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## NUCLEAR REFRACTIVE EFFECTS IN NUCLEUS-NUCLEUS SCATTERING AT INTERMEDIATE ENERGIES

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Résumé - Nous donnons des arguments théoriques qui montrent la présence d'un effet d'"arc-en-ciel" nucléaire dans la diffusion élastique  $C^{12} + C^{12}$  à 1,016 GeV.

Abstract - We present theoretical evidence for the occurrence of a nuclear rainbow effect in  $C^{12} + C^{12}$  elastic scattering at 1.016 GeV.

I - INTRODUCTION. The separation of the "positive" and "negative" angle components from the experimental data.

The angular distribution of  $C^{12} + C^{12}$  elastic scattering at 1.016 GeV /1,2/ has revealed a pattern similar to that already observed in  $\alpha$ -nucleus scattering at intermediate energies. In the latter case, the particular form of the angular distribution has been attributed /3/ to a refractive effect due to a nuclear rainbow. The qualitative identification of a rainbow is based on the similarity between the angular distribution pattern and that of an Airy function. This procedure is quite simple at low incident energies, because in this case only the Coulomb rainbow is present /4/. However, at intermediate energies, the nuclear transparency becomes large enough to reveal additional (nuclear) refractive contributions. As a result, the two contributing effects give rise to a complicated pattern due to inevitable wave-mechanical interferences and therefore the identification of the nuclear refractive contribution as a "rainbow", becomes ambiguous. In fact, in the resulting angular distribution only an exponential-like decrease, at large angles, is reminiscent of an Airy pattern. To avoid such difficulties, we shall proceed first to the elimination of the interference term from the angular distribution. This can be done by evoking the properties of the envelopes of the oscillatory pattern. These envelopes are defined by /5,6/

$$E_{\pm}(\theta) = [ \sigma_{+}(\theta)^{1/2} \pm \sigma_{-}(\theta)^{1/2} ]^2 \quad (1)$$

where  $\sigma_{+}$  and  $\sigma_{-}$  are the scattering cross-section through "negative" and "positive" angles, respectively. At each scattering angle  $\theta$ , the two values  $E_{\pm}$ , allows us to obtain  $\sigma_{\pm}$  from (1). The results we obtain /6/, are shown in figures 1 and 2, for elastic scattering of  $\alpha$  on  $Ca^{40}$  (104 MeV) /7/ and  $C^{12}$  on  $C^{12}$  (1.016 GeV)/1/. The upper part of these figures, shows the experimental data as well as the envelopes  $E_{\pm}$ . The two components  $\sigma_{\pm}$  are shown by open circles in lower part of the figures. In both cases, the exponential-like component can be unambiguously identified as due to a Coulomb rainbow, i.e., the "positive" angle component. For  $\alpha + Ca^{40}$ , we immediately recognize in the "negative" angle component, an Airy-type pattern, signature of a nuclear rainbow. However, for  $C^{12} + C^{12}$ , the interpretation of the  $\sigma_{+}/\sigma_{-}$  component is considerably less clear.

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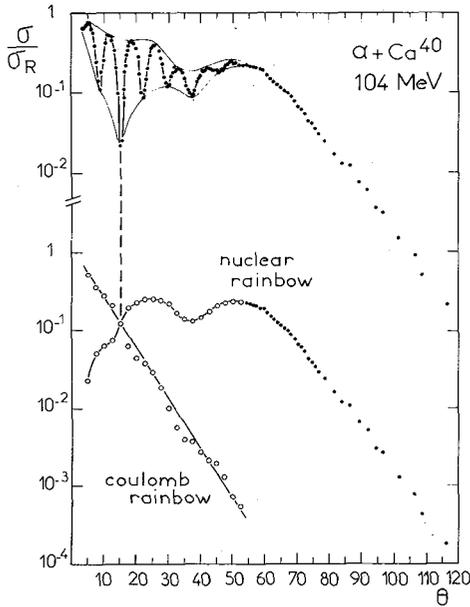


Fig.1

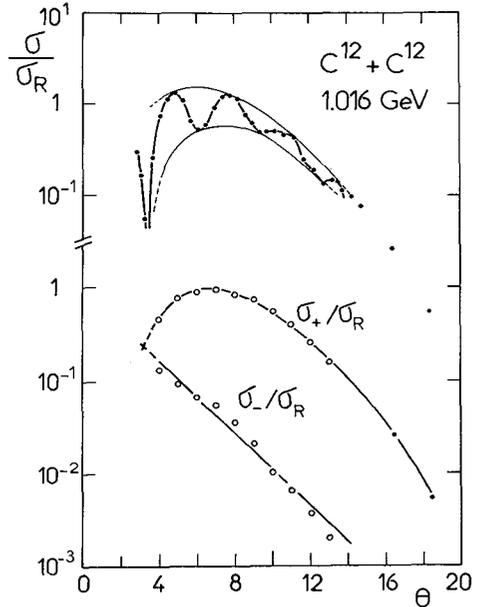


Fig.2

The aim of the present work is to clarify both qualitatively and quantitatively the origin of the "negative" angle component at intermediate energies  $E \approx 100$  MeV/A. Particular attention should be paid to the relative importance of refractive and absorptive effects.

II - The uniform asymptotic evaluation of the high energy scattering amplitude

Assuming  $V/E \ll 1$  ( $V$  is a measure of the absolute magnitude of the potential) and  $kR \gg 1$ , the scattering amplitude is written, in the  $l$ -representation /8/,

$$f(\theta) = \frac{1}{ik} \int_0^\infty dl \, l \, J_0(l\theta) \exp \left[ -\frac{i}{\hbar v} \int_{-\infty}^\infty v_{op} \left[ \left( \frac{l^2}{k^2} + z^2 \right)^{\frac{1}{2}} \right] dz \right] \quad (2)$$

where  $v_{op} = iW(r) + V(r)$  is the optical potential. The angular range of the approximation is  $\theta^{op} \sim V/E$ . Furthermore, we assume that the regime of the scattering process is a quasi-classical one, i.e., /8/

$$(V/E) \times kR \gg 1 \quad \text{or} \quad \theta kR \gg 1 \quad (3)$$

This condition allows the replacement of  $J_0(x)$  by  $(2/\pi x)^{\frac{1}{2}} \cos(x - \pi/4)$ . The scattering amplitude becomes,  $f(\theta) \sim f_+(\theta) + f_-(\theta)$ , where  $f_\pm(\theta)$  are given by

$$f_\pm(\theta) = \frac{1}{ik} \left( \frac{1}{2\pi\theta} \right)^{\frac{1}{2}} \int_0^\infty dl \, l^{\frac{1}{2}} e^{i\omega(l)} e^{i\varphi_\pm(l,\theta)} \quad (4)$$

$$\varphi_\pm(l,\theta) = \chi(l) \pm l\theta \mp \pi/4 \quad (5)$$

$$\omega(l) = \frac{1}{\hbar v} \int_{-\infty}^\infty W \left[ \left( \frac{l^2}{k^2} + z^2 \right)^{\frac{1}{2}} \right] dz, \quad \chi(l) = -\frac{1}{\hbar v} \int_{-\infty}^\infty V \left[ \left( \frac{l^2}{k^2} + z^2 \right)^{\frac{1}{2}} \right] dz \quad (6)$$

The square modulus of  $f_{\pm}(\theta)$  gives the scattering cross-section through "positive" and negative angles ;  $\sigma_{\pm}(\theta) = |f_{\pm}(\theta)|^2$ . The exponential factor,  $\exp \omega(\ell)$ , is the modulus of the  $S(\ell)$  matrix,  $|S(\ell)| = \exp \omega(\ell)$ .

The  $\sigma_{\pm}$  component is obtained through the integral (4), in which  $\omega(\ell)$  and  $\chi(\ell)$  are supposed to be smooth varying functions of  $\ell$ . As is well known, in the classical limit, the integral (4) can be evaluated with high degree of accuracy by stationary phase methods. However, in the present case, the points of stationary phase can be either well separated or coalesce, when  $\theta$  varies. To account for both situations, we use the uniform asymptotic approximation /9,10/. The integral (4) is of the general form /10/

$$I(\alpha) = \int g(x) e^{i f(\alpha, x)} dx \quad (7)$$

It is assumed that  $f(\alpha, x)$  has two stationary points,  $x = x^{(i)}(\alpha)$ ,  $i = 1, 2$ , which can close together or coalesce when  $\alpha$  varies. The uniform approximation of (7) is

$$I \sim 2^{\frac{1}{2}} \pi e^{iA} [ (G_1 + G_2) \xi^{\frac{1}{4}} Ai(-\xi) + i(G_2 - G_1) \xi^{-\frac{1}{4}} A'i(-\xi) ] \quad (8)$$

$$G_1 = g(x^{(1)}) / [ f''(\alpha, x^{(1)}) ] , \quad G_2 = g(x^{(2)}) / [ -f''(\alpha, x^{(2)}) ]$$

$$\xi \doteq \left\{ \frac{3}{4} [ f(\alpha, x^{(1)}) - f(\alpha, x^{(2)}) ] \right\} , \quad A = \frac{1}{2} [ f(\alpha, x^{(1)}) + f(\alpha, x^{(2)}) ]$$

where  $Ai$  and  $A'i$  are the Airy function and its derivative, respectively. When the  $x^{(i)}$  are real,  $\xi$  is positive, whereas when the  $x^{(i)}$  are complex conjugate,  $\xi$  is negative. The uniform approximation includes as special cases the non-uniform results : the stationary phase method ( $\xi \gg 0$ ) and the Airy approximation ( $\xi \sim 0$ ). Now, the stationary phase points of  $\varphi_{\pm}(\ell, \theta)$ , are those satisfying  $\varphi'_{\pm}(\ell, \theta) = 0$ , or

$$- \frac{1}{\hbar v} \frac{d}{d\ell} \int_{-\infty}^{\infty} V [ (\ell^2/k^2 + z^2)^{\frac{1}{2}} ] dz = \frac{1}{E} \int_0^{\infty} \frac{b}{r} [-V'(r)] dz = -\theta , \quad b = \ell/k \quad (9)$$

The left-hand side of (9) is the expression of the classical deflection angle /12/ in the limit  $V/E \ll 1$ . Equation (9) is written  $\theta(\ell) = -\theta$ . Therefore to evaluate  $\sigma_{\pm}(\theta)$  we only need the negative part of the deflection function  $\theta(\ell)$ . We parametrize  $\theta(\ell)$  by

$$\theta(\ell) = \theta_r + q(\ell - \ell_r)^2 , \quad \ell \sim \ell_r \quad (10)$$

where  $\theta_r$ ,  $q$  and  $\ell_r$  can be related to the parameters of the real part of the optical potential. The nuclear rainbow angle  $\theta_r$ , can be evaluated from (9). We take for  $V(r)$  a Wood-Saxon shape,  $V = -V_0 [ 1 + \exp(R-r)/a ]^{-1}$ . The force  $f = -V'_r$  is almost negligible outside the range  $R-a < r < R+a$ , where it remains close to  $-V'(r)r = -V_0/4a$ . Now, the maximum scattering angle, is reached by the trajectory of impact parameter  $b = R-a$ . Putting  $b/r \sim 1$  in (9), one obtains,

$$\theta_r \sim -\frac{1}{2} \frac{V_0}{E} \left( \frac{R}{a} \right)^{\frac{1}{2}} \quad (11)$$

The curvature of  $\theta(\ell)$  in  $\ell = \ell_r$ ,  $q$ , can be related to the thickness of the surface region  $\Delta r = 2a$ , through /5/,

$$q \sim \theta_r / (k \Delta r)^2 = \theta_r / 4 a^2 k^2 \quad (12)$$

Finally, in the limit  $V/E \ll 1$  one has approximately  $\ell_r = kR$ . At this stage, we are able to appreciate at what extent the different conditions of the high energy approximation are fulfilled in the case of  $C^{12} + C^{12}$  at 1.016 GeV. Taking for  $V(r)$  the parameters obtained by Buenerd et al. /2/ from a phase shift analysis of the data,

$$V_0 = 80 \text{ MeV}, \quad R = 3.66 \text{ fm}, \quad a = 0.47 \text{ fm},$$

one obtains

$$V_0/E \sim 0.16, \quad kR \sim 44, \quad V_0 R/\hbar v \sim 7, \quad \theta_r \sim 0.175$$

To proceed further, we now consider the particular form of  $|S(\ell)| = \exp \omega(\ell)$ . We shall use the familiar parametrization, /11/,

$$\exp \omega(\ell) = \{ 1 + \exp [(\ell_0 - \ell)/\Delta] \}^{-1} \quad (13)$$

within the angular range not too far from  $\theta_r$ , one has  $\ell \ll \ell_0$  and (13) can be approximated by

$$\exp \omega(\ell) \sim \exp [(\ell - \ell_0)/\Delta] \quad (14)$$

To remain consistent with the quasi-classical regime expressed by (3), the thickness ( $\sim 2\Delta$ ) of (13) should be much larger than the uncertainty  $\delta\ell$  on  $\ell$ . The  $\ell$ -values in the range  $|\ell - \ell_0| \sim 2\Delta$  correspond to classical trajectories deviated through  $\theta \sim \Delta p/p$ . As  $\delta\ell = k\delta b$  and  $\delta b \Delta p \sim \hbar$ , one has  $\Delta \gg 1/2\theta$ , i.e.,  $\Delta \gg 1$ .

Taking into account (10) and (14), one obtains from (8),

$$|f_+(\theta)|^2 = \sigma_+(\theta) \sim \frac{2\pi \ell_r}{k^2 \theta q^{\frac{2}{3}}} p^2 (\ell_r - \ell_0) F_\alpha(x) \quad (15)$$

$$F_\alpha(x) = A_i^2(x) + S_\pm^2(\alpha\sqrt{|x|}) \left\{ \frac{A_i^2(x)}{|x|} \pm A_i^2(x) \right\} \quad (16)$$

$$x = (\theta - |\theta_r|)/q^{\frac{1}{3}}, \quad \alpha = 1/\Delta q^{\frac{1}{3}}, \quad p^2 = e^{2(\ell_r - \ell_0)/\Delta} \quad (17)$$

and,

$$(+) \text{ for } \theta < |\theta_r|, \quad S_+ = \sinh(\alpha\sqrt{|x|})$$

$$(-) \text{ for } \theta > |\theta_r|, \quad S_- = \sin(\alpha\sqrt{x})$$

Further, we have taken for the classical cross-sections  $\sigma_1$  and  $\sigma_2$  associated with the two branches of  $\theta(\ell)$ , the mean value  $\bar{\sigma}$

$$\sigma_1 \sim \sigma_2 \sim \bar{\sigma} = \ell_r / \theta k^2 |\theta'|$$

and for  $\theta > |\theta_r|$ , we have put

$$|\ell| = [\ell_r^2 + (\theta - |\theta_r|)/q]^{\frac{1}{2}} \sim \ell_r$$

### III - Analysis of the angular distributions. Application to $C^{12} + C^{12}$ scattering at 1.016 GeV.

To analyze the angular distributions, we shall take the product  $[\theta \times \sigma_+(\theta)]$  rather than  $\sigma_+(\theta)$  itself. The advantage is that the resulting pattern depends only on the reduced variable  $x = (\theta - |\theta_r|)/q^{\frac{1}{3}}$ . Consider the function  $F_\alpha(x)$ , (16). The graphical representation of  $F_\alpha(x)$ , is plotted in figure 3, for different values of  $\alpha$ ;  $\alpha = 0., 0.5, 1., 1.5, 2., 2.5$ . It is clear from figure 3 that the pattern evolution of  $F_\alpha(x)$  is determined by the parameter,  $\alpha = 1/\Delta q^{\frac{1}{3}}$ , which depends on both absorptive and refractive contributions, through  $\Delta$  and  $q^{\frac{1}{3}}$ , respectively. For  $\alpha \rightarrow 0, \Delta \rightarrow \infty$ , one has,

$$F_\alpha(x) \rightarrow A_i^2(x)$$

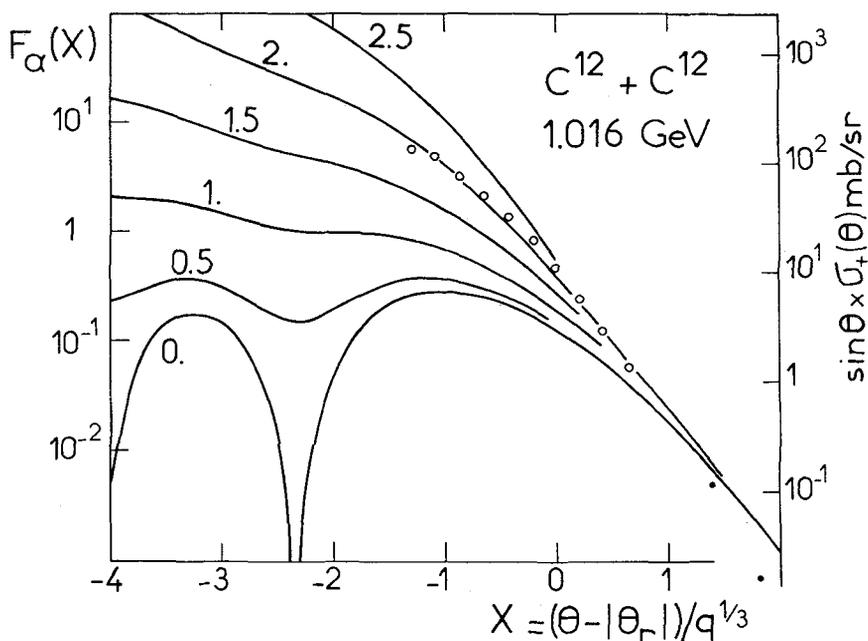


Fig.3

This limit corresponds to a transparent nucleus. The resulting pattern is that of an Airy function which exhibits the familiar structure due to interferences of classical trajectories near a "rainbow". As absorption is introduced,  $\alpha > 0$ , the oscillatory structure gradually disappears. However, it should be noted that for  $\theta \gtrsim |\theta_r|$  the effect of absorption is rather small. The asymptotic behaviour of  $F_\alpha(x)$  illustrates these qualitative analysis. One has,

$$\theta \ll |\theta_r|, \quad |x| \gg 1 \quad (x < 0) \quad \text{and} \quad \alpha \gtrsim 1$$

$$F_\alpha(x) \sim \frac{1}{4\pi} \frac{\exp(2\alpha\sqrt{|x|})}{\sqrt{|x|}} + \frac{1}{\pi\sqrt{|x|}} \sin^2\left\{\frac{2}{3}|x|^{3/2} + \pi/4\right\}$$

This expression shows the strong dependency of the angular distribution pattern on the parameter  $\alpha$ , in the angular range  $\theta < |\theta_r|$ .

$$\theta \gg |\theta_r|, \quad x \gg 1,$$

$$F_\alpha(x) \sim \frac{1}{4\pi} \frac{\exp(-\frac{4}{3}x^{3/2})}{\sqrt{|x|}}$$

This expression is independent of  $\alpha$ . This is an important result. It means that :

For  $\theta \gtrsim |\theta_r|$  the scattering cross section is weakly affected by absorption and therefore remains essentially determined by the nuclear rainbow effect.

We now consider the particular case of  $C^{12} + C^{12}$  scattering at  $E = 1.016$  GeV. With the parameters of Buenerd et al. /2/, for  $V(r)$ , one has from (11) and (12)  $\theta_r = 10^\circ$ , and  $q = 0.000547$ . These define the reduced variable  $x$ . We have plotted in figure 3 the points attributed to the  $\sigma_+(\theta)$  component (see figure 2) multiplied by  $\sin \theta$ . A reasonably good agreement is obtained with the curve  $F_\alpha(x)$ , for  $\alpha = 2$ . This leads to  $\Delta = 6.11$  and, as  $\ell_r \sim kR = 44.2$ , one obtains  $\ell_0 = 59$  and  $\exp \omega(\ell_r) = |S(\ell_r)| = 0.089 \sim 0.1$ . These values are very close to those obtained by a phase shift analysis of the data /10/.

In the light of the arguments given above we can conclude with two main remarks : For  $\theta \ll |\theta_r|$ , the absorption strongly reduces the contribution of the inner branch ( $\ell < \ell_r$ ) of the deflection function. As a result, the first "rainbow" maximum is masked.

For  $\theta \gtrsim |\theta_r|$ , the exponential-like decrease observed in the scattering cross-section of  $C^{12} + C^{12}$  at 1.016 GeV, is essentially a shadow effect due to a nuclear rainbow.

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