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Kinetic theory of magnetic field generation in the resonant absorption of light

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Résumé. — On utilise la théorie cinétique pour étudier la génération de champ magnétique dans l'absorption résonnante. Les résultats de théories antérieures sont retrouvés dans le régime non collisionnel, cependant qu'un régime collisionnel et un régime intermédiaire donnent lieu à des comportements nouveaux. Les lois d'échelle sont établies pour ces régimes. On met en évidence les hypothèses *ad-hoc* erronées de théories des fluides antérieures.

Abstract. — A kinetic theory of magnetic field generation in the resonant absorption of light is presented. Previous results are recovered in the purely collisionless regime, while a collisional and an intermediate regime are shown to give rise to new behaviours. Scaling laws are set up in those regimes. The misleading *ad-hoc* assumptions of previous phenomenological fluid theories are pointed out.

Magnetic field generation in resonant absorption of light has recently been studied for a steady state plasma, in the collisionless limit [1, 3]. The corresponding theory is achieved by solving the Vlasov equation for the electron distribution function in the incident electromagnetic field. Linear [1, 2] and *renormalized* [3] theories lead to the same analytical result :

$$\langle \mathbf{B} \rangle = \mu_0 e \langle n_e \rangle \langle \mathbf{u}_h \times \mathbf{s}_h \rangle / 2;$$

$\mathbf{u}_h(\mathbf{r}, t)$ is the linear velocity in the high-frequency electric field $\mathbf{E}_h(\mathbf{r}, t)$: $\partial \mathbf{u}_h / \partial t = -e \mathbf{E}_h / m_e$; \mathbf{s}_h is the corresponding displacement $\partial \mathbf{s}_h / \partial t = \mathbf{u}_h$, and n_e is the local electron density (we use $\langle \rangle$ to denote time averaging over a light wave period). It was demonstrated in reference [2] that the quantity of primary interest for the calculation of the magnetic field $\langle \mathbf{B} \rangle$ was the perturbed distribution function. It is also clear from reference [2] that *ad-hoc* assumptions of fluid models, such as modeling the collisionless absorption by an effective collision frequency and a phenomenological drag force, do not lead to the correct result.

It is the purpose of this Letter to show that a kinetic treatment is necessary not only in the collisionless limit, but also in the collisional limit. In fact, and this is the first failure of *ad-hoc* assumptions of fluid

models, the particle thermal pressure tensor is not independent of the high-frequency field. Indeed the main contribution for the magnetic field generation is related to the off-diagonal part of the thermal pressure tensor in the collisional limit. The second failure of fluid models is that the low-frequency nonlinear drag force does not take the simple form $\nu m_e \langle \mathbf{J} \rangle / e$, where ν is the electron-ion collision frequency. The standard drag effect in a static Lorentz force is only related to the average velocity (the current). Computed by expansion in the high-frequency electromagnetic field, the second order electron distribution function appears to have many velocity dependent different contributions [see eq. (15) below]. The effect of a collisional operator upon these terms has to be explicitly computed by a generalization of the Chapman-Enskog theory.

The method used in this Letter is similar to the method used by Bernstein *et al.* [4]. However our calculations are performed with a simpler hypothesis (we neglect large scale density and temperature gradients). On the other hand, they are performed to a higher order in the smallness parameters describing the thermal dispersion and the collisions. We find three different regimes for the dc magnetic field generation in the resonant absorption of light : a purely

collisionless regime where we recover previous results [1, 2, 3], a purely collisional regime, and an intermediate regime where the linear resonant absorption is collision dominated and the nonlinear calculations worked out in a weakly collisional regime. In the last two regimes we obtain new explicit formulas for the magnetic field.

To carry out the calculations, the electron distribution function $f^e(\mathbf{r}, \mathbf{v}, t)$ is assumed to be governed by the Fokker-Planck equation

$$\frac{\partial f^e}{\partial t} + \mathbf{v} \cdot \nabla f^e - \frac{e}{m_e} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f^e}{\partial \mathbf{v}} = \frac{\alpha}{v^3} \mathcal{C}(f^e), \quad (1)$$

$$\text{where} \quad \mathcal{C}(f) = \frac{\partial}{\partial v_i} (v^2 \partial_{ij} - v_i v_j) \frac{\partial}{\partial v_j} f \quad (2)$$

$\alpha = Z^2 e^4 n_i \ln \Lambda / 8 \pi \epsilon_0^2 m_e^2$; $\ln \Lambda$ is the Coulomb logarithm. Equation (1) corresponds to a completely ionized high Z material, where we have neglected the term for energy exchange between ions and electrons (of order m_e/m_i). We have neglected in eq. (2) the velocity of the centre-of-mass fluid. Note also that eq. (1) ignores the effects of the high-frequency field in the collision operator. The refined calculation taking into account the corresponding effect is left for a forthcoming publication. Then the present work appears as a first estimate of the collisions effect.

Following previous works [1-5], we write f^e as the sum of a slowly varying part f_i , and a part f_h which oscillates at the laser frequency, with a corresponding decomposition of the electromagnetic field

$$f^e(\mathbf{r}, \mathbf{v}, t) = f_i(\mathbf{r}, \mathbf{v}, t) + f_h(\mathbf{r}, \mathbf{v}, t), \\ f_i(\mathbf{r}, \mathbf{v}, t) = \langle f^e(\mathbf{r}, \mathbf{v}, t) \rangle.$$

To the lowest order in the high-frequency field strength, we require f_i to be a Maxwellian distribution function f_0 . This choice is natural since when there is no electromagnetic field, collisions relax the distribution function towards a Maxwellian form. In the collisionless limit, where this is no longer true, it can be shown [3, 6] that a renormalized collisionless theory does not actually require any assumption about f_i . In the high- (low-) frequency equation we keep only up to first (second) order terms in the high-frequency field strength. As the low-frequency electric and magnetic fields are presumably second order variables, we finally obtain two coupled equations for f_i and f_h

$$\frac{\partial f_h}{\partial t} + \mathbf{v} \cdot \nabla f_h - \frac{e}{m_e} \mathbf{E}_h \cdot \frac{\partial f_h}{\partial \mathbf{v}} = \frac{\alpha}{v^3} \mathcal{C}(f_h), \quad (3)$$

$$\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \nabla f_i - \frac{e}{m_e} \langle \mathbf{E} \rangle \cdot \frac{\partial f_i}{\partial \mathbf{v}} - \frac{e}{m_e} \times \\ \times \langle (\mathbf{E}_h + \mathbf{v} \times \mathbf{B}_h) \cdot \frac{\partial f_h}{\partial \mathbf{v}} \rangle = \frac{\alpha}{v^3} \mathcal{C}(f_i). \quad (4)$$

Equation (3) is solved by simultaneous expansion up to second order in the two smallness parameters $\epsilon_i = (\mathbf{v} \cdot \nabla)/\omega$, $\epsilon_c = \alpha/v^3 \omega$, where ω is the electromagnetic wave frequency. We easily obtain :

$$f_h = \left\{ (\mathbf{u}_h \cdot \mathbf{v}) - (\mathbf{v} \cdot \nabla) (\mathbf{s}_h \cdot \mathbf{v}) - \frac{2\alpha}{v^3} (\mathbf{s}_h \cdot \mathbf{v}) + (\mathbf{v} \cdot \nabla) \times \right. \\ \times (\mathbf{v} \cdot \nabla) (\boldsymbol{\sigma}_h \cdot \mathbf{v}) + \frac{2\alpha}{v^3} (\mathbf{v} \cdot \nabla) (\boldsymbol{\sigma}_h \cdot \mathbf{v}) + \frac{\alpha}{v^3} \\ \left. \times \left[6(\mathbf{v} \cdot \nabla) (\boldsymbol{\sigma}_h \cdot \mathbf{v}) - 2v^2 \nabla \boldsymbol{\sigma}_h + \frac{4\alpha}{v^3} (\boldsymbol{\sigma}_h \cdot \mathbf{v}) \right] \right\} f_0, \quad (5)$$

where $\boldsymbol{\sigma}_h = \int \mathbf{s}_h dt = -\mathbf{u}_h/\omega^2$. To simplify the writing of this and the following equations, we have chosen units in which $T_e/m_e = 1$, where T_e is the electron temperature.

Substitution of (5) in the cross term

$$\langle (\mathbf{E}_h + \mathbf{v} \times \mathbf{B}_h) \cdot \frac{\partial f_h}{\partial \mathbf{v}} \rangle$$

gives to the second order in the smallness parameters ϵ_i , ϵ_c only terms in ϵ_i^2 and $\epsilon_i \epsilon_c$. The terms in ϵ_c^2 vanish after averaging. By suitable transformations, the final equation for the low-frequency distribution function reads

$$\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \nabla (f_i - h_i) = \frac{\alpha}{v^3} \mathcal{C}(f_i - g_i) + \frac{2\alpha}{3v} \langle u_h^2 \rangle f_0, \quad (6)$$

where

$$h_i = \left[\frac{e}{m_e} \Phi - \langle u_h^2 \rangle + \frac{\langle (\mathbf{u}_h \cdot \mathbf{v})^2 \rangle}{2} \right] f_0 + \delta h_i, \quad (7)$$

$$g_i = \left[\left(\frac{1}{3} + \frac{1}{v^2} \right) \langle (\mathbf{u}_h \cdot \mathbf{v})^2 \rangle \right] f_0 + \delta g_i, \quad (8)$$

where Φ is the average electric potential

$$\langle \mathbf{E} \rangle = -\nabla \Phi,$$

which has to be included to satisfy charge neutrality. We suppose implicitly that the low-frequency magnetic field $\langle \mathbf{B} \rangle$ is stationary.

δh_i and δg_i are of higher order in the smallness parameters ϵ_i , ϵ_c , but play an important role in the understanding of the nonlinear interaction between the radiation and the plasma [6].

$$\delta h_i = \langle \mathbf{u}_h (\mathbf{v} \cdot \nabla) \mathbf{s}_h - (\mathbf{u}_h \cdot \mathbf{v}) (\mathbf{v} \cdot \nabla) (\mathbf{s}_h \cdot \mathbf{v}) + 2(\mathbf{u}_h \cdot \nabla) (\mathbf{s}_h \cdot \mathbf{v}) \rangle f_0, \quad (9)$$

$$\delta g_i = \left\langle \left(3 - \frac{2v^2}{3} \right) (\mathbf{u}_h \cdot \nabla) (\mathbf{v} \cdot \mathbf{s}_h) + \left(1 - \frac{2v^2}{3} \right) \mathbf{u}_h (\mathbf{v} \cdot \nabla) \mathbf{s}_h + \left(\frac{v^2}{3} - 1 \right) (\mathbf{u}_h \cdot \mathbf{v}) (\nabla \mathbf{s}_h) \right. \\ \left. - \left(\frac{2}{3} + \frac{2}{v^2} \right) (\mathbf{u}_h \cdot \mathbf{v}) (\mathbf{v} \cdot \nabla) (\mathbf{v} \cdot \mathbf{s}_h) \right\rangle f_0. \quad (10)$$

The successive velocity moments of eq. (6) establish the link with a fluid description [6]. In particular one can derive the evolution equation of the dc magnetic field from the momentum conservation equation. One obtains :

$$\begin{aligned} \frac{\partial \langle \mathbf{B} \rangle}{\partial t} + \nabla \times \frac{c^2}{\omega_{pe}^2} \nabla \times \frac{\partial \langle \mathbf{B} \rangle}{\partial t} - \frac{2 \alpha m_e}{e} \nabla \times \frac{1}{\langle n_e \rangle} \int \frac{\mathbf{v}}{v^3} f_1 d^3v = \\ = \nabla \times \frac{1}{e \langle n_e \rangle} \nabla \langle \vec{\pi} \rangle - \frac{2 \alpha m_e}{e} \nabla \times \frac{1}{\langle n_e \rangle} \int \frac{\mathbf{v}}{v^3} \delta g_1 d^3v \end{aligned} \quad (11)$$

where $\vec{\pi}$ is the particle pressure tensor.

The evolution equation (11) for the nonlinearly generated magnetic field differs significantly from the usual one (see for example ref. [7]). In particular, the resistive diffusion of the magnetic field does not take the familiar form $\mu_0^{-1} \nabla \times \vec{r} \cdot \nabla \times \langle \mathbf{B} \rangle$, where \vec{r} is the resistivity tensor. The right-hand side of (11) represents the source terms of the dc magnetic field. The first source term is the usual thermoelectric source, and the second one is the explicit nonlinear source term.

When $\varepsilon_i \gg \varepsilon_c$, we are in the weakly collisional case. The stationary solution of (6) is

$$\begin{aligned} f_1 = f_0 + h_1 + \frac{\alpha}{v^3} \times \\ \times \int \frac{dx}{v_x} \left[C(f_1 - g_1) + \frac{2v^2}{3} \langle u_n^2 \rangle f_0 \right]. \end{aligned} \quad (12a)$$

Here the density gradient is in the x direction. We also assume that a plane wave is obliquely incident on the inhomogeneous plasma, with its polarization in the x - y plane.

To the lowest order, we find that the main result is the modification of the distribution function, which accounts for the density depletion,

$$f_1 \simeq f_0 \left(1 + \frac{e}{m_e} \Phi - \langle u_n^2 \rangle + \left\langle \frac{(\mathbf{u}_h \cdot \mathbf{v})^2}{2} \right\rangle \right). \quad (13)$$

This result may be compared with the results of recent papers [8] and with the results of ref. [3]. To the next order, we have two contributions to the low-frequency electric current and magnetic field.

$$\begin{aligned} \langle B_z \rangle = \mu_0 e \langle n_e \rangle \langle u_{hx} s_{hy} \rangle + \\ + \frac{1}{3} \sqrt{\frac{2}{\pi}} \mu_0 e \langle n_e \rangle A' \alpha \int dx' \int^{x'} dx'' \langle u_{hx} u_{hy} \rangle \end{aligned} \quad (14)$$

with $A' = \int_{v_c}^{\infty} (6/v - v) \exp(-v^2/2) dv$, where v_c corresponds to a physical cut-off : $v_c \nabla$ of the same order of magnitude as α/v_c^3 . If we apply these results to the resonant absorption [9], we have to distinguish two cases. When $\varepsilon_i \gg \varepsilon_i^2 \gg \varepsilon_c$, we are in the purely colli-

sionless case, and the first term in (13) dominates. We recover the results of refs. [1-3], and $\omega_b/\omega \propto \omega/v_{eq}$, where $\omega_b = eB/m_e$, v_{eq} is the equivalent collision frequency in the resonance problem $v_{eq} = \omega(\lambda_D/L)^{2/3}$ where $\lambda_D^2 = 3T_e/m_e \omega^2$. This case corresponds to $v_{eq} \gg v$, where $v = 2\sqrt{2}/(3\sqrt{\pi}) \alpha(m_e/T_e)^{3/2}$ is the usual collision frequency. When $\varepsilon_i \gg \varepsilon_c \gg \varepsilon_i^2$, we are in the intermediate case, and the second term in (14) dominates, as we are already in the collisional regime for the linear resonant absorption phenomena. This case corresponds to $v_{eq} \ll v \ll \omega(v_{eq}/\omega)^{3/4}$. The scaling law for the dc magnetic field is

$$\omega_b/\omega \propto (v/\omega)^2 (\omega/v_{eq})^3.$$

One can point out that the magnetic field generated in those conditions is not localized, and penetrates inside the plasma, and therefore may affect the thermal transport.

When $\varepsilon_c \gg \varepsilon_i$, collisions dominate both phenomena. To solve (6), we encounter the usual difficulties of the Chapman-Enskog development [10]. The last term in eq. (6) gives rise to a secular variation of the distribution function, either in space or in time. However this secular variation does not affect the anisotropic part f_a of the distribution function which is responsible for magnetic field generation, and for off-diagonal terms of the electron pressure tensor,

$$\begin{aligned} f_a = g_1 + \frac{v^3}{12\alpha} \left(\frac{1}{6} - \frac{1}{v^2} \right) \nabla [\langle (\mathbf{u}_h \cdot \mathbf{v})^2 \rangle \mathbf{v} + \\ + v^2 \langle u_n^2 \rangle \mathbf{v} + 2v^2 \langle (\mathbf{u}_h \cdot \mathbf{v}) \mathbf{u}_h \rangle]. \end{aligned} \quad (15)$$

The off-diagonal terms of the electron pressure tensor reads (see also eq. (71) of ref. [4])

$$\pi_{ij} = \frac{1}{15} m_e \langle n_e \rangle \langle u_{hi} u_{hj} \rangle, \quad \text{for } i \neq j, \quad (16)$$

while in the purely collisionless case we find $\pi_{ij} = 0$ for $i \neq j$.

Coming back to eq. (11), one can see that these off-diagonal terms of the electron pressure tensor are here the dominant source term of the dc magnetic field,

$$\langle B_z \rangle = \frac{16}{15} \sqrt{\frac{2}{\pi}} \langle n_e \rangle \mu_0 e \frac{\langle u_{hx} u_{hy} \rangle}{\alpha}. \quad (17)$$

In the resonant absorption, the collision dominated limit corresponds to $\nu/\omega \gg (\nu_{eq}/\omega)^{3/4}$, and the dc magnetic field scales as $(\omega/\nu)^2$.

In summary, we have presented the main results of the dc magnetic field generation by nonlinear effects, in particular *via* resonant absorption. We have exhibited three physically different regimes, where the scaling laws for the dc magnetic field are not similar. In particular, the nonlinear phenomena depends explicitly on the exact physical mechanism of the linear resonant absorption. As an example, one cannot roughly estimate the dc magnetic field in the collisionless resonant

absorption with a phenomenological collision frequency. This conclusion would be the same in the more complex problem where electron-ion collisions are replaced by effective collisions such as ion acoustic turbulence.

In high intensity laser plasma experiments, and in the corresponding numerical simulations [1, 2, 5], one is in the purely collisionless limit, where our calculations agree with refs. [1] to [3]. In microwave plasma experiments, the collisional effects may become dominant, and the corresponding results given in this letter apply.

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