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► **To cite this version:**

Z. Huang, P. Cutler, T. Feuchtwang, E. Kazes, H. Nguyen, et al.. MODEL STUDY OF TUNNELING TIMES (SUMMARY). Journal de Physique Colloques, 1989, 50 (C8), pp.C8-27-C8-29. 10.1051/jphyscol:1989805 . jpa-00229903

HAL Id: jpa-00229903

<https://hal.science/jpa-00229903>

Submitted on 4 Feb 2008

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MODEL STUDY OF TUNNELING TIMES (SUMMARY)

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In classical mechanics, the traversal time, or the time interval for a particle with energy E to travel from point 'a' to point 'b' in a one-dimensional potential $V(z)$, is given by

$$t = \int_a^b \frac{dz}{\sqrt{2[E-V(z)]/m}} \quad (1)$$

In quantum mechanics, the value of a dynamical variable is the expectation value of the corresponding operator. However, an exact quantum extension of Eq. (1) is not available because there does not exist a convenient Hamiltonian operator whose eigenvalue is the elapsed time[1]. The determination of tunneling time is therefore based on model calculations and computer simulations.

A tunneling time can be calculated by studying the dynamics of a wave packet. The essence of the method is to relate the phase change ϕ between the incident and transmitted waves to the duration of the interaction with the barrier[2,3]. When applied to a one-dimensional square barrier of height V_0 , and width d , the tunneling time is given by[4,5],

$$\tau_p = \frac{m}{\hbar k} \frac{\partial \phi}{\partial k} = \frac{m}{\hbar k \kappa} \frac{k_v^4 \sinh(2\kappa d) - 2k^2 \kappa d (k^2 - \kappa^2)}{4k^2 \kappa^2 \cosh^2(\kappa d) + (k^2 - \kappa^2)^2 \sinh^2(\kappa d)} \quad (2)$$

where $k = \sqrt{2mE}/\hbar$, $\kappa = \sqrt{2m(V_0 - E)}/\hbar$, $k_v = \sqrt{2mV_0}/\hbar$, and m and E are the mass and the energy of the electron, respectively. For an opaque barrier ($\kappa d \gg 1$), Eq. (2) becomes

$$\tau_p \approx \frac{2/\kappa}{v_0} \quad (3)$$

where $v_0 = \hbar k/m$. The expression implies that the tunneling time is approximately equal to the decay length divided by the incident velocity. Also the tunneling time is independent of the barrier width.

Another approach to calculate a tunneling time is to use the Larmor spin precession as an atomic clock[6-8]. When an electron is in a homogeneous magnetic field B , it precesses in the direction perpendicular to the field with a constant angular velocity, $\omega_L = eB/(mc)$. By calculating the spin precession angle of a polarized electron beam, one can extract a tunneling time. In a previous paper[9], we have shown that if the magnetic field is applied throughout the space, the spin-precession tunneling time is identical to the phase time τ_p .

We have done computer simulations of a wave packet tunneling through a one-dimensional square barrier[10] using the algorithm of Goldberg, Schey, and Schwartz[11]. The results show that for a finite width wave packet, the tunneling time τ_d is linearly proportional to the barrier width. It is believed that the difference between τ_p and τ_d is due to the choice of wave function to model the electron. The phase method and the spin precession method both use a single wave component. This can be regarded as a wave packet with an infinite spatial extension. By contrast, the simulation uses a wave packet confined to a finite space region. Therefore, a direct confirmation of the analytical expression by the simulation is not available. If, however, one increases the spatial extension of the wave packet, the asymptotic limit of the simulation should be the analytical expression[10].

A tunneling time can also be determined with a time-dependent potential barrier. The following potential form has been considered by Buttiker and Landauer[12],

$$V(z,t) = V_0(z) + V_1(z)\cos\omega t, \quad (4)$$

where $V_0(z)$ and $V_1(z)$ are constants for $0 < z < d$, and zero otherwise. For small V_1 , the tunneling current is approximated by summing the three dominant terms, corresponding to the transmission energy of E and $E \pm \hbar\omega$. They found that the transmission probability T for the elastic tunneling term is the same as the one for the constant potential barrier $V_0(z)$. For the inelastic terms, $E \pm \hbar\omega$, the transmission probabilities are

$$T_{\pm} = T \left(\frac{V_1}{2\hbar\omega} \right)^2 \left[\exp \left(\pm \omega \tau_{BL} - 1 \right) \right]^2, \quad (5)$$

where $\tau_{BL} = (m/\hbar\kappa)d$. As seen from Eq. (5), T_{\pm} depends on the frequency ω . The response of the tunneling barrier is characterized by τ_{BL} , which may be regarded as the tunneling time in this process. This characteristic time is proportional to the barrier width d .

An experimental method has been proposed using a time-dependent barrier to determine the tunneling time[13]. Some preliminary results have been obtained[14], suggesting that tunneling time for a barrier with height of eV and width, of order Å is about a femtosecond.

The analyses of model and computer calculations of tunneling times have lead to the following conclusions:

- 1) Tunneling time can be classified into essentially two categories, depending on whether the time is linearly proportional to 'd' or independent of 'd' in the regime where $\kappa d \gg 1$, the opaque barrier limit. It is important to note, however, that for a thin barrier, there are still distinct tunneling times, characteristic of the model, but which differ from the tunneling times in the extrapolated thick barrier region.
- 2) The phase method yields a tunneling time independent of the barrier thickness for an opaque barrier. Numerical simulation of a Gaussian wave packet tunneling through a barrier should in principle agree with the phase method. However, the former has a wave packet localized in space while the latter uses a plane wave to obtain an analytical expression. Our simulations show that a localized wave packet has a tunneling time linearly proportional to d . The preliminary computer simulations also suggest that tunneling time depends on the width of the packet.
- 3) Spin precession in a magnetic field can be used as a clock. This can be related to the phase method. The Larmor spin precession tunneling time has been shown to be identical to the phase tunneling time, when the wave function and the barrier are immersed in the magnetic field.
- 4) By studying the tunneling through a time-dependent barrier, one can also define a tunneling time operationally. The analysis of such a tunneling process shows that the tunneling time is linearly dependent on d .

We may conclude that for a particle tunneling through a potential barrier, there is more than one characteristic time. The tunneling time can be determined only when the process it is supposed to characterize is specified. That is, one cannot define a tunneling time without specifying the experiment used to measure it.

Acknowledgement: This research was supported in part by the Office of Naval Research, Arlington, Virginia, Grant No. N00014-89J-1251.

References:

1. A. Peres, Am. J. Phys. **48**, 552 (1980).
2. D. Bohm, Quantum Theory, Prentice-Hall, New York, 1951, p.257-261
3. E. W. Wigner, Phys. Rev. **98**, 145 (1955).
4. T. E. Hartman, J. Appl. Phys. **33**, 3427 (1962).
5. E. H. Hauge, J. P. Falck, and T. A. Fjeldly, Phys. Rev. **B36**, 4203 (1987).
6. A. I. Baz', Sov. J. Nucl. Phys. **4**, 182 (1967), 161 (1967)
7. V. F. Rybachenko, Sov. J. Nucl. Phys. **5**, 635 (1967).
8. M. Buttiker, Phys. Rev. **B27**, 6178 (1983).
9. Z. H. Huang, P. H. Cutler, T. E. Feuchtwang, R. H. Good, Jr., E. Kazes, H. Q. Nguyen, and S. K. Park, J. de Phys. **49**, C6-17 (1988).
10. Z. H. Huang, P. H. Cutler, T. E. Feuchtwang, R. H. Good, Jr., K. Kazes, and H. Q. Nguyen, to be published in IEEE Trans. Elec. Dev. Part II Vacuum Microelectronics (1989).

11. A. Goldberg, H. M. Schey and J. L. Schwartz, Am. J. Phys. 35, 177 (1967).
12. M. Buttiker and Landauer, Phys. Rev. Lett. 49, 1739(1982), Physica Scripta 32, 429(1985)
13. P. H. Cutler, T. E. Feuchtwang, T. T. Tsong, H. Nguyen, and A. A. Lucas, Phys. Rev. B35, 7774 (1987).
14. P. H. Cutler, T. E. Feuchtwang, Z. Huang, T. T. Tsong, Y. Kuk, H. Q. Nguyen, P. H. Silverman, A. A. Lucas, and T. E. Sullivan, J. de Phys. 48, C6-101 (1987).