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To cite this version:
G. Pelletier. RENORMALIZATION GROUP METHOD APPLIED TO LARGE SCALE LANGMUIR TURBULENCE. Journal de Physique Colloques, 1979, 40 (C7), pp.C7-657-C7-658. 10.1051/jphyscol:19797319. jpa-00219310

HAL Id: jpa-00219310
https://hal.science/jpa-00219310
Submitted on 1 Jan 1979

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RENORMALIZATION GROUP METHOD APPLIED TO LARGE SCALE LANGMUIR TURBULENCE

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In 1972, V.E. ZAKHAROV [1] proposed a new approach of Langmuir turbulence based on the two following equations:

\[ \frac{3}{\beta} \frac{3}{\beta} + \frac{3}{\beta} \omega e^2 D \Delta \) \frac{E_j}{\omega e} = \frac{3}{\beta} \frac{\delta n}{\omega e} E_j \]  

\[ \frac{2}{\beta} \frac{2}{\beta} - C_R \) \frac{\delta n}{\omega e} = C_s \Delta \frac{E_0}{4n_0 T_0} \]  

In spite of its compatibility to the small amplitude perturbations \( \omega \ll n_0 T \), this system exhibits strong non-linear effects, since the "Reynolds number", which can be defined as the ratio between the non-linear time scale and the linear one i.e. \( R \sim \frac{\omega}{k^2 T_0 n_0} \), reaches large value for \( k^2 \omega^2 < 1 \).

This system has been intensively studied for its dynamical properties and to describe the various ways by which a cold Langmuir gas becomes unstable.

A statistical theory of the large scale dynamics cannot be elaborated using conventional perturbation expansions, which diverges and has an "infrared catastrophe". F.Kh. CHAKHINOV and V.N. TSYTOVICH [2] proposed a first statistical theory of the Langmuir condensate, based on a double averaging technique. The "renormalization group" method, introduced by K. WILSON in 1972 [3] to describe critical phenomena was fruitfully applied in various fields of physics. Initially formulated for static problems, the method has been generalised to dynamical problems, and has been recently used with success for large scale hydrodynamic turbulence [4].

The purpose of this communication is to indicate how this new method is handled on a system close to the system (1)-(2), why it is convenient for solving the statistics of the Langmuir condensate, and what are the first few results.

We complete the two equations (1)-(2) by introducing Landau dampings and two sources of excitations : random forces in (1) and random inhomogeneities in (2). So we are dealing with the following system, written with convenient reduced Fourier variables:

\[ U_k = \lambda_1 \frac{k}{\omega} \frac{k}{\omega} \phi + \phi \]  

\[ \phi = \lambda_2 \frac{k}{\omega} \frac{k}{\omega} U_k + U_k \]  

\[ \phi = \lambda_2 \frac{k}{\omega} \frac{k}{\omega} + \psi \]  

\( G_1 \) is the linear Green function which propagates the Langmuir envelopes \( G_1 = e^{i\omega t} \frac{\omega^2}{\omega^2} - ik \frac{\omega^2}{\omega^2} + i\gamma_1(k) \) \( \omega \) describes thermal dispersion, \( \gamma_1 \) is the Landau damping. \( G_2 \) is the sound wave Green function : \( G_2 = k^2 \left( \omega^2 + 2i\gamma_2 \omega - ak^2 \right)^{-1} \). The symbol \( * \) represents the convolution of the Fourier components in \( k \) and \( \omega \). \( \lambda_1 \) and \( \lambda_2 \) are the coupling parameters. The excitations \( \phi \) and \( \psi \) have spectra which are supposed to be truncated at an ultraviolet cut-off \( k_0 \), close to \( k \). The dynamics are determined by the two Green functions, the two couplings and the 2 excitation correlation functions.

A transformation \( T_k \) of the renormalisation group modifies all these six functions in two steps (sometimes it can also generate new couplings). The first step consists in eliminating the small scale degrees of freedom, the wave numbers of which are in the interval \( [k_0/2, k_0] \). The influence of small scale fluctuations on large scale dynamics in then described in renormalizing the six functions mentioned above, by a technique similar to that developed in ref [5]. In particular the Green functions are modified by the self-energy corrections \( \Sigma_1 \) and \( \Sigma_2 \) in accordance with Dyson equations:

\[ G_1^{-1} = G_0^{-1} - \Sigma_1 \]  

\[ G_2^{-1} = G_2^{-1} + \Sigma_2 \]  

Anyway it can be shown that the coupling parameters \( \lambda_1 \) and \( \lambda_2 \) are also changed in \( \lambda_1', \lambda_2' \); a renormalization of the excitation correlations is also necessary. The second step consists in performing scaling transformations; changing \( k \) in \( k' = sk \), the cut-off is again at \( k_0 \). All the quantities are then changed with appropriate scaling factors which are powers of \( s \).
The large scale properties of the system are found after many iterations of the renormalization group transformation. Obviously these properties can be found if the six above mentioned functions acquire a fixed form, with only few evolving parameters converging towards a stable limit (the fixed point).

a) Modification of the Green functions

Starting with the usual linear Landau damping in $G_1$, the renormalization group generates a non-linear damping rate, which reveals the existence of a dissipation on the large wavelenght. There are two scattering processes which play a role: on the one hand large $k$ sound waves modulate small $k$ Langmuir waves, and these modulations are absorbed by the fast electrons ($1$-$s$-$e$ scattering); on the second hand, a modified non-linear Landau effect causes an energy transfer towards large $k$, contrary to its effect in weak turbulence. So the renormalized damping rate can be expressed:

$$
\gamma_1 = \gamma_0 + \nu k^2 + o(k^4)
$$

(7)

and $G_1$ is characterised by four evolving parameters: $\omega_0$, $\nu$, $\gamma_0$, $\nu$.

The usual perturbation theory would give a second order modification of $G_2$ which leads to the well-known oscillating-two-streams, self-modulation and decay instabilities. But the selective renormalization of large scale perturbations by small scales alone does not destabilise $G_2$; only the parameter $\nu$ is modified.

b) Modification of the excitation correlations.

It can be shown that, starting from any excitation correlations, bounded in the large scale, the renormalization group generates white noise excitations, characterised by their amplitude parameters $D_1$ and $D_2$:

$$
\langle \phi(k,\omega) \phi(k',\omega') \rangle = 2D_1(k/k') (2\pi)^d \delta(k + k') \delta(\omega + \omega')
$$

(8)

$$
\langle \phi(k,\omega) \phi(k',\omega') \rangle = 2D_2(k/k')(2\pi)^d \delta(k + k') \delta(\omega + \omega')
$$

(9)

$D$ is the dimension of space.

So, after several iterations, the renormalization group makes the excitation correlations evolve in simply changing $D_1$ and $D_2$. The universal properties of the large scale proceed from that result.

c) Modification of the couplings

The renormalization group transforms the couplings simply by changing the two parameters $\lambda_1$ and $\lambda_2$; these modifications take properly into account differences with gaussian statistics, due to the quadratic couplings.

We indicate few results under the assumption that the "relevant" dynamics are subsonic for the largest wave length. Then there are two possible situations. The simplest situation is obtained when $\gamma_0$ is relevant; it means physically that the $1$-$s$-$e$ scattering is the main absorption for small $k$. The evolution of the set of parameters towards a fixed point is controlled by the following quantities:

the "Reynolds number" $y_1 = \frac{\lambda_1}{\nu} \frac{D_1}{\gamma_0}$

(10)

the "Kubo number" $y_2 = \frac{\lambda_2}{\nu} \frac{D_2}{\gamma_0}$

(11)

These two numbers scale as $s^{-d}$; so for every positive dimension of space the couplings vanish. The dynamics are therefore asymptotically free (trivial fixed point); the infrared modes are gaussianly distributed, with a Rayleigh-Jeans spectrum.

Another situation is obtained when $\gamma_0$ vanishes, the dissipation being described essentially by $k^2$ in (7). The convenient "Reynolds number" is now:

$$
y_1 = \frac{\lambda_1}{\nu} \frac{D_1}{\gamma_0}
$$

(12)

and the "Kubo number"

$$
y_2 = \frac{\lambda_2}{\nu} \frac{D_2}{\gamma_0}
$$

(13)

But now, $y_1$ scales as $s^{1-d}$ and $y_2$ as $s^{2-d}$, and then the asymptotic freedom is obtained beyond the crossover dimension $d_\infty = 4$, anyway $y_2$ vanishes for $d > 2$. In 3 dimensions, couplings are relevant and produce non-gaussian fields; but the investigation of the "non-trivial" fixed point is possible owing to an expansion in $\varepsilon = 4 - d$. The detailed theory will be published later on. At the transition $\gamma_0 = 0$, the energy spectrum is like $k^{d+1}$ and the density fluctuations is white noise, in agreement with KHAKINOV and TSTIYOVICH predictions for the condensate.

References


