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CHAOTIC RAY PATHS IN UNDERWATER ACOUSTICS

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ABSTRACT

We simulate and explore the properties of chaotic ray paths in a weakly range-dependent, deterministic ocean. Chaos results from nonlinearity of the differential equations that determine the ray paths, rather than from randomness in the model of the medium or from externally imposed noise. Extreme sensitivity to model details and initial ray conditions places a practical limit on the predictability of acoustic field properties, such as travel time. Chaotic ray paths are identified using the traditional Poincare section, power spectra and exponential sensitivity to initial conditions. We also discuss new diagnostic techniques that have some advantages over the traditional ones.

INTRODUCTION

The propagation of an acoustic wave in an ocean medium is usually described by a homogenous, partial-differential equation linear in the acoustic field. It is well known that the problem of solving such a wave equation in the eikonal or ray-theory approximation is mathematically equivalent to that of solving for the motion of a classical mechanical system. An approximate solution to the wave equation is constructed by integrating along ray paths which are obtained by solving equations analogous to Hamilton's equations of motion for the mechanical system. It is also known, as a result of work done in the last few years [1], that a classical mechanical system will have unstable or "chaotic" solutions unless the Hamiltonian is separable. Consequently, there will be chaotic ray paths in the acoustic propagation problem unless the wave equation is separable [2,3]. This instability exists even though the wave equation is deterministic. In this paper we present some aspects of our work to identify chaotic ray paths and to investigate their properties.

PROPAGATION IN A RANGE-DEPENDENT OCEAN

We consider first a situation where the non-separability of the wave equation is due to a range-dependent sound speed. The square of the index of refraction is modeled as the sum of two terms, one which depends only on depth and a second which represents a small perturbation that is periodic in range

$$(C_0/C(r,z))^2 = (C_0/C_M(z))^2 - 2u(r,z) \quad (1)$$

where  $C_M$  is the Munk canonical sound-speed profile [4] and

$$u(r,z) = A \exp[-(z/H)^2] \cos[2\pi(r/\lambda)] , \quad (2)$$

where  $C = 1.5$  km/s,  $H = 1$  km,  $\lambda = 10$  km, and  $A$  is a free parameter. The ray paths were obtained by solving the parabolic equation using the ray-theory approximation. This equation is thought to be a valid approximation to the Helmholtz equation for deep-ocean propagation.

In searching for chaotic rays in the sound channel a type of phase-space diagram called a Poincare section is often useful: it is a plot of ray depth vs. ray angle with the range dependence of the ray path suppressed. A Poincare section samples the ray path every cycle of the range-dependent perturbation  $u(r,z)$ , in our case, every 10 km, thus emphasizing resonances that occur because of phase locking between the double-loop range of the ray, about 50 km in the ocean sound channel, and the wavelength  $\lambda$  of the perturbation.

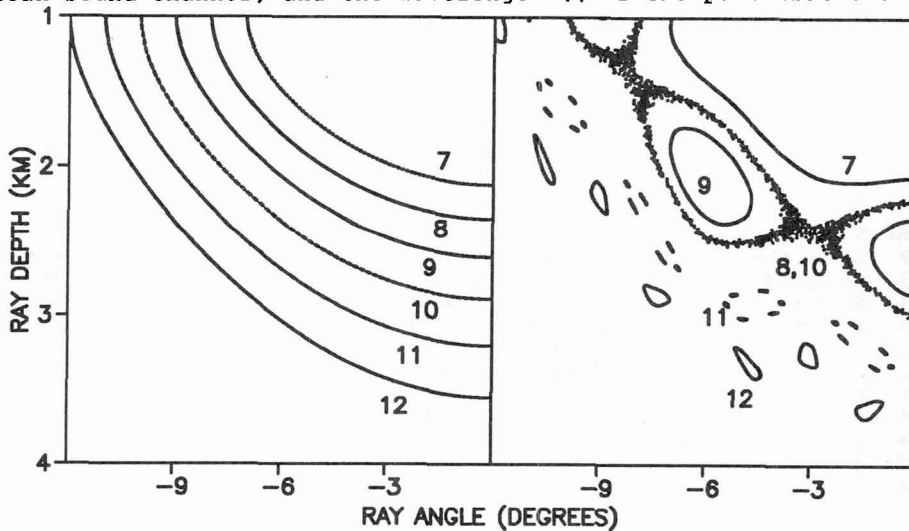


Figure 1

Figure 1 shows portions of two Poincaré sections for 6 rays having launch angles varying from  $7^\circ$  to  $12^\circ$  in  $1^\circ$  steps. For all the rays the source depth is 1 km. The two plots differ in the values taken for  $A$ , the amplitude of the perturbation. In the left-hand plot the amplitude is zero, corresponding to no range dependence in the sound speed. The sections are collections of dots whose loci are smooth, closed curves.

The right-hand plot corresponds to  $A = 0.01$ . The  $7^\circ$  curve is again continuous, though distorted. The  $9^\circ$ ,  $11^\circ$ , and  $12^\circ$  curves have a completely different topology. They are broken up into island structures which indicate that the rays are locked in resonance with the perturbation. Separating the two types of curves, the distorted, continuous curve and the island structures, is a boundary (not shown in the figure) called the separatrix. Ray paths having Poincaré sections near this boundary are chaotic. They have a characteristic pattern consisting of what appears to be a random distribution of dots in a bounded region of phase space. In this example, both the  $8^\circ$  and the  $10^\circ$  rays are chaotic. Their sections consist of intermingled dots occupying the same bounded region.

What is the significance of chaotic rays? Why are they unstable? A chaotic ray is unstable because it is not robust. Any small uncertainty in the initial conditions or the environment will create

a large change in the ray path. Consider, for example, two ray paths both having the same source depth but differing in launch angle by an infinitesimal amount. The difference in depths of the two rays, divided by the difference in launch angles, oscillates considerably with range. On the average, however, it grows linearly for a regular ray, whereas for a chaotic ray it grows exponentially, indicating physical instability. The coefficient in the exponent is the largest Lyapunov exponent, in the language of dynamic systems theory [1].

Sensitivity to the small-scale structure of the environment, i.e., to the parameters which appear in the wave equation, implies structural instability. Two ray tracings of a chaotic ray corresponding to values of a parameter which appears in the model for the sound speed differing by a fraction of a percent can show divergence after only a few hundred kilometers.

In closing this section we note chaotic ray paths have been found using other models for the range dependent part of the sound speed, including those for which the perturbation is not periodic, and using ray paths which satisfy the eikonal equation for the Helmholtz equation rather than that for the parabolic equation.

#### IDENTIFYING CHAOTIC RAYS

Three standard methods for identifying chaotic trajectories are the construction of Poincare sections, the construction of power spectra of the ray path, and the calculation of the largest Lyapunov exponent. Because they have some computational disadvantages for underwater acoustics, we have been developing more suitable methods [5].

Since a small change in the launch angle of a chaotic ray leads to a large change in its structure, a direct way of finding a chaotic ray is to plot some ray characteristic as a function of launch angle. The launch angles for which the curve rapidly changes define possible chaotic rays. Figure 2 shows two examples which illustrate this approach. Both plots in this figure correspond to the case  $A = 0.01$  for which rays in the vicinity of  $8^\circ$  and  $10^\circ$  are chaotic. In the lower plot, the depth of the ray at a range of 500 km has been plotted vs. launch angle. One clearly sees rapidly-varying structure associated with the two chaotic rays. The variation is so great that this structure cannot be resolved using an increment in launch angle as small as  $0.001^\circ$ .

In the upper plot, ranges to sound-channel-axis crossings have been plotted vs. launch angle. There are regions where the curves are flat indicating phase locking with the perturbation. Chaotic rays occur during the transition to phase-locking. We have plotted axis crossings which, for this example, correspond to source-depth crossings. One can construct similar plots by considering the ranges to surface or bottom reflections.

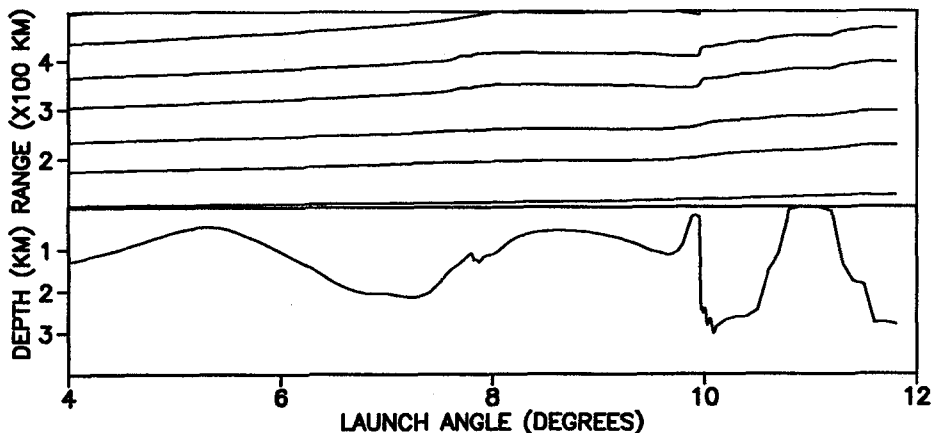


Figure 2

We have mentioned only two of a number of methods we have developed for identifying possible chaotic rays. With them we can search for chaotic rays in those situations where the standard methods are inappropriate or difficult to use.

In conclusion, we emphasize that our understanding of chaotic ray paths is still primitive and that many characteristics remain to be determined. The realization that ray paths can be unstable prompts a re-examination of acoustic remote-sensing techniques since full-wave solutions will also be sensitive to model details and initial conditions if ray paths are chaotic [6]. This re-examination has just begun. Our tools are numerical propagation simulations and establishing similarities to known canonical problems in nonlinear dynamics.

#### REFERENCES

1. A.J. Lichtenberg, and M.A. Lieberman, Regular and Stochastic Motion (Springer-Verlag, New York, 1982); J. Guckenheimer and P. Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields (Springer-Verlag, New York, 1983); I.C. Percival, Proc. R. Soc. Lond. A 413, 131 (1987).
2. S.S. Abdullaev, and G.M. Zaslavskii, Sov. Phys. JETP 53, 265 (1981); Sov. Phys. JETP 60, 435 (1984); Sov. Phys. Acoust. 34, 334 (1988).
3. D.R. Palmer et al., Geophys. Res. Letters 15, 569 (1988).
4. W.H.Munk, J. Acoust. Soc. Am. 55, 220 (1974).
5. D.R. Palmer, T.M. Georges, and R.M. Jones, J. Acoust. Soc. Am. 84, S91 (1988).
6. A. Peres, in Symposium on the Foundations of Modern Physics, Joensuu, Finland, August 1987, P. Lahti and P. Mittelstaedt, eds. (World Scientific, New Jersey, 1987)