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## THE HALL EFFECT IN KONDO SYSTEMS

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**Abstract.** – Initial analyses of the Hall effect in Kondo systems were based on the Boltzmann equation with skew scattering in the limit of weak magnetic fields. We obtain a more realistic assessment by using the Kubo formalism to include spin-flip and inelastic scattering processes, by considering the anomalous velocity mechanism and by accounting for nonlinearities.

The Hall effect provides details of the scattering mechanism and character of conduction electrons, that are inaccessible to other probes. This is particularly true for Kondo-ion systems, where the formation of a local moment-conduction electron bound state creates an anomalously large yet narrow peak in the density of states near the Fermi surface. This is reflected in the unusually large increase seen in the Hall constant at and below the Kondo temperature in mixed-valence and heavy fermion compounds [1]. The initial analyses of the data on the Hall effect in these Kondo-type compounds were based on independent *single-ion* scattering events and used the semi-classical Boltzmann transport equation [2, 3]. They assumed the anomalous or extraordinary Hall effect (EHE) is due to skew scattering and furthermore that the magnetic fields were sufficiently weak that the linear response of the system is adequate. The Hall constant predicted on the basis of this model has the *correct* sign and variation with temperature at high temperatures  $T \gg T_K$  ( $T_K$  is the Kondo temperature), i.e., in the limit of weak coupling. In contrast there is poor agreement with the experimental data at low temperatures,  $T < T_K$  (strong coupling). This discrepancy has generally been ascribed to the effects of coherence which are not taken into account in single-ion skew scattering models. However, some features observed at low temperatures can also be accounted for by further developments of the single-ion models. Without negating the necessity of ultimately taking into account interaction effects in dense Kondo systems we have included heretofore neglected features in the model based on independent scattering events in order to ascertain how well they reproduce the EHE observed. Here we outline these new features.

1) The original description of the EHE in Kondo systems was based on one electron scattering events; it does not include spin-flip and inelastic scattering processes [2, 3]. We used current-current correlation functions, i.e., the Kubo formalism, to calculate the EHE due to these two electron processes [4]. To obtain

these contributions for rare-earth impurities in metals we considered the spin and potential scattering in the nonresonant  $l = 2$  channel coming from the exchange and direct Coulomb interactions as well as the resonant scattering in the  $l = 3$  channel which comes from the Anderson mixing interaction. For elastic processes we reproduce the potential scattering contribution previously found [2, 3]; in addition we find the spin scattering in the  $l = 2$  channel produces a new contribution to the EHE which is proportional to the product of the resistivity and susceptibility.

The new contributions coming from *inelastic* skew scattering processes between the  $l = 2$  and  $l = 3$  channels are very interesting. They produce an EHE with the same sign as that due to elastic processes, but they go to zero much faster at high temperatures  $T \gg T_K$ , i.e., they are smaller than the elastic processes by a factor of  $(T_K / T)^2$ . However, at low temperatures, in particular at  $T = 0$  K, they are as large as the Hall effect due to elastic processes. In other words, the contribution to the EHE due to inelastic skew scattering has a temperature dependence which is very different from that of the elastic term.

2) Aside from the ordinary Hall effect which is due to the Lorentz force and the skew scattering mechanism which makes the scattering probability antisymmetric with respect to interchange of scattering vectors, there exists an additional contribution to the Hall effect known as the anomalous velocity contribution. This contribution is related to the so-called side-jump effect; it is due to a change in the expression for the current operator in the presence of spin orbit forces. It arises from the non-commutativity of the position operator of the conduction electron with terms in the Hamiltonian other than the kinetic energy operator. The anomalous velocity contribution to the EHE in Kondo systems exists for both the incoherent and coherent scattering regimes; we have focused our attention on the incoherent resonant scattering that is due to independent scattering events. Up till now we limited ourselves to one electron potential scattering,

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which perforce is elastic for an unsplit yet degenerate energy level. The hallmark of the anomalous velocity contribution to the EHE is that it goes as the square of the concentration of Kondo impurities [5], whereas the skew scattering contribution is linear in the concentration. Thus one expects this contribution to be important only for concentrated systems.

We find that at high temperatures ( $T \gg T_K$ ) the anomalous velocity contribution to the Hall constant is smaller than the skew scattering by a factor proportional to  $c\Gamma / |E_f|$  where  $c$  is the concentration of impurities,  $\Gamma$  the hybridization parameter used to characterize the width of the broadened  $4f$  local state, and  $E_f$  the position of this level relative to the Fermi surface [6]. For the Kondo regime of the Anderson model  $\Gamma / |E_f| \ll 1$  and the contribution from anomalous velocity is much smaller than skew scattering. On the contrary at low temperature, e.g., at  $T = 0$  K, we find the factor of proportionality is  $cE_F / k_B T_L$ , where  $E_F$  is the Fermi energy and  $k_B T_L$  is the inverse of reduced susceptibility at  $T = 0$  K [3]. For Kondo systems  $E_F / k_B T_L \gg 1$ , and we find the anomalous velocity contribution dominates over skew scattering. The reason is that the expression for the anomalous velocity contains one more derivative of the highly peaked  $4f$  density of states near the Fermi surface, i.e., the Kondo resonance, than the skew scattering expression. We conclude that for  $T \ll T_L \sim T_K$  the anomalous velocity contribution to the Hall effect is significant and exceeds that coming from skew scattering.

3) Finally, the magnetic field strength needed to obtain observable Hall voltages in some heavy fermion compounds is comparable in energy to their Kondo temperature [7]. The linearized theory of the extraordinary Hall effect in Kondo-type systems is inapplicable; in collaboration with D. L. Cox we have extended it to magnetic fields of arbitrary strength [8]. By starting with the quantum equation for electron transport in electric and magnetic fields, we find that the effect of the magnetic field on the conduction electron's self-energy introduces non-linearities, inter alia, in the Hall effect. By expanding the solution for the electron distribution function  $f(\mathbf{k}\sigma)$  in spherical harmonics  $Y_l^m(\hat{k})$  we find the first rank component of this function, which is needed to determine the current, is coupled to higher rank components  $l > 1$ . The solution for  $f(\mathbf{k}\sigma)$  is given as the inverse of an infinite matrix; it can be expanded in an ascending series in powers of  $H^2$ . When we keep only the lowest order antisymmetric term we retrieve the *linear* EHE due to skew scattering of conduction electrons. We have calculated the first few nonlinear terms and have found how they alter the skew scattering contribution to the EHE [8].

In summary, when the above features of the EHE are included in a calculation based on *independent* scattering events, we are better able to reproduce the variations with temperature of the Hall constants observed in Kondo-type alloys for which the scattering remains *incoherent* [9]. Indubitably for heavy fermion and other Kondo-lattice compounds it will be necessary to include the effects of coherency to completely understand the behavior of the Hall constant as one approaches  $T = 0$  K. However, our results indicate that a theory which includes the effects of coherency but is based solely on potential skew scattering will be inadequate.

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