CATHODE DENSITY WAVE IN AN ELECTRON BEAM CONTROLLED DISCHARGE

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Introduction. The energy input in a discharge is non uniform. Transient and non uniform gas heating results in acoustic waves which propagate in and out of the discharge. If the waves yield a significant density defect, they can drive the discharge into an arc. Moderate amplitude waves can also disturb the optical quality of the medium when the discharge is used to pump a laser. Two types of perturbations can generally be identified:

- Waves propagating in a direction perpendicular to the electric field are generated by bulk heating [1, 2].
- Waves propagating along the electric field mainly originate from the electrode vicinity, especially the cathode fall region [3, 4].

This paper only deals with the cathode wave in a discharge where the cathode is the solid electrode. This choice is made in order to analyze a single density disturbance. Holographic interferometry is used to display the density variations. A simple model permits an estimate of energy partition between electrode and gas heating.

Experimental arrangement. A schematic diagram of the discharge set-up is given in Fig. 1. The electron beam is produced by a plasma-anode electron gun. The electric field is continuously applied between solid cathode and grid anode. Experimental conditions are:

- Gas : N$_2$ (chosen in order to minimize bulk heating during wave propagation).
- Pressure : 0.15 bar.
- Electron beam energy : 110 keV.
- Electron beam current density : 2 mA/cm$^2$.
- Beam pulse width (FWHM) 25 µs.
- Discharge voltage : 2.6 kV along a 36 mm gap.
- Cathode fall deduced from the slope break of the current voltage characteristic, $V_c \alpha 440$ V.
- Discharge current : 230 A.
- Dimensions of electron window : 140 x 60 mm.

The method of a reference hologram is used to obtain a real time image of the phenomena. A picture of the interferogram is taken at a given time with a fast TRW converter. [5].

Results. The cathode wave is clearly displayed on the interferogram given in Fig. 2. The picture was taken 50 µs after discharge initiation with an exposure time of 5 µs. The density defect close to the cathode cannot be seen with the space and time scales considered. The fringe shift of wave disturbance is less than half an interferinge. It can be noted that the main fringe pattern (at rest fringes are perpendicular to electrodes) is not changed by bulk heating of the medium and that fringes decline towards anode should be due to the propagation of the rarefaction wave resulting from the anode field-free cavity. The maximum relative variation of density $\Delta \rho/\rho_0$ (Fig. 1) is lower than 4 %. The wave speed (ratio of wave front abscissa over time) is 340 m/s.

Discussion. Those results suggest the following main assumptions:

- linear approximation.
- In addition, it is assumed that the gas is perfect and at rest. The calculation is performed with no heat transfer to the cathode material. The appraisal of this effect will result from the comparison of the calculation with the experiment. The linear equation set is:

$$R(T_0 \rho' + \rho_0 T') = \rho'$$

$$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial u'}{\partial x} = 0$$

$$\rho_0 \frac{\partial u'}{\partial t} + \frac{\partial \rho'}{\partial x} = 0$$

$$\rho_0 c_v \frac{\partial T'}{\partial t} + \rho_0 \frac{\partial u'}{\partial x} = \frac{\partial \rho}{\partial t}$$

where $T_0$, $\rho_0$, $\rho_0$ and $u_0 (=0)$ are the equilibrium values for temperature, density, pressure and velocity and the primes denote the perturbations; $R$ is the constant of perfect gases, $\Delta \rho_c$ is the heat input per volume and time unit in the cathode region. Initial conditions are ($t=0$):

$$\rho' = 0$$

$$\frac{\partial \rho'}{\partial t} = (\gamma - 1) \frac{\partial \rho}{\partial t}$$

in the cathode region and 0 elsewhere, where $\gamma$ is the specific heat ratio.
Boundary conditions are:

\[ \frac{\partial \rho'}{\partial x} = 0 \text{ at the cathode surface } (x = 0) \text{ and } \rho' \rightarrow 0 \text{ for } x \rightarrow \infty. \]

The propagation equation is solved with Laplace transform in time and associated Green function for space variable. Heat input is assumed to be uniform in the cathode region of length \( d \). Time dependence is approximated by the functions (Fig. 4)

\[ \dot{\rho}_c = \dot{\rho}_0 \frac{t}{t_m} \quad \text{for } 0 \leq t \leq t_m \]

\[ \dot{\rho}_c = \dot{\rho}_0 \exp \left(-\frac{t}{\tau_c}\right) \quad \text{for } t > t_m + \tau_p \]

Finally, the analytic formula for the relative density variation is:

\[ \frac{\rho'(t,x)}{\rho_0} = \left(1 - \frac{t}{\tau_m} \cdot \frac{1}{\rho_0 \rho_0 c_0} \right) \left(\exp \left(-\frac{t}{\tau_m} \cdot \frac{x}{c_0}\right) - 1 \right) \]

\[ - \frac{1}{c_0 \tau_m} \left[ c_0 (t - \tau_m - \tau_p) - x \right] + \frac{1}{c_0 \tau_m} \left[ c_0 (t - \tau_m) - x \right] + U \left[ c_0 (t - \tau_m) - x \right] \]

where \( U \) is the unit step function and \( \rho_0 \) is the density at \( t = 0 \).

This function is displayed in Fig. 3 for \( \rho_0 = 670 \text{ W/cm}^2 \), this value being chosen in order to fit the experimental maximum of \( \rho'/\rho_0 \). The corresponding electric energy input (\( V_c \) times current density) is \( 1 \text{ 200 W/cm}^2 \) (Fig. 4). Accordingly, the electrical energy converted in gas heating is about 1/2. Most of the remaining energy is transferred into the cathode. This energy partition limit significantly the actual gas temperature increase close to the cathode.

**Conclusion.** A simple model permits an appraisal of energy partition in the cathode vicinity. Two experimental data (electrical energy input and \( \rho'/\rho_0 \)) are used to determine heat transfer into the cathode. Conversely, an appropriate model for this heat transfer would yield \( \dot{\rho}_0 \) directly from a single measurement of \( \rho'/\rho_0 \). More sophisticated work is in progress. In addition, the success in dealing with moderate amplitude perturbations (at short time scale and short exposure time) gives us confidence to study other problems, such as those involved in pulse repetition, with this interferometer.

**REFERENCES**


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**Fig. 1:** Schematic of discharge arrangement.

**Fig. 2:** Interferogram taken 50 \( \mu s \) after discharge initiation — Exposure time 5 \( \mu s \).

**Fig. 3:** Relative density versus distance from the cathode at \( t = 50 \mu s \) — experiment —— approximation

**Fig. 4:** Electrical power input per \( \text{cm}^2 \) in the cathode region — experiment —— approximation

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