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UNIFIED THEORY OF DISLOCATION DAMPING WITH A SPECIAL REFERENCE TO POINT-DEFECT DRAGGING

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**Abstract.**- The complete mathematical solution of dislocation damping for the equally spaced multi-dragging-point-defect case is investigated by the Laplace-transformation method combined with a variational procedure. It is shown that the dragging leads to an expression which is identical to the exact solution of the Koehler-Granato-Lücke (KGL) model with a modified damping constant. The apparent dissimilarity between the point-defect dragging model of Simpson and Sosin (SS) and the KGL theory is due to the retention of only the first term in the Fourier expansion by KGL and concomitantly the inertial term by SS.

1. Introduction.- In the present investigation, in order to elucidate the apparent discrepancy between the KGL theory (1,2,3) and the theory of Simpson and Sosin (4,5), the mathematical analysis of the equally spaced multi-dragging point defect which corresponds to the most probable distribution of defects along the dislocation line is formulated. The solution is found to be identical to the solution of the KGL model with a modified damping constant which is given by  $B_0 + B_d(N+1)/L$ , where  $B_0$  is the viscous damping constant in the absence of the point defects,  $B_d$  is the damping constant for each of the  $N$  dragging points, and  $L$  is the length of dislocation loops between firm anchor points. Using the compact form of the solution, the logarithmic decrement and the modulus defect are calculated. The expressions for  $\delta$  and  $\Delta G/G$  differ from those given by KGL due to their retention of only the first term in the Fourier-series solution of the problem, but they become identical to the results of SS (4) at the limit where the inertial constant approaches zero.

2. Dislocation Oscillation Under Dragging-Point Defects.- The displacement of the dislocation under the influence of an applied stress is given by the particular model chosen. The mathematical model for the equation of motion of a pinned-down dislocation loop is taken to be that used by Koehler (1) and later modified by Simpson and Sosin (4) to take into account the viscous drag effect of a discrete set of point defects (equally spaced):

$$A \frac{\partial^2 \xi}{\partial t^2} + \left[ B_0 + B_d \sum_{n=1}^{n=N} \delta \left( x - n \frac{L}{N+1} \right) \right] \frac{\partial \xi}{\partial t} - C \frac{\partial^2 \xi}{\partial x^2} = b \sigma_0 e^{i \omega t} \tag{1}$$

with the initial and boundary conditions

$$\xi(0, t) = \xi(L, t) = 0 \quad \text{and} \quad \xi(x, 0) = \frac{\partial \xi(x, 0)}{\partial t} = 0, \tag{2}$$

where  $\xi$  is the displacement of an element of the dislocation loop from its equilibrium position,  $A$  is the effective mass per unit length,  $C$  is the line tension in a

bowed-out dislocation, and  $b$  is the Burger's vector.

By the Laplace transformation, the following expression is obtained from Eqs.

(1) and (2):

$$A p^2 \bar{\xi} + p \left[ B_0 + B_d \sum_{n=1}^{2N} \delta \left( x - n \frac{L}{N+1} \right) \right] \bar{\xi} - C \frac{d^2 \bar{\xi}}{dx^2} = b \frac{\sigma_0}{p - i\omega}, \tag{3}$$

and similarly from Eq. (2) also the natural boundary conditions can be obtained  $\bar{\xi}(0) = \bar{\xi}(L) = 0$ . The overbar on any function denotes the Laplace transform of that function. The above boundary-value problem can be reduced to a proper variational problem (6), that is,

$$\int_0^L \left( \mathcal{L} \bar{\xi} - \frac{b\sigma_0}{p - i\omega} \right) \delta \bar{\xi} dx = 0, \tag{4}$$

where

$$\mathcal{L} \equiv A p^2 + p \left[ B_0 + B_d \sum_{n=1}^N \delta \left( x - n \frac{L}{N+1} \right) \right] - C \frac{d^2}{dx^2}. \tag{5}$$

The functions  $\phi_k(x) = \sin(k\pi x/L)$ , which satisfy the natural boundary conditions, are convenient coordinate functions. Therefore,  $\bar{\xi} = \sum b_k \sin(k\pi x/L)$  can be used in Eq. (4). Thus, by making use of the orthogonality properties of the harmonics, we obtain the following identity to first order in  $B_d$  for  $k > N$ , and rigorously otherwise:

$$b_k^{(1)} = \frac{4b\sigma_0}{\pi(p - i\omega)k} \frac{1}{(A p^2 + B p + C k^2 \pi^2 / L^2)} \delta_{k, 2m+1}, \tag{6}$$

where  $B = B_0 + B_d (N+1)/L$ ,  $N = 2M+1$ ,  $M = 0, 1, 2, \dots$  and  $B = B_0 + ((N+1)/L (1 - \delta_{k, (2M+1)(N+1)}))$ . Hence, the displacement of the dislocation loop can be written in the following form:

$$\xi^{(1)}(x, t) = \frac{4b\sigma_0}{\pi A} \sum_{m=0}^{\infty} \frac{\sin[(2m+1)\pi x/L]}{(2m+1)} L^{-1} \times \left( \frac{1}{(p - i\omega)(p^2 + dp + \omega_m^2)} \right), \tag{7}$$

with the substitutions  $d = B/A$ ,  $\omega_m = (2m+1) \frac{\pi}{L} (C/A)^{1/2}$ , where  $L^{-1}$  denotes the inverse Laplace transformation. One should point out that Eq. (7) gives not only the steady-state solution of the problem, but also the transient part equally well. The steady-state part of the solution has the analytical form (7):

$$\xi_{st}^{(1)}(x, t) = \frac{4b\sigma_0}{A\pi} \sum_{m=0}^{\infty} \frac{\sin[(2m+1)\pi x/L] e^{i\omega t - i\delta_m}}{(2m+1) [(\omega_m^2 - \omega^2)^2 + (\omega d)^2]^{1/2}}, \tag{8}$$

where  $\tan \delta_m = \omega d / (\omega_m^2 - \omega^2)$ . The above expression is identical to the one obtained by Granato and Lücke (2). A closed form is also obtained which has the following expression:

$$\xi_{st}^{(1)}(x, t) = \frac{b\sigma_0 e^{i\omega t}}{A i \omega (\omega + d)} \times \left( 1 - \frac{\cosh\{[i\omega(\omega + d)A/C]^{1/2}(x - L/2)\}}{\cosh\{[i\omega(\omega + d)A/C]^{1/2}L/2\}} \right). \tag{9}$$

**3. The Decrement and the Modulus Defect.** - To find the decrement and the apparent modulus change we will follow the general procedures of Nowick (8). The dislocation strain produced by a loop of length  $L$  in a cube of unit dimensions is usually given by  $\langle \xi \rangle L b$ , where  $\langle \xi \rangle$  is the average displacement of a dislocation of length  $L$ . Thus, if  $\Lambda$  is the total length (the density of dislocations) of the movable dislocation

line,

$$\epsilon_d = \Delta b \frac{1}{L} \int_0^L \xi(x, t) dx. \quad (10)$$

The relationship between the total strain and the applied stress can be written  $\epsilon_t = J(\omega)\sigma$ , where  $\epsilon_t = \epsilon_e + \epsilon_d$  and  $\sigma = \sigma_0 \exp(i\omega t)$ . The elastic strain  $\epsilon_e$  is given by the elasticity theory,  $\epsilon_e = \sigma/G$ . With these definitions of stress and strain, the appropriate measure of the internal friction is the logarithmic decrement  $\delta = J_2(\omega)/J_1(\omega)$ , where  $J(\omega) = J_1(\omega) - i J_2(\omega)$ . Similarly, the modulus defect is  $\Delta G/G = GJ_1(\omega) - 1$ .

In the calculation of the average displacement of a dislocation loop of length  $L$ , we will intentionally use the closed form which is given by Eq. (9). However, first let us introduce the parameter  $\alpha$  as  $\alpha^2 = i C/\omega (i\omega + d) A$ , which upon substitution in Eq. (9) yields

$$\xi_{st}(x, t) = -\frac{b\sigma_0 e^{i\omega t}}{C} \alpha^2 \left( 1 - \frac{\cos[(x-L/2)/\alpha]}{\cos(L/2\alpha)} \right). \quad (11)$$

With the usage of the above expression in Eq. (10), one finds that

$$\epsilon_d = -\frac{2\sigma_0 e^{i\omega t} \Delta b^2 \alpha^3}{CL} [L/2\alpha - \tan(L/2\alpha)]. \quad (12)$$

On the other hand it is very useful to introduce the identity which can be obtained from the definition of the parameter  $\alpha$  as  $\alpha = L \exp(i\theta)/\mu M$ , where  $M^2 = 2(1 + \omega^2 A^2/B^2)^{1/2}$ ,  $2\theta = \tan^{-1}(B/A\omega)$ , and  $\mu^2 = \omega BL^2/2C$ . Here, the last parameter  $\mu$  is a dimensionless universal parameter. If one substitutes newly defined parameters in Eq. (12), the following results can be obtained (9) after rearranging the terms:

$$\delta = \frac{\pi G b^2 \Delta L^2}{2C \mu^3} \{2\tilde{\mu} \sin 2\theta + 4[\cos 3\theta \sinh(\tilde{\mu} \sin \theta) - \sin 3\theta \sin(\tilde{\mu} \cos \theta)] / [\cosh(\tilde{\mu} \sin \theta) + \cos(\tilde{\mu} \cos \theta)]\} \quad (13)$$

and

$$\frac{\Delta G}{G} = \frac{G b^2 \Delta L^2}{2C \mu^3} \{2\tilde{\mu} \cos 2\theta - 4[\cos 3\theta \sin(\tilde{\mu} \cos \theta) + \sin 3\theta \sinh(\tilde{\mu} \sin \theta)] / [\cosh(\tilde{\mu} \sin \theta) + \cos(\tilde{\mu} \cos \theta)]\}, \quad (14)$$

where  $\tilde{\mu} = \mu M$ . If the inertial term  $A$  is taken to be equal to zero as done by Simpson and Sosin (4), one can immediately deduce  $M = \sqrt{2}$  and  $\theta = \pi/4$ , which upon substituting into above equations yields

$$\delta = \frac{\pi G b^2 \Delta L^2}{2C \mu^3} \left( \mu - \frac{\sinh(\mu) + \sin(\mu)}{\cosh(\mu) + \cos(\mu)} \right), \quad (15)$$

and

$$\frac{\Delta G}{G} = \frac{G b^2 \Delta L^2}{2C \mu^3} \frac{\sinh(\mu) - \sin(\mu)}{\cosh(\mu) + \cos(\mu)}. \quad (16)$$

The above expressions are identical to the results obtained by SS for the uniformly distributed point defects along a dislocation segment. Eqs. (15) and (16) were also deduced by Oen, Holmes, and Robinson (10) for the KGL model dislocation damping assuming that  $A = 0$ .

4. Discussion.— In Fig. 1, the decrement is plotted with respect to the newly defined normalized frequency  $\Omega_B = \omega/\omega_B^0$  for various values of the damping constant  $D$

by using Eq. (14) and the plotter facilities of an HP-9821A minicomputer. Here,  $\omega_B^0$  is really the fundamental oscillation frequency of a vibrating dislocation loop without inertia, and it is given by  $\omega_B^0 = (\pi/L)^2 C/B$ . Thus, from the definition of the parameter  $\mu$ , it is clear that  $\mu^2 = \pi^2 \cdot \Omega_B/2$ . The frequency response has two main branches depending upon whether the damping is large  $D > 1/2$  or small  $D \leq 1/2$ . However, in this plot it is clearly demonstrated that the maximum loss occurs at  $\omega = \omega_B^0$  or  $\mu = \pi^2/2$  for large dampings.

In order to illustrate the general behavior of the decrement which is measured at any given driving frequency as a function of  $B$  or dragging defect concentration, we decided to introduce a new plotting procedure in Fig. 2, where the logarithmic decrement obtained from Eq. (15) is shown with respect to the normalized frequency  $\Omega_B$  for various values of  $\omega/\omega_A^0$  where  $\omega_A^0$  is the fundamental resonance frequency which is given by  $\omega_A^0 = (\pi/L)(C/A)^{1/2}$ . This new plot reveals that, with the exception of the resonance damping, the decrement indicates a well-defined maximum with respect to the viscous drag constant  $B$ . In addition, with a driving frequency less than one tenth of the resonance frequency, the maximum in the decrement corresponds to a value of  $\Omega_B$  which is about equal to units. Consequently, an increase in the decrement due to the addition of point defects can be observed if  $\mu_0$  is less than  $\pi^2/2$ , where  $\mu_0 = (\omega_B^0 L^2/2C)^{1/2}$ . In other words,  $B_0$  which is the viscous drag constant for the defects that produce the background damping (pre-irradiation damping) should be less than  $\pi^2 C/L^2$  for the "peaking effect" to be observed.

The mean decrement resulting from an exponential dislocation loop distribution is also calculated by the author (9) and plotted in Fig. 3 using an HP-9821A. This original plot reveals the fact that with the exception of  $\omega \gg \omega_A^0/2$ , the mean decrement shows a well-defined maximum with respect to the viscous drag constant  $B$  for an exponential loop distribution. In addition, with a driving frequency less than one one-hundredth of the mean resonance frequency, the expectation value of the decrement depends solely upon the current value of  $\bar{\Omega}_B$  and the maximum in the decrement corresponds to a value of  $\bar{\Omega}_B$  which is about equal to 0.1.

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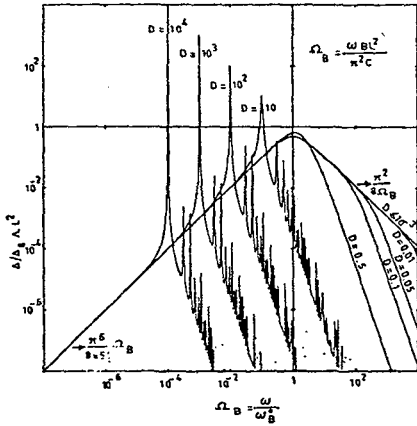


FIG. 1 Decrement plotted as a function of the renormalized frequency  $\Omega_B = \omega/\omega_0^2$  for various values of the damping constant for a delta-function distribution of the loop length, where  $\omega_0^2$  is the fundamental frequency of the dislocation loop in the absence of the inertial term and is given by  $C\pi^2/L^2B$ .

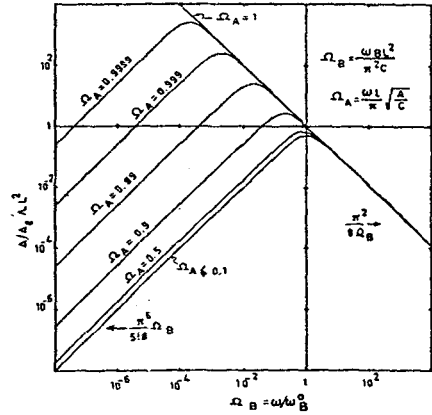


FIG. 2 Graphical representation of the decrement as a function of the renormalized frequency  $\Omega_B = \omega BL^2/\pi^2 C$  for various values of the normalized driving frequency  $\omega/\omega_0^2 < 1$ , where  $B$  is the modified viscous damping constant which is given by  $B = B_0 + B_d(N+1)/L$ .  $N$  is the number of defects added per dislocation loop and  $B_d$  is the viscous drag constant due to these defects.

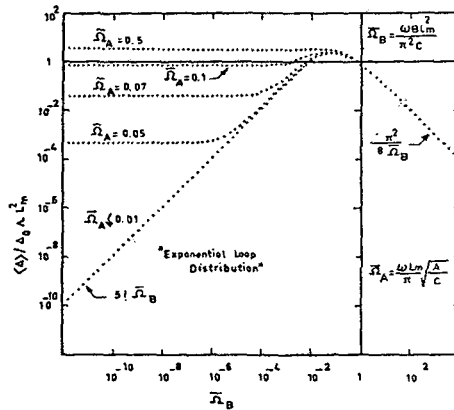


FIG. 3 Graphical representation of the decrement as a function of the renormalized frequency  $\Omega_B$  for an exponential distribution of loop lengths.