NON PERTURBATIVE EFFECTS AND QCD SUM RULES

H. Rubinstein

JOURNAL DE PHYSIQUE
Colloque C3, supplément au n° 12, Tome 43, décembre 1982 *page* C3-249

NON PERTURBATIVE EFFECTS AND QCD SUM RULES

H.R. Rubinstein

Weizmann Institute of Science, Rehovot, Israel
Chalmers University of Technology, Göteborg, Sweden

Introduction. Consider a polarization operator generated by some current, physical or unphysical, that has the general form $j(\Gamma)=q\Gamma q$. The correlation function of this current generates, to zeroth order a quark loop in a given partial wave with quantum numbers determined by Γ. At very short distances because of asymptotic freedom the amplitude is determined by this term. Separating the quarks (by taking derivatives in the conjugate variable) one must include corrections. Gluon exchanges cannot keep quarks from escaping. Therefore, after normal ordering other operators besides the unit operator survive. Their Wilson coefficients are calculated perturbatively while the matrix element, which cannot be evaluated, parametrizes our ignorance of what goes on a long distance . These operators like $\langle GG\rangle$ and $\langle\bar{q}q\rangle$ are gauge and renormalization group invariant and therefore universal. The expression obtained is now matched with a sum of resonances and eventually a continuum. One therefore obtains a determination of masses and couplings in terms of fundamental Lagrange parameters.

Since earlier work has been described in earlier conferences [1] I will make a critical evaluation of these results and discuss new calculations [2,3,4,5,7,8,14].

Heavy quark systems. In this case the mass of the quark fixes the scale. The derivatives of the polarization function become:

$$M_n^J=1/n!(-d/dQ_0^2)^n\pi^J(Q^2)_{Q_0^2}=1/\pi\int \mathrm{Im}\pi(s)/(s+Q_0^2)^{n+1}ds$$

where Q_0^2 is a spacelike reference point upon which the results should not depend. In the QCD side we have $M_n^J(\xi)=A_n^J(1+a_n(J,\xi)\alpha_s+b_n(J,\xi)\phi)$, where $\xi=Q_0^2/4m_h^2$ and $\phi=\frac{4\pi^2}{9}<\frac{\alpha s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}>(4m_c^2)^{-1}$ is the famous gluon condensate. On the resonance side $\mathrm{Im}\pi(s)=9m_R^2/4g^2\delta(s-m_R^2)$ + continuum. Saturating only with one resonance one obtains the formula: $r_n^J(\xi)=(M_R^2+Q_0^2)^{-1}$. In figure 1 one sees the result for a charmonium state [2]. The breakdown for high n signals as it can be checked that the theory is no longer valid. For small n the disagreement has to do with the breakdown of the one resonance approximation. The smooth matching works too well. For the vector current and n=4 the $\Psi(3100)$ is sufficient to saturate as it can be checked directly from experiment. One may ask which are the parameters involved. For every flavour there is the mass of the quark ($m_u=m_d=0$, etc). For all partial waves and flavours one has Λ_{QCD} and the values of the matrix elements like $\langle\bar{q}q\rangle$ and $\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle$. Λ_{QCD} agrees with present estimates and the others have been established by other methods like current algebra and lattice simulations. [6]

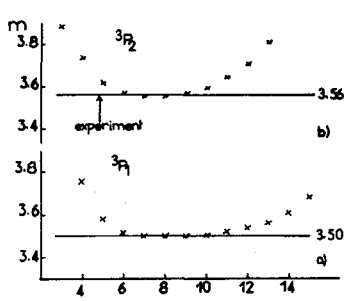

Figure 1

Though there is no systematic study of higher dimensional operators and many times their Wilson coefficients are not known. In all cases when estimates are available their effect is small. Other hidden parameters are: 1) threshold values for the continuum. In charmonium these effects are very small, in baryons too. For massless quark systems these effects are essential but physical assumptions or experimental information unables the practitioner to make accurate predictions. 2) In the case of baryons there is ambiguity in the choice of current (7,8).

$^1S_0$ = 3.01 .02 Gev
$^3S_1$ = 3.10 .01 Gev
$^3P_0$ = 3.40 .01 Gev
$^3P_1$ = 3.50 .01 Gev
$^3P_2$ = 3.56 .01 Gev
$^1P_1$ = 3.51 .01 Gev
$m_\gamma - m_{\eta_B}$ = 60 Mev
m_c = 1.25 Gev($p^2=-m^2$)
m_b = 4.26 Gev($p^2=-m_b^2$)

As an example we see the masses predicted for the charmonium states and some results on the bottonium spectrum. In the case of the bottom quark there are problems that require some further assumptions (9) but different techniques yield very similar results and one is therefore confident on the results. These calculations are the best known method to establish the mass of the heavy quarks (10). The $^1P_1$ state has not been observed but its position seems to be predicted at 3.51 by all models (11).

Light mesons with I=1. The lowest lying mesons were studied in Ref. 3. Here I describe the calculations of Ref. 3 which have completed the knowledge of the spectrum up to spin 2. In the case of massless quarks one must introduce a scale into the problem. This is accomplished by calculating at q^2 large and taking a Borel transform.

$$L_M\pi^J(Q^2)=\lim_{\substack{Q^2\to\infty \\ n\to\infty}} \text{ and } Q^2/n=M^2 \text{ of } (n-1)!^{-1}Q^{2n}(-d/dQ)^{2n}\pi^J(Q^2) \text{ giving}$$

$$L_M\pi^J(q^2)=(\pi M^2)^{-1}\int \exp(-s/M^2)\ \mathrm{Im}\pi^J(s)ds$$

In the case of the A_1 meson which we discuss as an example the polarization operator

$$\pi(Q^2)=\frac{1}{8\pi^2}(1+\frac{\alpha_s}{\pi})Q^2\ln\frac{Q^2}{\mu^2}+\frac{1}{2Q^2}\langle 0|m_u\bar{u}u+m_d\bar{d}d|0\rangle-\frac{1}{24Q^2}\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle+$$

$$+\frac{\pi\alpha_s}{2Q^n}\langle 0|\ \bar{u}\gamma_\mu\lambda_a u-\bar{d}\gamma_\mu\lambda_a d\ ^2|0\rangle+\frac{\pi\alpha_s}{9Q^n}(\langle 0|\bar{u}\gamma_\mu\lambda_a u+\bar{d}\gamma_\mu\lambda_a d)(\sum_{u,d}\bar{q}\gamma_\mu\lambda^a q)|0\rangle$$

where the operators discussed earlier are shown to appear. The others can be expressed in terms of these or estimated since their contribution is small. After a Borel transform the sum rule (including the continuum which is important) reads

$$m_R^2=M^2\ \frac{2(1+\frac{\alpha_s}{\pi})[1-(1+\frac{S_0}{M^2}+\frac{S_0^2}{M^4})]e^{-S_0/M^2}+.1/M^6}{(1+\alpha_s/\pi)[1-(1+\frac{S_0}{M^2})]e^{-S_0/M^2}-\frac{0.46}{M^4}-0.1/M^6}$$

A similar sum rule can be obtained using the axial current. The results seen in Fig. 2 are quite satisfactory. In these cases the continuum is important but the value that gives best results is very reasonable. Large number of states can be calculated except for the B meson where accidentally the presence of subtractions makes the calculation impossible. Couplings as shown can also be calculated. For completeness

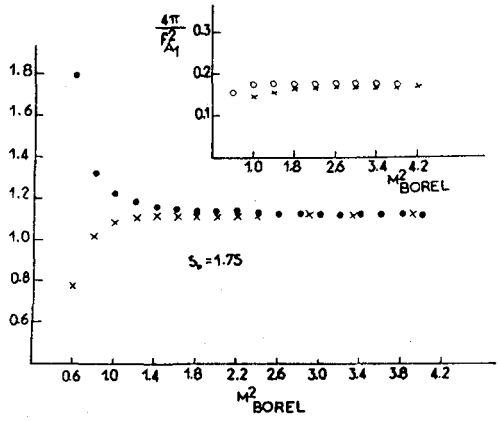

Figure 2

ρ	1^{-1}	750±30	776
ω	1^{-0}	750±30	780
K*	$1^{-\frac{1}{2}}$	920	892
φ	1^{-0}	1070	1020
f	2^{+0}	1270	1270
f*	2^{+0}	1500	1516
A_2	2^{+1}	1320	1317
K**	$2^{+\frac{1}{2}}$	1420	1434
δ	0^{+1}	1000	981
S*	0^{+0}	1000	980
ε	0^{+0}	1350	1300
A_1	1^{+1}	1150	1100-1300
D	1^{+0}	1270	1285
E	1^{+0}	1460	1418
B	1^{+1}	not available	1231
A_3	2^{-1}	1630	1660
		Theory	Exp

Table 1

$f_\pi = \frac{m_\rho}{2\pi}$ =125 Mev (133 Mev), $\frac{g_\rho^2}{4\pi}$ =2.3±.1 (2.36±.18), f_c^{2+}=0.037±.003 (0.04) (5)

$\frac{4\pi}{f^2_{A_1}}$ =0.16±0.02 (.16±.02)

Couplings for ℓ=0 and 1 light quark mesons

---

we include the results of Shifman et al without discussing them.(12)

Notice that the parameters are Λ_{QCD}, $m_n=m_d=0$, m_s=150 Mev, $\langle\bar{q}q\rangle$ (its breaking is unimportant) and φ. The small differences like f, A_2 can be understood (different thresholds), but overall the theory predicts I=0,1 degeneracy in agreement with experiment. There are still conflicting models for QCD sum rules on scalars (14). Our results seem confirmed by lattice simulations (6).

Light heavy quark systems.(16) (open bottom) Here we only mention that $f_p \sim$200 Mev (16) which is imcompatible with models to explain τ_{D^+}/τ_{D^0} and $m_{0^{++}} - m_{0^{-+}} \simeq$800 Mev. This result is due to the term $m_b\langle\bar{u}u\rangle$ which splits strongly opposite parity states. This term seems "non-potential".

Baryons. For the baryon octet there are two possible currents. The natural choice is the one that has an SU(6) non relativistic limit and couples to the non perturbative operators. For the decuplet the choice is unique.

The nucleon polarization function is given by

$$\int d^4x\, e^{ip.x}\langle 0|T\eta_n(x)\bar{\eta}_n(0)|0\rangle = \not{p}F_1(p^2)+1.F_2(p^2).$$

On dimensional counting it follows that F_1 is even and F_2 is odd. As a consequence the function F_2 is proportional to $\langle\bar{q}q\rangle$ without a mass factor and is furthermore enhanced by the elimination of one loop integral. It completely overwhelms the bare loop term. The other sum rule is standard and several terms including the two condensates and bare loop compete. After Borel transform the two sum rules become:

$$M^6 + 4/3\ a^2 - bM^2 = 2.(2\pi)^4\lambda_N^2\ e^{-M_N^2/M^2}$$

$$2\ a\ M^4 = 2(2\pi)^4\lambda_N^2\ M_N\ e^{-M_N^2/M^2} \quad \text{where } a = -(2\pi)^4\langle\bar{q}q\rangle,\ b = {}^2\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle,$$

and M_N is the mass of the nucleon and λ_N the coupling to three quarks, a quantity that appears in proton decay calculations [15]. M is the Borel variable as usual.

Solving for the nucleon mass one obtains:

$$M_N = 2aM^4/M^6 + 4/3a^2 - bM^2$$

and a slightly more complicated formula if the continuum is allowed. In Figure 3 we see the results as a function of M. The results show how for massless quarks the chiral condensate generates the nucleon mass. The continuum improves the results but it is a minor effect. Analogous formulae can be written for the octet and decuplet and the agreement is excellent. In the decuplet there is a new operator contributing: $\langle\bar{q}G^a_{\mu\nu}\lambda^a q\rangle \equiv m_0\langle\bar{q}q\rangle$ and its value has been estimated elsewhere. It is interesting that these calculations depend crucially on $\gamma = (\langle u\bar{u}\rangle - \langle\bar{q}q\rangle)/\langle\bar{u}u\rangle$. [17]

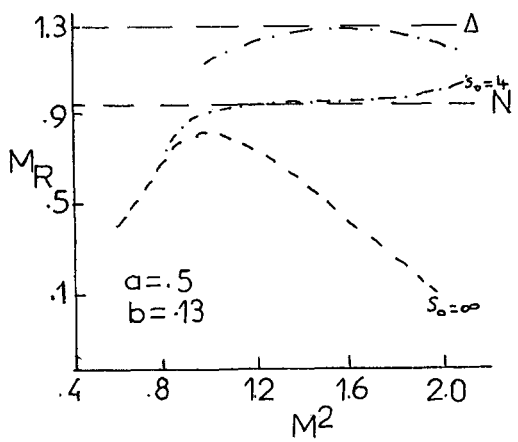

Figure 3

M_N=900(940)	M_Λ=1070(1115)	M_Σ=1170(1185)
M_Ξ=1370(1320)	$M_N{}^*$=1240(1235)	$M_Y{}^*$=1370(1385)
$M_\Xi{}^*$=1510(1520)	M_{Ω^-}=1650(1670)	

Table 2

Allowing for different strange and u quark masses but setting γ=0 drives the Ξ below the Σ. The calculation also yields a correction to proton lifetime that seems to rule out simplest SU(5).

It is also possible to apply the theory to three point functions. though we cannot discuss these results here [18], one can compute the pion nucleon coupling constant and obtain

$$g_{NN\pi} = \frac{e}{(2\pi)^2}\frac{M_N^4}{2}\ f_\pi\ m_\pi^2\ \frac{1}{\lambda_N^2}\ (m_u+m_d)^{-1}$$

in remarkable agreement with experiment. Calculations of higher partial waves for baryons show the negative parity states far above the 56 representation as desired.

Conclusions. A theory based on QCD and some dynamical assumptions about the convergence of dispersion integrals can reproduce remarkably well the spectrum of hadrons with very few parameters. Moreover it predicts the appearance of interesting terms in the spectrum of these states. In particular: establishes the spin dependence of

the confining forces are short distances and unables to demonstrate how chiral breaking endows the proton with mass. There are also non local terms that are necessary and their presence would be a decisive element in proving its validity. Lattice simulations of these systems are of great interest. There is some work on lattices that might explain the success of the theory [19]. Chiral breaking seems to occur at much shorter distances than confinement dressing the quarks and establishing the properties of the bound states at short distances.

Because of time I cannot discuss other interesting issues like the Schwinger Smilga Cronstrom gauge [20], further calculations on Wilson coefficients [21], form factors and many other topics. References are for guidance only and certainly incomplete.

I would like to thank my friends at Weizmann for discussions and my collaborators L.H. Reinders and S. Yazaki that played a crucial role in all our results.

References

1. V. Zakharov, High Energy Physics Conference, Wisconsin 1980, p. 1235.
M.A. Shifman, International Symposium on Electromagnetic and Weak Interactions, Bonn 1981.
2. L.J. Reinders, H.R. Rubinstein and S. Yazaki, Nucl. Phys. B186 (1981) 109.
3. L.J. Reinders, H.R. Rubinstein and S. Yazaki, Nucl. Phys. B196 (1982) 125.
4. S. Narisson and E. de Rafael, Phys. Lett. 103B (1981) 57.
5. M.A. Shifman et al, Phys. Lett. to appear.
6. H. Hamber and G. Parisi, Phys. Rev. Lett. 47 (1982) 1792, and preprints.
T. Banks, R. Horsley, H.R. Rubinstein and U. Wolf, Nucl. Phys. B140 (FS3) (1981) 692.
7. B.I. Ioffe, Nucl. Phys. B188 (1981) 317, B191 (1981) 591.
A.V. Smilga, preprint.
8. Y. Chung, H.G. Dusch, M. Kremer and D. Schall, Phys. Lett. 102B (1981) 175.
Pascual and Tarrach, preprint, Madrid 1982.
9. For a discussion, see M.A. Shifman, Ref. 1.
10. See Ref. 2. For all quarks masses: P. Leutwyler and S. Mallik, Physics Reports to be published.
11. This state should be observed by isospin violation decay. See N. Isgur, H.J. Lipkin, H.R. Rubinstein and A. Schwimmer, Phys. Lett. 89B (1979) 79.
12. M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385.
13. V.A. Novikov, M.A. Shifman, A.I. Vainshtein, M.B. Voloshin and V.I. Zakharov, Phys. Rep. 41 (1978) 1.
14. E.V. Shuryak and collaborators, for example, discuss instanton effects in these channels.
15. V.S. Berezinsky, B.L. Ioffe, Ya I. Kogan, Phys. Lett. 105B (1981) 33.
16. L.J. Reinders, H.R. Rubinstein and S. Yazaki, Phys. Lett. 104B (1981) 305.
E.V. Shuryak, Novosibirsk preprint. See Shifman, Ref. 1.
17. L.J. Reinders, H.R. Rubinstein and S. Yazaki, to be published.
18. L.J. Reinders, H.R. Rubinstein and S. Yazaki, Weizmann Institute preprint 1982.
19. J. Kogut, M. Stone, H.W. Wyld, J. Shigemitsu, S.H. Shenker and D.K. Sinclair, Phys. Rev. Lett. 48 (1982) 735.
20. J. Schwinger, Particles, Sources and Fields, 1970, p. 271; M.J. Dulobikov, A.V. Smilga, Nucl. Phys. B185 (1981) 109; C. Cronstrom, Phys. Lett. 90B (1980) 267.
21. W. Hubschmid and S. Mallik, preprint butp 10/1982.