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COFOCAL CONICS IN SMECTICS C

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Résumé. — On a observé, au-dessous de la température de transition $A \rightarrow C$, des coniques focales formées dans une phase smectique A. Dans le cas d'un défaut isolé torique, il apparaît une nouvelle ligne joignant le centre à deux points opposés sur le cercle singulier. On montre par des considérations topologiques qu'il s'agit d'une ligne de disclinaison d'ordre $S = -1$ pour le τ -directeur.

Abstract. — Cofocal conics formed in a smectic A phase are observed below the $A \rightarrow C$ transition temperature. In the case of an isolated toric defect a new line appears joining the centre to two opposite points on the singular circle. It is shown from topological considerations, that this line is a $S = -1$ disclination line for the τ -director.

1. **Introduction.** — It has long been known that the most usual defects in smectics A are cofocal structures [1, 2]. The layers are Dupin cyclides and the defect is constituted by two cofocal conics. The simplest case is the toric one, where the cofocal conics reduce to a circle and its axis. This defect can occur as an isolated defect, in an otherwise perfectly monocrystalline domain. The focal domain is constituted by a cylinder of revolution built upon the singular

circle, and one easily sees that, far from the circle, the axial singularity of the layers decreases and becomes negligible (Fig. 1).

Focal conics are much less usual defects in smectics C. They can however be observed when cooling a smectic A, but their structure has to be more complex. The reason can easily be understood by symmetry considerations in the toric case; in the A-phase one has an axial symmetry ($D_{\infty h}$) for the layer structure, compatible with the smectic A basic symmetry (also $D_{\infty h}$).

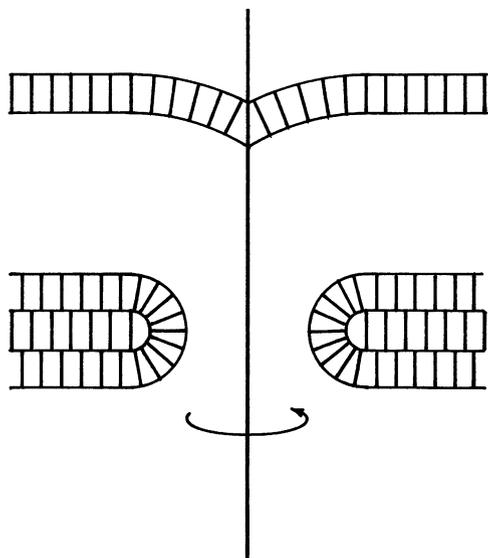


FIG. 1. — Schematic representation of a toric domain in a smectic A. Far from the centre the axial discontinuity vanishes.

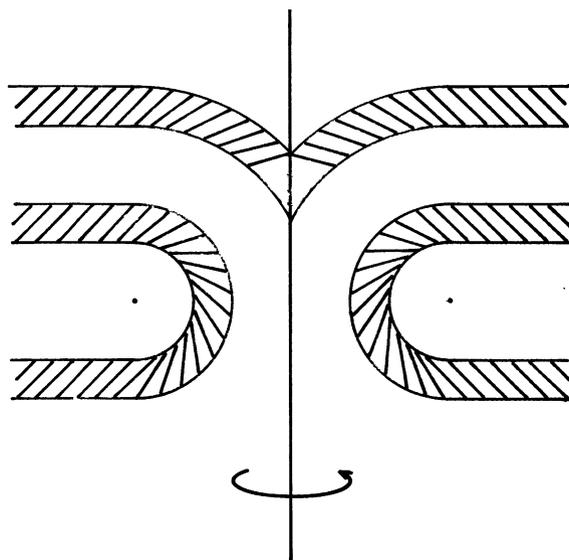


FIG. 2. — A toric domain in a smectic C under the assumption of an axial symmetry. Far from the centre, the layer discontinuity vanishes but the axis remains as a $S = +1$ disclination line.

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In smectics C, the basic symmetry is C_{2h} , no longer compatible with the toric one. The result is that, if one assumes an axial symmetry, the tilt plane rotates around the axis, and hence that, far from the circle, the axis appears as a disclination line of order $S = +1$ (Fig. 2). If such was the case, the toric defect would no longer be an isolated defect, and its axis, which in smectics A is visible only over a short length (of order of the circle diameter), would, in the C phase be visible over the whole domain width. The same argument can be applied to the general case of focal conics, where the hyperbolas are topologically equivalent to the toric axis.

Another possibility exists : far from the defect, a uniform orientation can be achieved. Then the axis is no longer an axis of revolution and the defect appears more complex than in smectics A. As shall be seen later, this can occur and cofocal structures in smectics C have configurations which are not obtained by a mere trivial extension of the A-case.

2. Experimental. — The experimental process is classical and very simple. The liquid crystal is placed

between two non-treated parallel glass plates. The sample width is of order 100μ . The sample is placed in an oven, ensuring a stabilized temperature, and examined through a polarizing microscope.

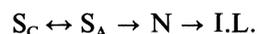
Several smectic materials were used. They can be divided into two classes :

a) The material has no A phase : it exhibits the following transitions



In this class, no focal structures could be observed.

b) The C-phase could be obtained by cooling an A-phase : the phase succession was either

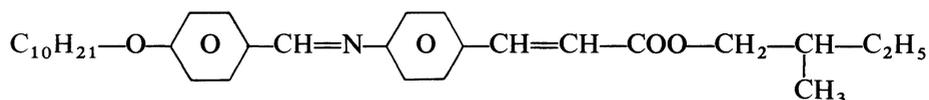


or

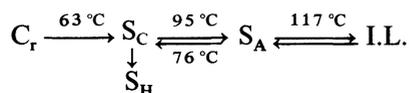


In both cases focal conics were obtained in the A-phase and some of them remained in the C-phase.

The best results were obtained with a racemic mixture of



which exhibits the following phase transitions :



3. Results. — Three geometries could be observed in the A-phase :

a) Horizontal elongated ellipses, with vertical hyperbolas (Fig. 3a).

b) Horizontal hyperbolas, with vertical ellipses (Fig. 4a).

c) Oblique toric domains (Fig. 5a).



a)



b)

FIG. 3. — A group of ellipses. a) In a smectic A-phase. b) In the smectic C-phase. Some of the ellipses are vanishing. A new line crossing the hyperbola is clearly visible on one ellipse.

In all cases, we have observed focal conics remaining in the C-phase. These three cases lead to different observations.

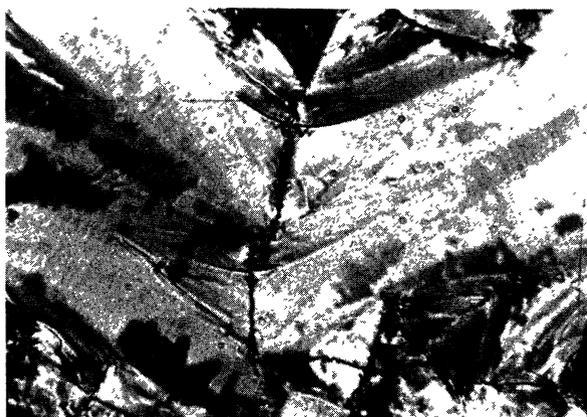
a) Ellipses are much less stable in smectics C. Isolated ones tend to disappear. On the other hand ellipses belonging to a polygonal domain generally persist in the C-phase. The director field, inside the ellipse, exhibits a strong variation. The results is



a)



a)



b)



b)

FIG. 4. — A group of hyperbolas. a) In a smectic A. b) In the C-phase. The general texture appears as somehow broken. Helices corresponding to twist walls are visible. Hyperbolas are clearly shorter than in the A-phase.

FIG. 5. — A quasi toric domain. The hyperbola is nearly a straight line, and the ellipse eccentricity is very small. a) In a smectic A. b) In a smectic C. The visible part of the hyperbola is much shorter. A new line joins the centre of the hyperbola to two nearly opposite points on the ellipse.

somehow analogous to a conoscopic effect, and one observes a darkening inside the ellipse which tends to mask any detail in the C-phase (Fig. 3b). Note on one of the ellipses an unusual line crossing the hyperbola.

b) Ellipses are observed as lines, which forbids the observation of any detail in their planes. A shortening of the hyperbolas is again observed (Fig. 4b).

c) One observes a shortening of the visible part of the axis. A new line joins the axis to two nearly opposite points on the circle (Fig. 5b).

4. Discussion. — The general shortening of the visible parts on the toric axes obviously shows that they are not prolonged by disclination lines : the toric defect remains an isolated defect inside an otherwise regular smectic domain.

We shall essentially discuss this toric defect. The general case of cofocal structure is topologically equivalent.

Start from the free-energy density which can be written in a condensed form as

$$F = F_0 + \frac{B_{\parallel}}{2} \varepsilon^2 + \frac{B_{\perp}}{2} (\theta - \theta_{\varphi})^2 + \frac{1}{2} \nabla \mathbf{v} \cdot \mathbf{K}_1 \cdot \nabla \mathbf{v} + \frac{1}{2} \nabla \mathbf{v} \cdot \mathbf{K}_2 \cdot \nabla \varphi + \frac{1}{2} \nabla \varphi \cdot \mathbf{K}_3 \cdot \nabla \varphi \quad (1)$$

where ε is the relative dilation of the layers, θ the tilt angle, \mathbf{v} the normal to the layer and φ the azimuthal angle of the tilt plane. Note that if φ cannot be defined in a non-planar structure, $\nabla \varphi$ can be defined as $\Delta \tau_y$ in a local frame of reference $(X(\tau), Y, Z(\mathbf{v}))$ where τ is the unit-vector along the projection of \mathbf{n} on the layer (τ -director) — \mathbf{K}_1 , \mathbf{K}_2 and \mathbf{K}_3 are respectively fourth, third and second rank tensors.

It must now be realized that these elastic terms are of different orders. The first two terms are first order, and the B 's are of order 10^7 cgs (except very close to the transition temperature). On the other hand,

the K 's are curvature terms, of order 10^{-7} cgs, like Frank elastic constants in nematics.

At the A-C transition, B_{\parallel} has a non-vanishing value. It follows that any new distortion of the layer will need a large activation energy. This is the reason why cofocal structures formed in an A-phase tend to persist in the C-phase, in spite of the fact that these structures do not spontaneously form, as can be seen in the case of the N-C transition.

The second important point is that, except in cores where the molecular orientation exhibits a strong variation over a very short distance, the tilt angle variations will be very small. All the defect structure must, except in a core of radius $\xi_{\perp} \simeq B_{\perp}/K$, therefore be described as a continuous variation of the azimuthal angle φ at $\theta = \theta_0$, in the geometry of toric layers.

Make now a map of a layer, with the following conventions : \mathbf{v} is oriented from the singular circle towards the axis and $\mathbf{n} \cdot \mathbf{v} > 0$. This completely defines τ .

The following step is to project the half-torus on a cylinder. It is best to use for each point a local stereographic projection in its half meridian plane (Figs. 6, 7), which keeps the azimuthal angle φ . The cylinder is then open along one of its generatrices.

The result is shown on figure 7c. τ is directed downwards along AA' and upwards along BB' . It exhibits a 2π rotation along BAB and a -2π one along $B'A'B'$. From symmetry considerations, it must be vertical along the equatorial line and horizontal along CC' and DD' . It must therefore have two singularities of order $S = -1$ at points C'' and D'' .

These singular points on each layer lead to a $S = -1$ disclination line along the diameter of the singular circle normal to the external tilt plane. Note that this diameter is a twofold axis of rotation for the whole configuration. If the torus is slightly deformed, the two anchoring points of the ellipse shift toward the real focus of the hyperbola, as can be seen on figure 5c, and will be discussed later.

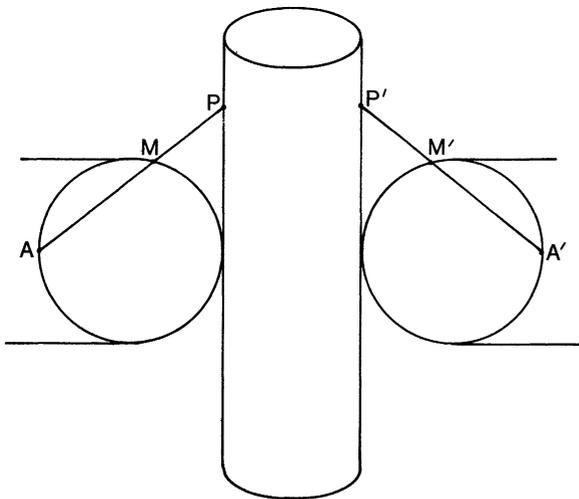


FIG. 6. — Local stereographic projection of the torus on a cylinder. The τ -director angle with the meridian is conserved.

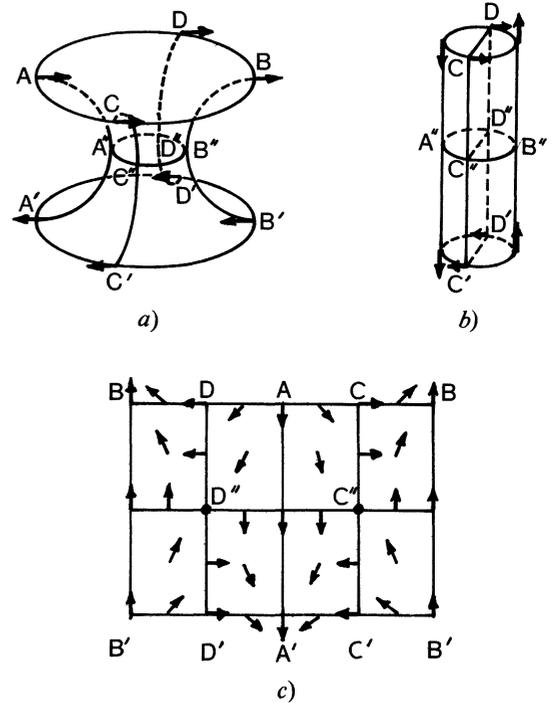


FIG. 7. — Mapping of the τ -director from the torus (a), to the cylinder (b) and then to a plane (c). The general configuration of the τ -director is shown on figure 7c. Two singular points ($S = -1$) appear in C'' and D'' .

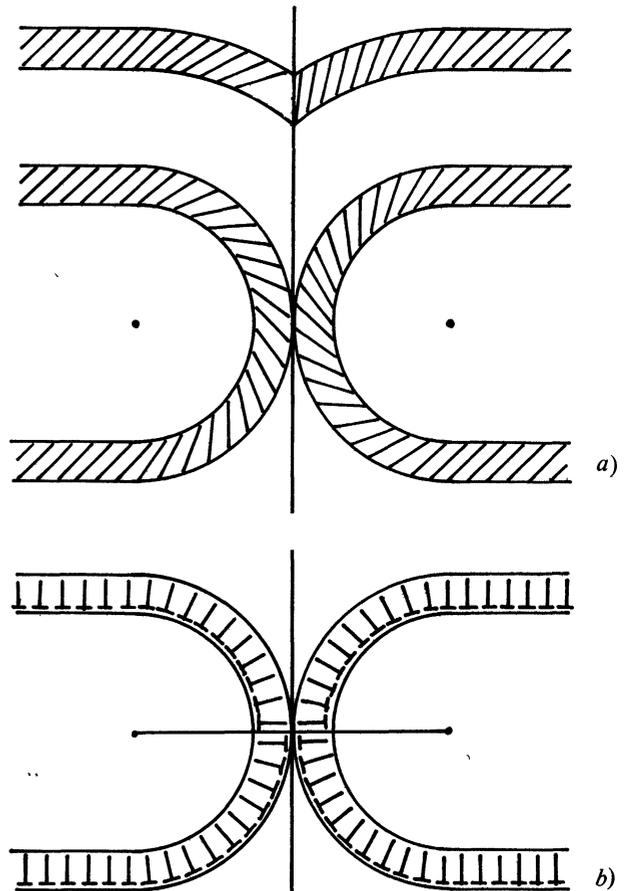


FIG. 8. — The director configuration along two meridian planes : a) $AA' B' B$ and b) $CC' DD'$. The nails point at 45° towards the observer.

The shortening of the toric axis can also be understood. At a given point on the toric axis, one has, in a A-phase, a conical singularity with an apex angle ψ_0 . In the meridian plane AB, in the C-phase, the angular discontinuity is still $2\psi_0$; but, along the meridian plane CD, the directors \mathbf{n} are tilted towards the normal to this meridian plane and the angular discontinuity is reduced to $2\psi'$, with

$$\sin \psi' = \sin \psi \cos \theta_0 .$$

It follows that the average value of the angular singularity is reduced and that the visible part of the axis is shortened.

The general case of focal conics can be easily deduced from the toric one. The configuration must be topologically equivalent. The disclination line is no longer a diameter : a balance must occur between the distortion elastic energy and the core energy which probably tends to shift the singular points on the ellipse towards the real focus of the hyperbola.

5. Conclusion. — In the half-torus situation, the singular circle is a ($S = \frac{1}{2}$) rotation dislocation for the layers. As shown by Kléman [3], in a smectic C, such a rotation dislocation is associated with a Néel wall for the τ -director, except in the symmetry plane $ABB'A'$. Néel walls with opposite bendings join along a Bloch line which is in fact a disclination line ($S = -1$) for the τ -director.

Acknowledgments. — These results were first presented at the 1978 *Annual meeting of the Association Française de Cristallographie*. Some days later, one of us (O. P.) met Pr. Kléman, and learned from him that similar results had been independently obtained by J. Sommeria and himself at Orsay. It is a pleasure to thank Pr. Kléman for this information and for the useful discussion that followed.

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