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**VERTICAL-SLICE OCEAN-ACOUSTIC TOMOGRAPHY - EXTENDING ABEL INVERSION TO
NON-AXIAL SOURCES AND RECEIVERS**

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Abstract - We discuss inversion of vertical-slice ocean-acoustic tomography (travel-time) measurements, in which we use the adiabatic-invariant approximation to convert multi-loop measurements to single-loop ray properties before using an Abel transform. We demonstrate the inversion by applying it to a simulated pulse-arrival sequence for a uniform sound channel generated by a ray tracing program, and compare the recovered sound-speed profile with that used for the simulation. For a uniform sound channel, the inversion gives both the symmetric and antisymmetric parts of the sound channel, including the vertical displacement of the sound-channel axis.

1 - INTRODUCTION

The need to measure heat transport in the ocean and supplement *in situ* measurements of temperature distribution has long been known /1,2/. Because the sound speed in water depends on temperature, measuring the travel time of acoustic pulses over known distances in the ocean can provide reasonable estimates of ocean temperature distribution over large regions. This method, known as ocean acoustic tomography /3/ has been shown to work, at least in the deep ocean, for measuring both temperature distribution /4,5/ and current flow by reciprocal transmission /6/. Although the main information about horizontal temperature variations comes from comparing travel-time measurements among several source-receiver pairs (called horizontal-slice tomography), it has been found that some range dependence of the ocean sound channel can be detected from a single source-receiver pair (vertical-slice tomography) /7,8/.

Of course, tomography inversion requires that the measured pulse arrivals can be correctly identified as to their number of ray loops and whether they were transmitted up or down.

Although comparisons with *in situ* measurements have shown that ocean acoustic tomography is successful, several questions remain that have not been answered until now.

1. What part of the sound channel is determined by tomography, and what part by *a priori* information?
2. How much information is available from tomography travel-time measurements for each source-receiver pair, i.e., vertical-slice tomography?
3. In what sense does tomography measure a range-averaged sound channel?
4. How much information about sound-channel range dependence is available from vertical-slice tomography?

2 - APPROACH

Examining the Abel inversion provides insights about what tomography actually measures, first, because vertical- and horizontal-slice tomography are performed separately, and second, because it explicitly computes single-loop ray properties before transforming them to sound-speed profiles.

For a uniform sound channel, this procedure leads to a useful inversion method, both for the symmetric part of the sound channel /9/ and for the antisymmetric part /10,11/. In that procedure, one determines from the measured pulse travel times the dependence of single-loop range (range between crossings of the sound-channel axis) on the sound slowness at the ray turning point, \hat{S} , for both upper and lower ray loops. For a uniform sound channel, \hat{S} is the same for all loops of a raypath, whereas in a range-dependent sound channel, \hat{S} varies from one loop to the next. The extension of this procedure to elevated source and receiver and to a range-dependent sound channel offers insights into the tomography-inversion process that apply to any method for extracting information from tomography travel-time measurements /12/. The remainder of this section deals with vertical-slice tomography.

Adiabatic-invariant approximation

The simplest approximation that can be made about a range-dependent sound channel is the adiabatic-

ic-invariant approximation /12,13/, which assumes that the sound channel changes by only a small amount in the horizontal distance of one ray loop. In that case, \hat{S} varies only slightly from one ray loop to the next in the sound channel.

The adiabatic-invariant approximation allows us to separate the pulse travel time of a sound-channel raypath into two contributions, one from the up-and-down motion of the raypath in the sound channel, and the other from the horizontal motion. The first is the ray-action contribution, the sum of contributions for each loop or partial loop in the sound-channel raypath. The second equals the product of the source-receiver range and the range average of \hat{S} .

Ray-action contribution

The action integral is essentially the integral of the vertical component of the sound slowness along the ray. Although it is cumulative along the raypath, it is useful to consider the contribution from each ray loop or part of a ray loop, and add up the contributions. In the adiabatic-invariant approximation, however, the action integral J for a double loop (from one upper turning point to the next, for example) is the same for each double loop of a raypath from the source to the receiver. (Each raypath has a different value of J , however.) Because of this, the total ray action contribution to the pulse travel time is just J times the number of double loops plus the contributions of the parts of loops at the source and receiver on the ends of the raypath.

The values of the double-loop action J and the action integrals for the partial loops at the source and receiver for the raypaths for which pulse travel time has been measured can be estimated by subtracting the measured travel times for appropriate pairs of raypaths, if careful interpolation methods are used.

Range average of \hat{S} (sound slowness at ray turning points)

As we would expect, the measured pulse travel times for propagation in the sound channel depend on the range average of \hat{S} , and therefore inversion determines at most this range average, but nothing more detailed about the range dependence of \hat{S} .

Once we have determined all of the ray action contributions for each raypath for which we have measured the pulse travel time, we can subtract the ray action contribution from the measured travel time to determine the range average of \hat{S} . However, overestimating the source-receiver range will proportionally underestimate the range average of \hat{S} .

Elevated source and receiver

Because the source and receiver are rarely exactly on the sound-channel axis, we would expect there to be parts of ray loops on the ends of each raypath whose contributions to the pulse travel time must be added to that of the complete double loops. For a range-dependent sound channel, however, even in the adiabatic-invariant approximation, the action for an upper loop (from one crossing of the sound-channel axis to the next) is not the same at the two ends of the raypath, even though the action for a complete double loop is the same from one end of the raypath to the other. Because of that, the contributions of the actions from the parts of loops at the ends of the raypath must be considered whether or not the source and receiver are on the sound-channel axis. For the range-dependent sound channel, therefore, elevating the source and receiver does not contribute an additional problem.

Symmetric part of sound channel

The symmetric part of the sound channel is determined by the Abel transform of the double-loop range (the range from one upper turning point of the ray to the next) /9/. The double-loop range is, in turn, equal to the negative of the derivative of J with respect to \hat{S} . Thus, the dependence of J on \hat{S} determines the symmetric part of the sound channel.

Because J is constant along the raypath in the adiabatic-invariant approximation, the value determined from measured travel times applies to the whole raypath for which it was determined. Because \hat{S} as determined from the measured travel times is a range-averaged value, the resulting dependence of J on \hat{S} determines a range-averaged profile for the symmetric part of the sound channel when the Abel transform is applied to it.

Antisymmetric part of sound channel

The antisymmetric part of the sound channel is determined by the Abel transform of the difference between upper-loop range (between crossings of the sound-channel axis) and lower-loop range. These single-loop ranges are, in turn, equal to the negative of the derivative of the corresponding action integrals with respect to \hat{S} . Because the adiabatic-invariant approximation does not require single-loop action to be constant along the raypath, the single-loop action integrals determined by the pulse travel times do not apply to the whole raypath, but only to the end near the source or receiver. (In general, the single-loop action at the source will be different from that at the receiver.)

Thus, it is not possible to determine a range-averaged antisymmetric part of the sound channel from pulse travel-time measurements. At most, it would be possible to determine the antisymmetric part of the

sound channel at the source and receiver, but only if the local value of \hat{S} at the source and receiver for each raypath could be determined. However, there is no way to determine that from pulse travel-time measurements for a single source-receiver pair without *a priori* information.

The single ray loops used to determine the antisymmetric part of the sound channel do not have to be between crossings of the sound-channel axis. They can be between crossings of any convenient height, such as the source or receiver depth. In fact, it is useful to choose those depths, first, because the corresponding action integrals are more easily determined by the travel-time measurements, and second because the resulting profiles will be relative to the source or receiver depth (which are presumed known) instead of relative to the sound-channel axis (whose depth is not known beforehand).

Abel transform

Once the dependence of the double-loop action J on \hat{S} has been determined, an Abel transform gives the range-averaged symmetric part of the sound channel. However, overestimating the source-receiver range will proportionally underestimate the range-averaged \hat{S} and overestimate the thickness and sound speed of the sound channel. Using *a priori* information (that the lower part of the sound channel does not change much) allows us to detect an erroneous value for source-receiver range /8,14/.

An Abel transform can also be applied to the dependence of the difference of upper- and lower-loop actions on \hat{S} , but the resulting antisymmetric part of the sound channel would have physical significance only for a uniform sound channel. (We show how this works for a uniform sound channel in section 4.)

It is well known, however, that the Abel transform yields a unique profile only when the sound-speed profile above (and below) the sound-channel axis is monotonic. This is not a failing of the Abel transform, but is inherent in the measurements. Without using *a priori* information, the measurements of pulse travel time in the sound channel do not determine unique profiles if the upper and lower parts of the profile are not monotonic, no matter what method is used to invert the measurements.

3 - INVERSION FROM SIMULATED MEASUREMENTS

We applied the inversion method to the simulated pulse-arrival sequence in figure 1. Because this example is for a uniform sound channel, both the symmetric and antisymmetric parts of the profile are independent of range. The resulting profile in figure 2 agrees well with the profile used for the simulation.

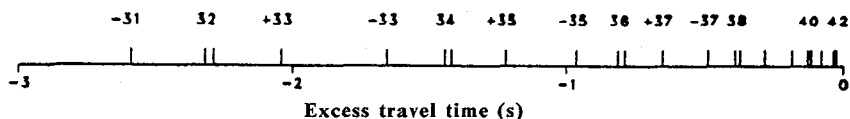


Figure 1. Pulse-arrival sequence simulated by ray tracing for a horizontally uniform sound channel using the same canonical sound speed profile as used in /12,15/. The travel times are relative to that for an axial ray. The depths of the source, receiver, and sound-channel axis are 1200 m, 1100 m, and 1300 m, respectively.

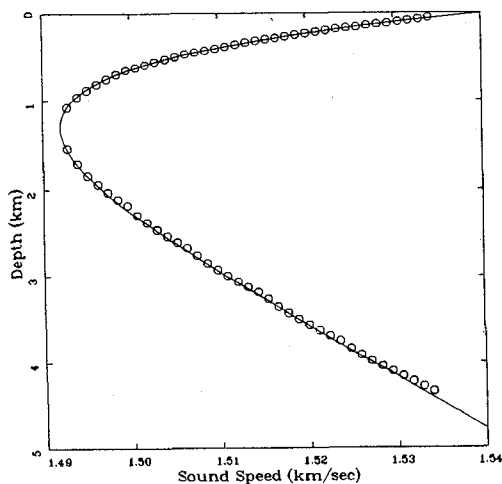


Figure 2. Comparison of the tomographic inversion (circles) with the sound-speed profile used for the simulation of the pulse-arrival sequence.

4 - SUMMARY OF VERTICAL-SLICE TOMOGRAPHY RESULTS

1. For each source-receiver pair, the measured pulse travel times determine a range average of the symmetric part of the sound-speed profile in which \hat{S} (the sound slowness at the ray turning point) is averaged in range.
2. For each source-receiver pair, the measured pulse travel times determine the action (phase integral) for the partial loops at the source and receiver at the ends of the raypath. For a uniform sound channel, this allows calculation of the antisymmetric part of the sound channel. This explains why it is possible to get some range information from vertical-slice tomography, and agrees at least qualitatively with previous results /7,8/
3. Overestimating the source-receiver range will proportionally overestimate the thickness and sound speed of the sound channel.

Our results are valid whenever the adiabatic-invariant approximation is valid, but that approximation may sometimes neglect the effects of scale sizes on the order of the size of a ray loop (about 50 km). It is apparently possible to detect such small eddies in tomography measurements, however /16/.

5 - IMPLICATIONS FOR HORIZONTAL-SLICE TOMOGRAPHY

It is possible to do horizontal-slice tomography with Abel inversion if we use the adiabatic-invariant approximation, in which the action J for a double loop of a ray is constant along the ray.

For a fixed J , we thus have a measure of the integrated value of \hat{S} for each source-receiver pair. This can be inverted by the usual methods of horizontal-slice tomography to yield \hat{S} as a function of horizontal position (x,y) for that value of J . Because that can be done for each value of J , from $\hat{S}(J, \text{vertical slice})$ we get $\hat{S}(J,x,y)$. Thus, for each x,y we get $\hat{S}(J)$, allowing us to calculate the symmetric part of the sound channel for any horizontal location (x,y) .

Thus, tomography determines the symmetric part of the sound channel everywhere in the interior (within a certain horizontal resolution) of the tomographic region, but determines the antisymmetric part of the sound channel only on the periphery. at the moorings?

A physical picture of this is as follows. Imagine contours of constant sound speed throughout the tomographic region. These contours are fixed at the periphery of the region by the tomography measurements. Also, the separation of the same contour above and below the sound-channel axis is fixed by the measurements (the symmetric part of the sound channel). However in the interior, the sound channel can move up and down (within limits) like a drum head without affecting the tomographic measurements.

A priori information is required to reduce this ambiguity; that is, we can use our knowledge that most of the variation (both in time and space) in the temperature (and therefore sound speed) in the ocean occurs in the upper part of the sound channel to fix the vertical motion of this "drum head." (In the objective tomography-inversion method, this is accomplished by expressing the solution in terms of ocean dynamic modes.)

6. REFERENCES

- /1/ Munk, W. and Wunsch, C., Phil. Trans. R. Soc. Lond. A307 (1982) 439-464.
- /2/ Munk, W. and Forbes, A. M. G., Global ocean warming: an acoustic measure?, J. Phys. Oceanography, accepted for publication (1989)
- /3/ Munk, W. and Wunsch, C., Deep-Sea Res. 26 (1979) 123-161.
- /4/ Ocean tomography group, A demonstration of ocean acoustic tomography, Nature 299 (1982) 121-125.
- /5/ Cornuelle, B., Wunsch, C., Behringer, D., Birdsall, T., Brown, M., Heinmiller, R., Knox, R., Metzger, K., Munk, W., Spiesberger, J., Spindel, R., Webb, D., and Worcester, P., Tomographic maps of the ocean mesoscale. Part 1: Pure acoustics, J. Phys. Oceanography 15 (1985) 133-152.
- /6/ Worcester, P., Spindel, R., and Howe, B., IEEE J. Ocean Eng. OE-10 (1985) 123-137.
- /7/ Howe, B., Worcester, P., and Spindel, R., J. Geophys. Res. 92 (1987) 3785-3805.
- /8/ Howe, Bruce M., J. Geophys. Res. 92 (1987) 9479-9486.
- /9/ Munk, W. and Wunsch, C., Rev. Geophys. and Space Phys. 21 (1983) 777-793
- /10/ Jones, R. M., Georges, T. M., and Riley, J. P., Deep-Sea Res. 33 (1986) 601-619.
- /11/ Munk, W. and Wunsch, C., Deep-Sea Research 29 (1982) 1415-1536.
- /12/ Munk, W. and Wunsch, C., Geophys. Astrophys. Fluid Dynamics 39 (1987) 1-24.
- /13/ Wunsch, C., Rev. Geophys. 25 (1987) 41-53.
- /14/ Cornuelle, Bruce D., J. Geophys. Res. 90 (1985) 9079-9088.
- /15/ Munk, W., J. Acoust. Soc. Am. 55 (1974) 220-226.
- /16/ Cornuelle, B. and Howe, B., J. Geophys. Res. 92 (1987) 11680-11692.