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## COOPER PAIRING IN SPIN-POLARIZED FERMION SYSTEMS

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Résumé.- On discute la formation de paires de Cooper dans les systèmes de Fermi à spin polarisé, tels que  $^3\text{He}\uparrow$  et  $\text{D}\uparrow$ , en insistant particulièrement sur les questions suivantes :

- (1) Quelles sont les conditions de formation des paires de Cooper, et à quelles températures cette formation est-elle probable pour les systèmes envisagés ?
- (2) Quelle est la relation entre les paires de Cooper et des molécules diatomiques ?
- (3) Quels sont les phénomènes qualitativement nouveaux que l'on attend dans un système à paires de Cooper, et qu'apparaîtra-t-il probablement de nouveau si la formation de paires a lieu dans un système à spin polarisé ?

Abstract.- I discuss the phenomenon of Cooper pairing in strongly spin-polarized Fermi systems, such as  $^3\text{He}\uparrow$  and  $\text{D}\uparrow$ , with particular attention to the questions (1) what are the conditions for Cooper pairing to occur, and at what temperatures is this likely to happen for the systems of practical interest ? (2) what is the relationship between Cooper pairs and diatomic molecules ? (3) what are the qualitatively new phenomena we expect in a Cooper-paired system, and what new physics is likely to emerge if the phenomena occurs in spin-polarized systems ?

In this talk I shall discuss, informally and without detailed derivation, the questions: What is Cooper pairing and under what conditions do we expect it to occur in spin-polarized (and some other) quantum systems? What are the similarities and differences between Cooper pairs and diatomic molecules? What are the consequences of Cooper pairing and what can we use it for? It should be emphasized at the start that the experimental relevance of the phenomenon to spin-polarized systems, particularly the hydrogen isotopes, is extremely sensitive to the maximum density at which they can be stabilized, which at the time of writing is an unknown quantity.

Cooper pairing is, in the crudest terms, a phenomenon which occurs in degenerate Fermi systems and which involves the formation by two fermions of a sort of giant diatomic molecules ("Cooper pairs") which automatically undergo Bose condensation. It generally leads to the complex of phenomena which go under the generic name of superfluidity, and the

resulting system is called a "Fermi superfluid".

The conditions for its occurrence are: a fairly high degree of degeneracy, a (weakly) attractive interaction between the fermions, and the absence of too much incoherent scattering. It is not necessary that the paired fermions be identical, or even that they have the same mass, but they must have at least approximately the same Fermi momentum. An important consequence of this is that pairing of fermions with opposite spin is suppressed by even a fairly weak spin polarization, since this will increase the up-spin Fermi surface at the expense of the down-spin one.

Taking for the moment a naive view of the Cooper pairs as simply giant diatomic molecules, one would expect them to be described by some "molecular" wave function of the type

$$\Psi(\underline{r}_1 - \underline{r}_2, \underline{\sigma}_1, \underline{\sigma}_2)$$

where  $\underline{\sigma}$  indicates the spin of a fermion and  $\underline{r}_1 - \underline{r}_2$  the relative separation of the two fermions. (Here and in the subsequent discussion we assume for sim-

plicity that the centre of mass of the pairs is at rest). In accordance with the Fermi statistics, the wave function should be antisymmetric under exchange of particles 1 and 2. Moreover, for almost all the systems which are of interest in the present context the total spin of a pair is likely to be conserved to a very good approximation. Thus, in the case of two fermions of spin  $\frac{1}{2}$ , one has the possibilities:

- a)  $S = 0, \quad \ell = \text{even}$
- b)  $S = 1, \quad \ell = \text{odd}$

where  $\ell$  is the relative orbital angular momentum. It turns out that for  $\ell = 0$  (s-wave pairing) the properties of the Cooper pairs, and hence of the whole system, are isotropic, whereas for  $\ell \neq 0$  (with one exception which is not relevant in the present context) the properties are anisotropic and one speaks of an "anisotropic superfluid".

One finds that in dilute unpolarized systems the energetics will always favour  $\ell = 0$  pairing (just as the groundstate of a diatomic molecule in a  $\Sigma$ -state always has angular momentum zero). On the other hand, for a spin-polarized system, even a dilute one,  $\ell = 0$  pairing (which is associated, as above, with total spin zero) is suppressed and generally speaking the pairs will form in an  $\ell = 1$  state, thereby giving rise to anisotropic superfluidity.

In Table 1, I review some actual and possible (laboratory) Fermi superfluids. The last column indicates the critical temperature at which one might expect the onset of Cooper pairing. Under the heading of "actual" superfluids one might perhaps also include the Bose-condensed excitations reported by A. Mysyrowicz at this Conference, since after all an exciton is nothing but a bound state of two fermions (electron plus hole). /1/.

Two aspects of the numbers in Table 1 deserve comment. First, it should be emphasized that the calculation of the critical temperature of a Fermi

Table 1

Actual and possible Fermi superfluids

<u>System</u>	$S^*$	$\ell$	$T_c (^{\circ}\text{K})$
(a) <u>Actual</u>			
electrons in superconductors	0	0	$\leq 20$
$^3\text{He-A, B, A}_1$	1	1	$\sim 2 \times 10^{-3}$
(b) <u>Possible</u>			
$^3\text{He}$ in $^4\text{He}$	0	0	$\sim 10^{-4} - 10^{-3} ?$
$^3\text{He}\uparrow$ in $^4\text{He}$	0	0	?
$^3\text{He}\uparrow$	1	1(3?)	$10^{-3} - 10^{-2} ?$
$D_{\uparrow 1}$	2	1	$\leq 10^{-6}$
$D_{\uparrow 3}$	1	0	

\* In all cases, except the first,  $S$  is the nuclear spin. In the case of deuterium there is also an electronic spin contribution, which in the "spin-polarized" state is by definition always 1.

superfluid is in general an extremely tricky business, since it depends exponentially on parameters such as the effective interaction at the Fermi surface which themselves are often not well known. Hence one should treat the numbers quoted for  $^3\text{He}\uparrow$ , and to a lesser extent for  $^3\text{He}$  in  $^4\text{He}$  (at the maximum concentration,  $\sim 10\%$ ) with considerable caution. However, an exception to the general rule is the case of a very dilute gas where the two-particle s-wave scattering length  $a_s$  is known; in this case the critical temperature for s-wave pairing should be given to a very good approximation, by the formula

$$T_c \approx 1.6 T_F \exp - \pi / (2 k_F |a_s|) \quad (a_s < 0) \quad (1)$$

where  $T_F$  is the Fermi temperature and  $k_F$  the Fermi momentum.

For a dilute system with p-wave pairing, the factor  $k_F |a_S|$  in the exponent is replaced by a factor of order  $k_F^3 |b|$ , where  $b$  is a quantity with the dimensions of volume which is the p-wave analogue of the scattering length. Because of the much sharper dependence of the exponent on density in the p-wave case, p-wave pairing is likely to occur in dilute systems, if at all, only at presently unattainable temperatures.

The second comment concerns spin-polarized deuterium. This is clearly a special case in the context of the above discussion, in that the deuterium atom, though a fermion, has nuclear spin 1. This invalidates the considerations given above for fermions of spin  $\frac{1}{2}$ . Assuming that the electronic spins are completely polarized, we can consider two main cases: (a)  $D_{\uparrow 1}$ , in which only the lowest nuclear Zeeman state is appreciably populated. In this case the (nuclear) spin of the Cooper pair is 2 (the total spin is 3!) and, bearing in mind the Fermi statistics, we see that the orbital angular momentum must be odd. For a dilute system the energetically favoured pairing state is a p-state, but it is likely that this will occur only at unattainably low temperatures for the reason given above. (b)  $D_{\uparrow 3}$ , in which all three nuclear Zeeman states are (nearly) equally populated. Depending on the density and field this may be the equilibrium state or possibly a long-lived metastable state. In this case the favoured pairing is with  $\ell = 0$  but nuclear spin 1 - a unique case. Since the system is likely to be very dilute, we can use the formula (1) and substitute the experimental value of the  $^3\Sigma_u^+$  scattering length for deuterium,  $-3.7 \text{ \AA}$ . This gives approximately

$$T_c \sim 100n^{2/3} \exp - 1/(6n^{1/3}) \quad (2)$$

where the number density  $n$  is measured in  $\text{\AA}^{-3}$ . (It is necessary, here, to remember that the relation

between  $k_F$  and  $n$  is modified from the familiar one because of the triple spin degeneracy). Thus, for example, for  $n = 10^{19} \text{ cm}^{-3}$ , the critical temperature is unobservably low ( $< 10^{-8} \text{ K}$ ) but for  $n = 10^{21} \text{ cm}^{-3}$  it would be already of the order of  $1^0 \text{ K}$ .

Let me now turn to the second topic of this talk: in what ways are Cooper pairs like and unlike diatomic molecules which have suffered Bose condensation? There are a number of obvious qualitative differences: in all known Fermi superfluids, the pair "radius" is very much larger than the mean spacing between particles, whereas the naive concept of a diatomic molecule would seem to imply the opposite assumption; the standard BCS theory /2/ of Cooper pairing invokes heavily the degeneracy of the Fermi sea, whereas for diatomic molecules this plays no role; and, in the anisotropic case, the excitation spectrum of the paired system is generally anisotropic /3/, whereas for a diatomic molecule it is isotropic whatever the angular momentum state. Nevertheless I believe that there may be some sense in which it is legitimate to view diatomic molecules and Cooper pairs as the two ends of a continuous spectrum of possible behaviour of a Fermi system with attractive interactions. To investigate this point, let us consider the following model system (for a more detailed account of this model and the calculations based on it, see ref. /4/.) We imagine a system of  $N$  fermions of spin  $\frac{1}{2}$  contained in unit volume, and with (for the moment) no spin polarization and no external magnetic field. The potential between the fermions has a core which is fairly strongly repulsive, plus a weakly attractive tail which effectively cuts off at some characteristic radius  $r_0$  (which might in practice be a few  $\text{\AA}$ ); the overall potential is repulsive (i.e.  $\int V(\mathbf{r}) d\mathbf{r} > 0$ ). However, the details of the potential are such that it is either just enough, or not quite enough, to bind two particles

in free space into a diatomic molecule. In either case the s-wave scattering length  $a_s$  is very much larger than the characteristic range  $r_0$  of the potential. ( $a_s > 0$  for a bound state,  $< 0$  if the state is not quite bound). If the two-particle state is bound, its energy is given approximately by  $E = -\hbar^2/m a_s^2$ . We now assume that  $N$  is such that the mean spacing between particles, say  $l$ , is large compared to  $r_0$ ; however, we make no assumptions about the ratio  $l/a_s$  and moreover imagine that by varying the details of the potential we can vary this quantity continuously from positive to negative values. The virtue of this model is that, when suitably scaled, most of the properties of the system should be insensitive to the details of the potential and functions only of the single dimensionless variable  $l/a_s$ .

We can now write down an ansatz /1/ for the wave function of the  $N$ -body system which reduces to the description of a set of noninteracting, Bose-condensed diatomic molecules in the limit  $l/a_s \rightarrow +\infty$  and to that of a Cooper-paired system in the opposite limit  $l/a_s \rightarrow -\infty$ . It is the following:

$$\begin{aligned} \bar{\Psi}(r_1\sigma_1, r_2\sigma_2, \dots, r_N\sigma_N) = \\ A\psi(r_1\sigma_1 r_2\sigma_2) \varphi(r_3\sigma_3 r_4\sigma_4) \dots \dots \\ \varphi(r_{N-1}\sigma_{N-1} r_N\sigma_N) \end{aligned} \quad (3)$$

where  $A$  is an antisymmetrization operator. Whether or not the wave function (3) is a reasonable approximation to the true ground-state wave function of the system, it is of some interest to study how the transition between the two limits takes place.

Using our knowledge of the correct form in these limits, we assume that the "molecular" wave function is of the form

$$\varphi(\underline{r}_1 \underline{r}_2 \sigma_1 \sigma_2) = \frac{1}{\sqrt{2}} (\uparrow\uparrow - \downarrow\downarrow) \varphi(|\underline{r}_1 - \underline{r}_2|) \quad (4)$$

in an obvious notation, i.e. it corresponds to a spin singlet,  $l = 0$  state, with the centre of mass at rest.

To do any useful calculations with the wave function (3) it is necessary to use the standard BCS trick of relaxing particle number conservation and minimizing, instead of  $H$ , the quantity  $H - \mu N$  where  $\mu$  is the chemical potential. (For the subsequent steps, see e.g. ref. /5/). If we then introduce the Fourier transform,  $c_k$ , of the function and define the complex quantities  $u_k, v_k$  such that

$$|u_k|^2 + |v_k|^2 = 1, \quad c_k = v_k/u_k, \quad (5)$$

then it turns out that the function (3) is just the  $N$ -particle projection of the particle-nonconserving BCS-type function

$$\begin{aligned} \bar{\Psi}_{\text{BCS}} &= \prod_k \phi_k \\ \bar{\phi}_k &= u_k |0,0\rangle + v_k |1,1\rangle \\ &\equiv (u_k + v_k a_{k\uparrow}^+ a_{-k\downarrow}^+) \text{vacuum} \end{aligned} \quad (6)$$

where the function  $\phi_k$  is a state vector in the "occupation" space associated with the pair of plane-wave states  $(k\uparrow, -k\downarrow)$ . This space is four-dimensional and is spanned by the basis vectors  $|0,0\rangle, |1,1\rangle, |0,1\rangle$  and  $|1,0\rangle$ , where for example  $|1,0\rangle$  labels the state in which the plane-wave state  $(k\uparrow)$  is occupied and the state  $(-k\downarrow)$  is empty. The linear combination  $\bar{\phi}_k$  (eqn.(6)) is the groundstate within this space; the excited states are the two "broken-pair" states  $|1,0\rangle$  and  $|0,1\rangle$  and the "excited-pair" state  $v_k^* |0,0\rangle - u_k^* |1,1\rangle$ .

A many-body wave function of the form (3) (with  $\varphi$  given by the singlet, s-wave form (4)) is completely parametrized by the set of quantities

$$F_k \equiv u_k^* v_k \quad (7)$$

It is the Fourier transform of this quantity,  $F(\underline{r})$ , (rather than  $(\underline{r})$ ) which plays the role of a wave function for the relative motion of the Cooper pairs.

Indeed the expectation value of any two-particle operator of the general form  $A = \frac{1}{2} \sum_{ij} A(\underline{r}_i - \underline{r}_j)$  (e.g. the potential energy) is given, apart from Hartree-Fock-type terms which are of no great inte-

rest in the present context, by the expression

$$\langle A \rangle = \int A(\underline{r}) |F(\underline{r})|^2 d\underline{r} \quad (8)$$

which may be compared with the corresponding expression for an isolated diatomic molecule ( $F(\underline{r}) \rightarrow \psi(\underline{r})$ ).

When  $F(\underline{r})$  has its equilibrium value, the energies of the excited states of the pair ( $\underline{k}\uparrow, -\underline{k}\downarrow$ ) are given by  $E_{\underline{k}}$  (broken pair) and  $2E_{\underline{k}}$  (excited pair) where  $E_{\underline{k}}$  and the associated quantity  $\Delta_{\underline{k}}$  are implicitly defined by the equations

$$F_{\underline{k}} = \Delta_{\underline{k}}/2E_{\underline{k}} \quad (8a)$$

$$E_{\underline{k}} = ((\epsilon_{\underline{k}} - \mu)^2 + |\Delta_{\underline{k}}|^2)^{\frac{1}{2}} \quad (8b)$$

$$(\epsilon_{\underline{k}} \equiv \hbar^2 \underline{k}^2 / 2m)$$

All the above statements are quite generally true once we assume the ansatz (3) for the many-body wave function, irrespective of whether or not we are in the usual "Cooper-pair" limit. However, it should be strongly emphasized that once we are outside this limit we can no longer assume that the chemical potential  $\mu$  is simply the free Fermi energy  $\epsilon_F$ ; it must, in fact, be determined self-consistently from the equation  $\sum_{\underline{k}} n_{\underline{k}} = N$ , using the fact that the number of particles  $n_{\underline{k}}$  in the plane-wave state  $\underline{k}$  (with either spin) is given (in the groundstate (6)) by

$$n_{\underline{k}} = 2|v_{\underline{k}}|^2 = 1 - \frac{\epsilon_{\underline{k}} - \mu}{E_{\underline{k}}} \quad (9)$$

Now for the equation determining  $F(\underline{r})$ ; we first note for orientation that the familiar Schrödinger equation for a diatomic molecule can be written after Fourier transformation in the form

$$(2\epsilon_{\underline{k}} - E_0) \psi_{\underline{k}} + \sum_{\underline{k}'} V(\underline{k} - \underline{k}') \psi_{\underline{k}'} = 0 \quad (10)$$

It turns out that the quantity  $F_{\underline{k}}$  obeys the equation (the familiar BCS gap equation lightly disguised)

$$2E_{\underline{k}} F_{\underline{k}} + \sum_{\underline{k}'} V(\underline{k} - \underline{k}') F_{\underline{k}'} = 0 \quad (11a)$$

$$E_{\underline{k}} \equiv [(\epsilon_{\underline{k}} - \mu)^2 + |\Delta_{\underline{k}}|^2]^{\frac{1}{2}} \quad (11b)$$

This is a nonlinear equation because, by (3) and (11a),  $\Delta_{\underline{k}}$  depends on  $F_{\underline{k}}$  itself by the relation

$$\Delta_{\underline{k}} = - \sum_{\underline{k}'} V(\underline{k} - \underline{k}') F_{\underline{k}'} \quad (12)$$

Now, it is fairly obvious by inspection of eqns.(11) and (9) and the use of some simple renormalization tricks /4/ that both the chemical potential  $\mu$  and the quantity  $\Delta_{\underline{k}}$  are at most of order of magnitude of the free Fermi energy  $\hbar^2 k_F^2 / 2m$  or the quantity  $\hbar^2 / 2ma_S^2$ . If we now consider values of  $k$  of the order of  $r_0^{-1}$ , then by our initial hypothesis ( $k_F r_0 \sim r_0 / \ell \ll 1$ ,  $r_0 / a_S \ll 1$ ) we find that  $E_{\underline{k}}$  is very much larger than either of these two energies and is hence large compared to  $\mu$  and  $|\Delta_{\underline{k}}|$ , and also to  $E_0 \sim \hbar^2 / 2ma_S^2$ . Under these conditions the BCS gap equation (11a) simply reduces to the Schrödinger equation (10), so we find the important result that the short-range behaviour of the pair wave function  $F(\underline{r})$  is exactly the same as that of the wave function of an isolated diatomic molecule. This result is probably qualitatively valid for cases more general than the simple model considered here (cf.ref. /3/).

Let us now consider the solution of equations (11a) and (9) in the two limiting cases. Quite generally it turns out that for  $k \ll r_0$  the quantity  $\Delta_{\underline{k}}$  tends to a constant,  $\Delta$ . In the case  $\ell / a_S \rightarrow +\infty$  (two-particle state bound, very dilute system) we find that  $\Delta \rightarrow 0$ ,  $\mu \rightarrow -\hbar^2 / 2ma_S^2$  (half the binding energy of the molecule) and the BCS equation reduces to the Schrödinger equation for all  $\underline{k}$ . Thus in this limit our wave function simply describes a Bose condensation of noninteracting diatomic molecules, as indeed we should expect a priori. In this limit the pair radius (which is just  $a_S$ ) is by hypothesis much less than the interparticle spacing.

In the opposite limit,  $\ell / a_S \rightarrow -\infty$  (dilute system with very weak attraction) we obtain the standard

BCS results:  $\mu$  tends to the free Fermi energy  $\epsilon_F \equiv \hbar^2 k_F^2 / 2m$ , while  $\Delta$  becomes exponentially small:

$$\Delta = \text{const. } \epsilon_F \exp - \pi / (2 k_F |a_S|) \quad (13)$$

We find that in this limit the pair "radius" is of order  $\hbar v_F / \Delta$  ( $v_F$  = Fermi velocity) and hence is very much greater than the interparticle spacing.

We can, of course, solve eqns.(11a) and (9) for quite general values of  $\ell/a_S$  and so study the transition between the two limits. Is there any point at which a qualitative change occurs? One might at first sight expect that such a point might occur at  $\ell/a_S = 0$ , which is the point at which the two-particle state in free space becomes bound; but in fact nothing special happens at this point. Indeed, the formal solutions to eqns.(11a) and (9) are quite continuous throughout the whole range. However, there is in fact a point at which at least the physical significance of some of the results changes, namely the point at which the chemical potential  $\mu$  passes through zero. To see this we go back to eqn.(8b) and note that  $E_{\mathbf{k}}$  is the (minimum) energy of excitation of the pair state ( $\mathbf{k}\uparrow, -\mathbf{k}\downarrow$ ). It follows that the minimum excitation energy of the system as a whole ("energy gap") is the minimum value of  $E_{\mathbf{k}}$  as  $\mathbf{k}$  varies. Now, for  $\mu > 0$  this minimum value (which always occurs in the region  $k \ll r_0^{-1}$ , where  $\Delta_{\mathbf{k}} = \Delta$ ) is just  $|\Delta|$  itself - hence the conventional name "energy gap" for the quantity  $\Delta$ , in BCS theory. On the other hand, for  $\mu < 0$  the energy gap is not  $|\Delta|$  but rather the quantity  $(|\mu|^2 + |\Delta|^2)^{1/2}$ . We would therefore expect some of the high-order thermodynamic derivations to have singularities at the point  $\mu = 0$ , and it is no doubt quite possible that the ansatz (3) breaks down completely in the neighbourhood of this point.

It should be added that the whole situation becomes a great deal more complicated at finite temperatures. In the BCS limit the temperature at

which pairs are formed is identical to the temperature at which they undergo Bose condensation. In the opposite limit of diatomic molecules, however, it is obvious that the molecules dissociate only around a temperature very much higher than that at which they Bose-condense. (Dissociation does not correspond to a phase transition in the usual sense).

It is possible to generalize the model to the case of p-wave pairing; for example, if we consider  $N$  fermions all with spin  $+\frac{1}{2}$  in unit volume, then it is clear that the orbital wave function of the pair must be odd, so that the energetically favoured pairing state is a p-state, and by a suitable choice of potential it is possible to arrange that the system be close to the onset of the two-particle bound state. The problems of renormalization of the potential, etc., are rather more complicated than in the s-wave case, but the general pattern of the results is similar; in particular eqns.(11) and (9) still apply. The "gap"  $\Delta_{\mathbf{k}}$  is now no longer constant in the region  $k r_0 \ll 1$ , but is of the general form  $\Delta_0 \mathbf{k} \cdot \mathbf{c}$ , where  $\mathbf{c}$  is a real or complex unit vector. It immediately follows that the excitation energy  $E_{\mathbf{k}}$  is anisotropic and for  $\mu > 0$  has nodes at the points where  $\mathbf{k} \cdot \mathbf{c} = 0$ . For  $\mu < 0$ , on the other hand, the energy gap is finite for all directions; for small departures from the diatomic-molecule limit the main effect of the many-body interactions is to give the excitations on anisotropic effective mass. The quantity  $F(\mathbf{r})$  is always anisotropic (with approximately p-wave symmetry) and reduces in the diatomic-molecule limit to the Schrödinger wave function of a molecule in a p-state, as we expect.

Finally, let me discuss briefly some of the more striking manifestations and consequences of Cooper pairing in a Fermi system. First there are phenomena associated with the centre-of-mass motion

of the pairs; most of these occur for any spin and angular momentum of the pairs. The most spectacular phenomenon is that of superfluidity (persistent currents, frictionless flow through "superleaks", and the associated phenomenon of anomalous rotational inertia); in addition such systems are expected to show anomalously low entropy, convective heat transfer and the phenomenon of second sound. In addition, if the spin of the pairs is nonzero, one would expect metastable "spin supercurrents" and if the orbital angular momentum is nonzero, "orbital supercurrents" associated with situations in which the orientation of the anisotropic wave function varies in space, and possibly a finite orbital angular momentum in equilibrium./6/.

A second class of striking effects occurs only in Cooper-paired systems where the pairs have  $S \neq 0$  and/or  $\ell \neq 0$ , and is associated with the internal structure of the pair wave function. Because the pair function picks out a particular orientation or set of orientations, one gets a variety of phenomena similar to those observed in liquid crystals, e.g. the occurrence of various types of topological singularities. In addition, even when the orientation is spatially uniform, many of the properties of the system will be anisotropic. Another property peculiar to the anisotropic case is the existence of various types of collective excitation corresponding to deformation of the internal structure of the pair wave function; some of these excitations may play a very important role in nuclear magnetic resonance or in the absorption of ultrasound.

But perhaps the most fascinating prospect opened up by the existence of new types of system with Cooper pairs formed in an anisotropic state is the possibility of amplification of ultra-weak effects, which is a direct consequence of the fact that Cooper pairs are by their very nature automatically Bose-condensed. Let me finish by illustrating this

phenomenon briefly with three examples from our only existing anisotropic superfluid, liquid  $^3\text{He}$  below 3 mK. (Of these three examples, the first is well established experimentally, the second may have been observed and the third is as yet speculative).

(1) The nuclear dipole-dipole interaction. For a gas of ordinary diatomic molecules in a relative p-state this would tend to orient the nuclear spins perpendicular to the direction of orbital angular momentum (two magnets have lower energy when they lie in the plane of relative motion). However, the associated energy advantage is at most of order  $10^{-7}\text{K}$ , which is tiny compared to the thermal energy at  $T \sim 1$  mK. Hence in an ordinary gas the nuclear dipole energy is a very small perturbation indeed. However, in superfluid  $^3\text{He}$  we are dealing not with ordinary diatomic molecules but with Cooper pairs, and the latter, being Bose-condensed, must all have the same relative motion as well as the same centre-of-mass motion. Hence the energy advantage gained by the "right" configuration is not  $\sim 10^{-7}\text{K}$ , but  $\sim 10^{-7}\text{K} \times N$ , the total number of pairs in the system. This is very large compared to  $kT$ , so the pairs do indeed orient their spins perpendicular to their orbital angular momentum.

(2) Electronic ferromagnetism. If a homopolar diatomic molecule rotates, it generates a small magnetic moment which is proportional to the extent to which the average position of the electrons on one of the atoms fails to coincide with that of the nucleus. (An intrinsically chemical effect). For a rare-gas dimer, this magnetic moment is extremely small, and in fact the energy of orientation in any attainable magnetic field is tiny compared to the thermal energy  $kT$ . Consequently an ordinary gas of rotating molecules would have the individual molecules oriented at random in even the strongest fields. In  $^3\text{He-A}$ , however, the rotating "molecules"

are Cooper pairs and therefore Bose-condensed, so they all have the same axis (and sense) of rotation, and the liquid therefore acquires a magnetic moment proportional to the total number of pairs, that is it behaves like a ferromagnet.

(3) Parity violation. If one is looking for the effects of the parity violation characteristic of the weak interaction, an obvious line is to search for an electric dipole moment, on an elementary particle, atom or molecule in a stationary state. By the Wigner-Eckart theorem such a dipole moment would have to lie along the total angular momentum vector  $\underline{J}$  :  $\underline{d} = c\underline{J}$ , and such a relationship between the polar vector  $\underline{d}$  and the axial vector  $\underline{J}$  would certainly require violation of parity conservation (P). Unfortunately it would also violate time-reversal invariance (T) and it is generally believed that the strength of that part of the weak interaction which violates P and T is only  $\sim 10^{-3}$  of that which violates P alone. However, suppose that an atomic or molecular system were characterized by two independent angular momentum vectors  $\underline{L}$  and  $\underline{S}$  (say, an orbital and spin angular momentum). Then we can form the hypothesis  $\underline{d} = c\underline{L} \times \underline{S}$ , and this violates P but not T. Now, calculation shows that any such dipole moment would have to be very weak indeed, so that even in the strongest possible electric fields its orientation energy could not compete with  $kT$ . So a gas of independent atoms or molecules having this characteristic would be completely disoriented (the vectors  $\underline{L}$ ,  $\underline{S}$  and  $\underline{L} \times \underline{S}$  would point in random directions) and no effect would be observable. Once again, however, Bose condensation makes an essential difference in  $^3\text{He-B}$ ; the Cooper pairs turn out to have a finite expectation value of the vector  $\underline{L} \times \underline{S}$ , and because of the Bose condensation the direction of this vector must be the same for all pairs. Consequently one predicts a total electric

dipole moment due to parity-violating effects which, although certainly very small, is macroscopic in the sense of being proportional to the total mass of liquid.

If the spin-polarized systems  $^3\text{He}\uparrow$  and  $\text{D}\uparrow$  do indeed become superfluid at attainable temperatures, one would expect a number of similar amplification effects. In particular, these effects which depend strongly on the "chemistry" of the Cooper pairs should in principle be much stronger in  $\text{D}\uparrow$  than in the much more chemically inert  $^3\text{He}$ . However, against this must be set the likely reduced density of the former system, as well as the fact that the mere existence of a strong electronic polarization may tend to mark more subtle orientational effects. Clearly, a great deal depends, here as elsewhere, on the maximum density at which spin-polarized systems can be stabilized.

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