\delta f \ll F_0$, the time dependent average distribution function, $F(\mathbf{v}, t)$, which results from the ensemble average of (2) can be put in the form

$$F(\mathbf{v}, t) = \int d^3p \langle \exp i\mathbf{p} \cdot \mathbf{v}(t) \rangle H(p) \quad (3)$$

where we have used a Fourier analysis, $H(p)$ being the Fourier transform of $F_0(\mathbf{v})$. Calculation of (3) can now be carried out by using an expansion into cumulants. The relevant quantities in our problem are $\langle \Delta \mathbf{v} \rangle$ and $\langle \Delta \mathbf{v} \Delta \mathbf{v} \rangle$ where $\Delta \mathbf{v} = \mathbf{v}(t) - \mathbf{v}$. This can be done by assuming a W.K.B. variation of the wave field during the relaxation process in the form

$$\varphi_{\mathbf{k}}(t) = \varphi_{\mathbf{k}} \exp \int dt \gamma_{\mathbf{k}}(t),$$

giving in the lowest approximation

$$\begin{aligned} \langle \Delta \mathbf{v} \Delta \mathbf{v} \rangle &= \frac{e^2}{M^2} \int d^3k \frac{\mathbf{k} \mathbf{k} |\varphi_{\mathbf{k}}|^2}{(\omega - \mathbf{k} \cdot \mathbf{v})^2 + \gamma_{\mathbf{k}}^2} \\ \langle \Delta \mathbf{v} \rangle &= \frac{1}{2} \frac{\partial}{\partial \mathbf{v}} \cdot \langle \Delta \mathbf{v} \Delta \mathbf{v} \rangle. \end{aligned} \quad (4)$$

It is worth noticing that the usual quasilinear equation can be recovered by linearising the exponential term in (3) and identifying \mathbf{p} with $-i \partial / \partial \mathbf{v}$. We further assume an isotropic spectrum, i.e., $|\varphi_{\mathbf{k}}|^2 = I(k)$ and integrate (4) over angles, this gives

$$\begin{aligned} \langle \mathbf{p} \cdot \Delta \mathbf{v} \rangle &= pU \\ \langle (\mathbf{p} \cdot \Delta \mathbf{v})^2 \rangle &= p_{\parallel}^2 a_{\parallel}^2 + p_{\perp}^2 a_{\perp}^2 \end{aligned} \quad (5)$$

where subscripts refer to the direction of \mathbf{v} and the various quantities appearing in the RHS of (5) take different values accordingly to the value of v . The two limiting cases are :

1. $v \ll c_s$. In this domain calculations are made by neglecting \mathbf{v} in (4), this gives

$$\begin{aligned} U &= \frac{\alpha c_s}{2} \\ a_{\parallel}^2 &= a_{\perp}^2 = \frac{2 \alpha c_s^2}{3} \end{aligned} \quad (6)$$

where $\alpha = e^2 \varphi^2 / T_e^2$ is the characteristic energy ratio.

2. $v \geq c_s / \sqrt{2}$. In this domain, calculations are greatly simplified by replacing the kernel of (4) by a delta function we then obtain

$$\begin{aligned} U &= \frac{\pi c_s^7}{\eta v_d^2} \int_K^{\infty} dk \frac{k^6 W(k)}{\omega_k^4} \\ a_{\parallel}^2 &= \frac{\pi c_s^7}{\eta v_d v^2} \int_K^{\infty} dk \frac{k^4 W(k)}{\omega_k^2} \\ a_{\perp}^2 &= \frac{\pi c_s^7}{2 \eta v_d} \int_K^{\infty} dk \frac{k^6 W(k)}{\omega_k^4} \left(1 - \frac{\omega_k^2}{k^2 v^2} \right) \end{aligned} \quad (7)$$

where the notations

$$\eta = \sqrt{\pi m / 8 M}, \quad W(k) = e^2 I(k) / (4 \pi^2 T_e^2)$$

have been used and the lower limit of integration which originates from the resonance condition has to be taken as $K = 0$ for $v \geq c_s$ and

$$K = k_d \sqrt{\frac{c_s^2}{v^2} - 1} \quad \text{for } v < c_s.$$

The background function obtained by integrating (3) over \mathbf{p} now takes the form

$$F(v, t) = \frac{n_0 \exp(-(v+U)^2/a^2)}{\pi^{3/2} ab^2} \quad (8)$$

where

$$\begin{aligned} a^2 &= (2 T_i / M) + 2 a_{\parallel}^2 \\ b^2 &= (2 T_i / M) + 2 a_{\perp}^2. \end{aligned} \quad (9)$$

Referring to the expressions of a_{\parallel} and a_{\perp} given previously we conclude that the ion distribution is heated to an effective temperature

$$T_{\text{eff}} = T_i + \frac{2}{3} \alpha T_e$$

in the body part and it contains now a finite population with an effective temperature $T_h = T_e(\alpha/\eta)^{1/2}$ in the range of velocity $|\mathbf{v}| \simeq c_s$. The population of ions at $v = \omega_k/k$ which determines the damping rate according to (1) is obviously given by (8) and (7) in which $K = k$. Balancing the so-obtained damping rate with the linear electron growth rate, we obtain at saturation the equation

$$\int_K^{\infty} dk k^2 (1 + k^2 \lambda_e^2) W(k) = \beta (1 + k^2 \lambda_e^2)^{-5/2} \quad (10)$$

where $\beta = \eta / \text{Log}(c_s / \eta v_d)$.

It is easily seen from the above formula that the wave spectrum, i.e., $W(k)$, behaves as k^{-2} in the limit of very small k ; for larger values of k , this function strongly damps to zero. In addition the reduced turbulent energy deduced from the above formula is

$$\alpha = 0.25 \eta / \text{Log}(c_s / \eta v_d). \quad (11)$$

As a conclusion we have developed a simple method which enables us to explain the formation of energetic ions which have been observed experimen-

tally [3]. Under the hypothesis that these particles are responsible of the mechanism of saturation, the wave spectrum does not necessitate to be cut-off at some value of k and the total turbulent energy depends little on the temperature ratio.

It is also tempting to compare our method with those developed previously in the field of renormalized theory.

1. In the Dupree [4]-Weinstock [5] formalism the action of the operator U upon the velocity space was considered as weak, the fluctuating distributed function given by this theory in the lowest approximation as

$$\delta f = \frac{e}{M} \int dt' \bar{U}(t, t') \mathbf{E}(\mathbf{r}, t') \frac{\partial F}{\partial \mathbf{v}}(\mathbf{v}, t') \quad (12)$$

was calculated by expanding the electric field in terms of its Fourier components and considering the background function as a constant. This imperfection has been remedied on the one hand by Misguich and Balescu [6] using the operator formalism and by Horton and Choi [7] on the other hand, using the formalism of Green's function. As a result, the particle propagator in the k -space, i.e., $G_k(\mathbf{v}, \mathbf{v}')$ defined as

$$\delta f_k = -\frac{e}{M} \int d^3v' G_k(\mathbf{v}, \mathbf{v}') \mathbf{E}_k \cdot \frac{\partial F}{\partial \mathbf{v}'}(\mathbf{v}') \quad (13)$$

has been found to be non local in the velocity space. This feature may be easily recovered by our method if we use a double Fourier expansion (in real and velocity spaces) of the integrand of (12) before operating U and using the cumulant expansion.

2. As for the dynamical description of the background function which leads to the appearance of a tail population, our calculations differ with those of Horton and Choi mainly in the following points :

(i) We have considered the production of energetic particles as occurring during the relaxation phase itself but not in a steady state. (ii) The background function appeared in our work quite naturally, unlike in that of Horton and Choi who have been led to require that the particle propagator be a function of the difference of its arguments in order to solve the quasi-linear equation. In our opinion this subsidiary assumption is rather difficult to justify.

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