



HAL
open science

REVIEW ON RECENT WORKS ON VORTICES IN SUPERCONDUCTORS

E. Brandt

► **To cite this version:**

E. Brandt. REVIEW ON RECENT WORKS ON VORTICES IN SUPERCONDUCTORS. Journal de Physique Colloques, 1978, 39 (C6), pp.C6-1426-C6-1432. 10.1051/jphyscol:19786583 . jpa-00218075

HAL Id: jpa-00218075

<https://hal.science/jpa-00218075>

Submitted on 4 Feb 2008

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

REVIEW ON RECENT WORKS ON VORTICES IN SUPERCONDUCTORS

E.H. Brandt

Ames Laboratory - USDOE and Dept. of Physics, Iowa State University, Ames, Iowa 50011, U.S.A.*

Résumé.- On passe en revue : a) les solutions numériques de la théorie de Gorkov pour le réseau idéal de vortex dans le domaine entier de champ et de température, et b) les solutions de la théorie de Ginzburg-Landau pour le réseau déformé et leurs applications à l'ancrage des vortex.

Abstract.- We review : a) the numerical solutions of Gorkov's theory for the ideal vortex lattice in the entire field-temperature plane, and b) the distorted-lattice solutions of the Ginzburg-Landau theory and their application to flux pinning.

1. THE IDEAL VORTEX LATTICE.- Following Gorkov's extension of the microscopic BCS theory of superconductivity /1/ the properties of the ideal flux line lattice (FLL) initially were calculated for about 15 years only in the special cases $t = T/T_c \approx 1$ (Ginzburg-Landau (GL) and Neumann-Tewordt approximations) and $b = B/B_{c2} \approx 1$ (linearized Gorkov theory). Solutions for the entire temperature-induction range have been obtained only recently using two numerical methods (I and II), which complement each other. This is visualized in figure 1 and in table I.

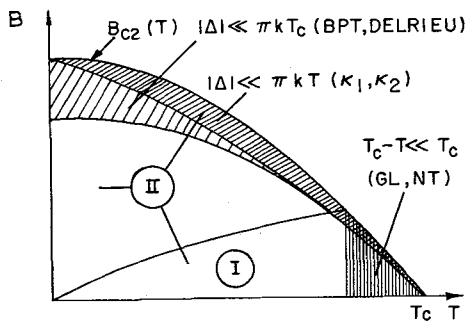


Fig. 1 : Range of validity in the temperature-induction-plane of various solution methods for Gorkov's equations

Method I, developed mainly by L. Kramer and W. Pesch, uses Eilenberger's energy-integrated Green functions of the electrons /2/. This version of Gorkov's theory applies to arbitrary electron mean free path, ℓ . It is suitable for calculations of the thermodynamic /3,4,5/ and transport /6/ properties and the density of states /5,7/ of type-II

superconductors. Since this method requires a large numerical effort /8/ it has been applied so far only to rotationally symmetric solutions. i.e. to the isolated vortex or, within a circular cell method, to the FLL with sufficiently large lattice spacing, $d \gg \text{Min}(\hbar v_F / \pi kT, \ell)$.

Method II, developed by the author, starts from Gorkov's free energy expression for clean superconductors ($\ell = \infty$) and extends the BPT-approach /9-12/ to arbitrary induction. The Green functions are eliminated analytically using the spatial periodicity of $|\Delta|$ (energy gap) and H (magnetic field). The resulting free energy is an explicit functional of $|\Delta|$ and H and may be minimized by a Ritz variational method. We use a finite Fourier series with 5 independent coefficients as a trial function for H, and various periodic trial functions for $|\Delta|^2$. This method yields Δ , H, and the magnetization curves with good accuracy even on a small computer.

We summarize some of the results :

The magnetic field of an isolated vortex at low temperatures exhibits a rather narrow and high peak at the vortex center. Such a sharp peak, degenerating even to a cusp at $T = 0$, was first calculated by Delrieu /13,14/ for a clean superconductor near H_{c2} . He found that the Fourier coefficients of H decrease slowly, as $|\underline{K}|^{-3}$ where \underline{K} is a reciprocal FLL vector, and to have alternating signs. The saddle points and the minima of H interchange their positions at $t = 0.62$ for the clean superconductor. At $t = 0$, H is zig-zag-shaped along the nearest neighbor directions. Method II confirmed this surprising behavior in the large induction range $0.5 \lesssim b < 1$. At $b \approx 0.5$ the sharp minima of H almost touch the plane $H = 0$ and then flatten out. A further interesting result is that some of the Fourier

* On leave from Max-Planck-Institut für Metallforschung Institut für Physik, Stuttgart, Germany

Table I

	METHOD I	METHOD II
STARTS FROM	ENERGY-INTEGRATED GREEN FUNCTIONS	FREE ENERGY FUNCTIONAL FOR PERIODIC $ \Delta $ AND H
APPLICABLE TO	ALL MEAN FREE PATHS, STATIC AND TRANSPORT PROPERTIES	ONLY PURE MATERIALS (?) ONLY STATIC PROPERTIES (?)
SOLUTION METHOD	SOLVES SYSTEM OF DIFF. EQS.	RITZ VARIATIONAL METHOD
COMPUTATIONAL EFFORT	LARGE	MODEST
FORM OF SOLUTIONS	CIRCULAR CELL	PERIODIC TRIAL FUNCTIONS
INDUCTION RANGE	SMALL INDUCTIONS	ARBITRARY INDUCTION
<u>METHOD YIELDS :</u>		
DENSITY OF STATES	YES	NO
TRANSPORT COEFFICIENTS	YES	NO
MAGNETIZATION CURVES	YES	YES
INDUCTION JUMP B_0	NO	YES
LATTICE SYMMETRY	NO	YES
SHEAR MODULUS	NO	YES
Δ AND H NEAR VORTEX CORE	YES	YES
Δ AND H NEAR CELL BOUNDARY	NO	YES

coefficients of H are negative at $b = 1$ but become positive as b is lowered. Their vanishing at a certain value of b should be observable by neutron diffraction.

The gap parameter is found by both methods to exhibit a narrowing of the vortex core at very low t . This effect is most pronounced at small b and was investigated in more detail in /15,16/, where it was found that in a clean superconductor at $t = 0$ the slope $d|\Delta|/dr$ becomes infinite at the vortex core. For the spatial average of $|\Delta|^2$ method II gives the simple relationship $\langle |\Delta|^2 \rangle / \Delta_{BCS}^2 = 1 - b \pm 0.05$ for all values of b , t , and for all GL-parameters κ .

In the magnetization curves a first-order phase transition occurs at the penetration field H_{c1}^1 in low- κ clean materials. This is consistent with experiments. The curves $B(H_a)$ (H_a is the applied field) obtained by method II and shown in figure 2 have a slightly S-shaped, almost vertical part from which H_{c1}^1 and the induction jump B_0 are determined by a Maxwell construction or by equating the area under the curve to $\mu_0 H_c^2$. A small change in the curves $B(H_a)$ can, therefore, result in a large change of B_0 , even if H_{c1}^1 is almost unchanged. A quantitative agreement with experimental values for B_0 , which have been measured with high accuracy by decoration, magnetic and neutron scattering experiments /17/, cannot be expected, mainly since the calculation is based on the simple BCS-model-inter-

action between electrons on a spherical Fermi surface and thus neglects "real metal effects".

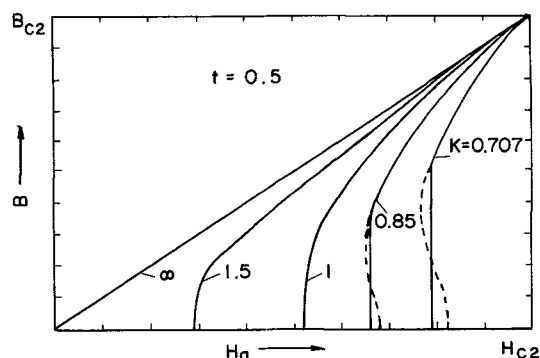


Fig. 2 : Magnetization curves of a type-II superconductor at $t = 0.5$ for various values of κ . The dashed lines belong to metastable states

Finally we remark on cubic FLLs and on "FL attraction", an interpretation of the first order transition at H_{c1}^1 . These features have been of great interest, since they are clearly seen in some experiments on low- κ material, but could be explained by GL theory. Several authors tried to derive Fl attraction analytically from the Gorkov equations and, at the same time, explain its origin. The numerical methods yield the attraction but do not give a simple explanation for it. The analytical calculations showed that a field reversal of the iso-

lated FL is not necessary for FL-attraction. It also would give too large a value for the equilibrium distance if the neighboring FL is assumed to sit in the field minimum. It is rather an additional potential term which compensates the magnetic repulsion and leads to a net attraction of FLs. This term, which may be called the "gap parameter", "core", or "condensation energy" attraction, is present even in the GL theory. For $\kappa > 1/2^{1/2}$ it is always smaller than the repulsive term, but at $\kappa = 1/2^{1/2}$ it exactly compensates the magnetic repulsion such that all FL-configurations have equal free energy. For κ close to $1/2^{1/2}$ any small correction to the GL free energy could possibly lead to a domination of the attractive term.

A similar statement holds for the symmetry of the FLL: the GL-theory and method II yield a stable triangular FLL for all values of b , κ , t . However, the difference in the free energies of the cubic and hexagonal symmetries is extremely small. Any small additional energy term could, therefore, suffice to stabilize the cubic lattice. Such a term, for example, may arise from coupling between the FLL and an anisotropic crystal lattice, but, as observed recently, a cubic FLL may occur also in a hexagonal matrix /18/.

2. ON A RIGOROUS TREATMENT OF FLUX PINNING.- Very recently some progress has been made towards a rigorous treatment of flux pinning at imperfections of the crystal lattice. As pointed out in various review articles /19-24/ there is still a discrepancy between experimental values of the volume pinning force P_V obtained, for example, from flux-density gradients or from critical currents, and theoretical values obtained by a statistical summation of elementary pinning forces estimated from GL theory. This discrepancy may amount to several orders of magnitude if P_V itself is compared and not, as is often done, just its square root. A particularly troublesome problem is the theoretically predicted but experimentally unobserved threshold for the elementary pinning forces, below which summation theories yield $P_V = 0$. Furthermore, the observed dependence of P_V on magnetic history is not explained by existing summation theories /21/. These challenging problems are still unsolved, and a rigorous treatment starting from GL theory seems to be required. The work reviewed here is one step in this direction.

Up to the present, nearly all theoretical work on flux pinning used the concept of individual flux lines (FLs) interacting with each other and with the pinning centers by a two-body interaction. This picture is very useful. It can be derived from GL theory for sufficiently low induction. The interaction between FLs turns out to be composed of a magnetic repulsion and a condensation-energy attraction, which is most important for small κ /25,26/. Unfortunately, the interacting-FL picture breaks down as soon as the vortex cores begin to overlap. This happens already at rather low inductions $b \sim 0.3$ as can be seen from figure 3 in reference /26/. In contrast to widespread opinion, this breakdown also occurs in superconductors with large κ . The proper range of validity for the London model is, therefore, $b \ll 1$ and not $\kappa \gg 1$ (if the generalized interaction potential is used).

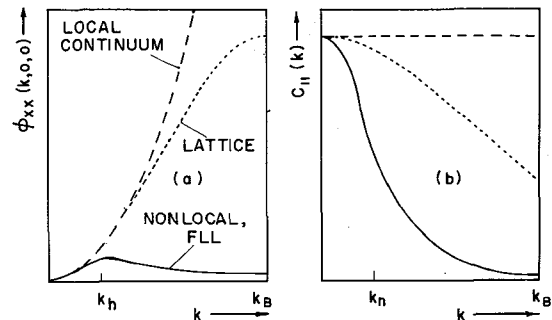


Fig. 3 : a) The elastic matrix component $\tilde{\phi}_{xx}(k,0,0)$.
 b) The compressional modulus $c_{11}(k) = n\tilde{\phi}_{xx}(k,0,0)/k^2$. Arbitrary units. Dashed lines : local continuum, dotted lines : lattice with short range forces, solid lines : FLL with $k_B/k_h = 4$ corresponding to $\kappa = 4$, $b = 0.4$, or $\kappa = 0.9$, $b = 0.75$

A rigorous treatment of the pinning problem /27-31/ should start from a free energy functional of the form /28/

$$F = F_0 + F_1 + F_d \quad (1)$$

where F_0 describes the homogeneous material and in the following shall be chosen as the GL functional; F_1 is the perturbation caused by inhomogeneity, and F_d introduces a driving force causes, e.g., by a transport current. In what follows we omit all surface phenomena by considering an infinite FLL, and we assume the perturbations F_1 and F_d to be small. In principle the solutions Ψ (GL function) and \underline{A} (vector potential) could be obtained by standard perturbation methods using an expansion in terms of some eigenfunctions. We do not follow this path but

rather use a perturbation method which retains, and thereby generalizes, the useful concepts of "FL positions" (defined as the zeros of Ψ and denoted by \underline{r}_ν where ν is a FL index, or by $\underline{r}_\nu(z)$ for bent FLs, where z is a line parameter for which we choose the coordinate z along H_a) and "Force densities on FLs" (denoted by \underline{P}_ν or by $P_\nu(z)$, a force per cm). Both vectors $\underline{r}_\nu(z)$ and $\underline{P}_\nu(z)$ have only two independent components since we deal with lines.

The main trick of our method is the introduction of the "distorted-lattice solutions" Ψ_s and \underline{A}_s . They are defined as the solutions which minimize the unperturbed functional F_0 with the constraint that Ψ_s have first order zeros along the FL positions $\underline{r}_\nu(z)$. After insertion of Ψ_s and \underline{A}_s the free energy functional (1) reduces merely to a function of all FL positions \underline{r}_ν (or to a functional of the $\underline{r}_\nu(z)$ if the FLs are bent). The minimization of F with respect to the FL-positions is equivalent to solving the force balance equation (virtual displacements !)

$$\frac{dF_0}{d\underline{r}_\nu(z)} + \frac{dF_1}{d\underline{r}_\nu(z)} + \frac{dF_d}{d\underline{r}_\nu(z)} = -\underline{P}_\nu^{el}(z) - \underline{P}_\nu^{pin}(z) - \underline{P}_\nu^{driv}(z) = 0 \quad (2)$$

for all values of ν and z . Here the first term reproduces the normal definition of the elastic force, the second term defines the pinning force, and the last term is the driving force per unit length of each FL element.

3.-THE DISTORTED-LATTICE SOLUTIONS.- Until recently, solutions of the GL equations for a distorted FLL were known only for low inductions, where $|\Psi|$ is essentially constant, \underline{H} is a linear superposition of isolated vortex fields, and the free energy is a sum over the two-body interactions between FLs (or FL-elements if bending is allowed). At larger inductions, only periodic solutions were known. Periodic solutions, as a matter of principle, do not tell anything about the inhomogeneously distorted lattice. Furthermore, they cannot give the forces on each FL in the FLL, but these are required in order to calculate the strain caused by a pinning center. The force on one FL has to be calculated from the free energy of a FLL in which the given FL is slightly displaced.

One, therefore, needs distorted-lattice solutions which apply to the entire induction range

$0 < b < 1$. We find these by the following method /28/: first, we calculate the correction to the periodic solutions Ψ_0 and \underline{A}_0 linear in the displacements $\underline{s}_\nu(z) = \underline{r}_\nu(z) - \underline{R}_\nu$ from the regular positions $\underline{R}_\nu = (X_\nu; Y_\nu; z)$. For convenience we do this for periodic displacement fields, $\underline{s}_\nu(z) = \text{Re}\{\underline{s}_0 \exp(i\underline{k}\underline{R}_\nu)\}$; the generalization to non-periodic displacements is by linear superposition. A divergence at $k/(1-b) \rightarrow 0$ is removed by means of the nonlinear terms in the GL equations. In the results for Ψ , \underline{A} and F appear two characteristic wave vectors,

$$k_\Psi^2 = (1-b)2/\xi^2 = k_B^2(1-b)/b, \quad (3)$$

$$k_h^2 = \langle |\Psi|^2 \rangle / \lambda^2 \approx k_\Psi^2 / 2\kappa^2, \quad (4)$$

where ξ is the coherence length, $\lambda = \kappa\xi$, and $k_B = (2b)^{1/2}/\xi$ is the radius of the circularized Brillouin zone (BZ). The results can be transcribed into expressions which apply even to large displacements if the strain is small, e.g.,

$$|\Psi(\underline{r})| = |\Psi_0(\underline{r})| \left[1 + \frac{b}{2\xi^2} \sum_{\underline{K}} \text{Re}\left\{ \frac{i\underline{s}_0(\underline{k}+\underline{K})}{k_\Psi^2 + (\underline{k}+\underline{K})^2} e^{i(\underline{k}+\underline{K})\underline{r}} \right\} + 0(\underline{s}_0^2) \right] \quad (5)$$

becomes :

$$|\Psi(\underline{r})| = |\Psi_0(\underline{r}-\underline{s}(\underline{r}))| \left(1 + \frac{b}{2\xi^2} \frac{\text{div } \underline{s}(\underline{r})}{k_\Psi^2 + k^2} \right) + 0(k^2 \underline{s}_0^2). \quad (6)$$

where $\underline{s}(\underline{r}) = \text{Re}\{\underline{s}_0 \exp(i\underline{k}\underline{r})\}$ and $k = |\underline{k}| < 0.7 k_B$.

As a next step, the corresponding expressions are calculated for the limit $b \ll 1$. They turn out to be very similar to the results for $b \approx 1$ if appropriate representations are chosen. The high induction results approximately apply, therefore, to the entire induction range $0 < b < 1$. One can quite generally state that extensions to low b of results derived at high b give ordinarily much better approximations than extensions of the London-model to larger b .

Looking carefully at the small-strain results, one finds that they can be reproduced from the following more transparent expressions, which we assume to approximately apply to arbitrary distortions (if b , $\langle |\Psi|^2 \rangle$, k_Ψ , and k_h are allowed to vary spatially) and to arbitrary b and κ (ϕ is the phase of Ψ):

$$|\Psi(\underline{r})|^2 = \langle |\Psi|^2 \rangle \cdot \exp\left\{ \frac{b}{1-b} \right\} \prod_{\nu} \exp\left\{ - \int d\underline{z}_\nu \frac{\exp\left[-|\underline{r}-\underline{r}_\nu(\underline{z}_\nu)| k_\Psi \right]}{|\underline{r}-\underline{r}_\nu(\underline{z}_\nu)|} \right\}, \quad (7)$$

$$\text{grad } \phi(\underline{r}) = \text{rot } \int_V d\underline{r}' \frac{1}{2|\underline{r}-\underline{r}'|}, \quad (8)$$

$$\underline{H}(\underline{r}) = \int_V d\underline{r}' \frac{\phi_0 k_h^2 \exp\{-[(\underline{r}-\underline{r}')^2 + \xi^2/2]^{1/2}\}}{\mu_0 4\pi [(\underline{r}-\underline{r}')^2 + \xi^2/2]^{1/2} C}, \quad (9)$$

where $C \lesssim 1$. Equations (7) to (9) reduce to the correct solutions in all special cases investigated so far, (7) and (8) exhibit the correct singularities along the FLs, and (9) satisfies $\text{div } \underline{H} = 0$. These expressions should be useful trial functions in numerical calculations going beyond linear elasticity theory.

4. THE ELASTIC RESPONSE OF THE FLL.- The elastic energy of the strained FLL has the general form

$$F_{el} = \frac{1}{2} \int_{BZ} \frac{dk_x dk_y}{4\pi^2 n} \int \frac{dk_z}{2\pi} \{ \tilde{\phi}_{xx} |\tilde{s}_x|^2 + \tilde{\phi}_{xy} (\tilde{s}_x \tilde{s}_y^* + \tilde{s}_x^* \tilde{s}_y) + \tilde{\phi}_{yy} |\tilde{s}_y|^2 \}, \quad (10)$$

where $n = B/\phi_0$, $\tilde{\phi}_{\alpha\beta}(\underline{k})$ is the elastic matrix of the FLL,

$$\tilde{\underline{s}}(\underline{k}) = \int_V dz \exp\{i(k_x X_V + k_y Y_V + k_z z)\} \underline{s}_V(z) \quad (11)$$

$= (\tilde{s}_x; \tilde{s}_y; 0)$ is the Fourier transform of the displacement field, $\tilde{\phi}_{\alpha\beta}(\underline{k})$ and $\tilde{\underline{s}}(\underline{k})$ are periodic in the k_x - k_y -plane, and the integration is over the first BZ of the FLL. For the FLL with hexagonal symmetry one has within local elasticity theory

$$\tilde{\phi}_{\alpha\beta}(\underline{k}) = \frac{1}{n} \begin{cases} c_{11} k_x^2 + c_{66} k_y^2 + c_{44} k_z^2; & (c_{11} - c_{66}) k_x k_y \\ (c_{11} - c_{66}) k_x k_y; & c_{66} k_x^2 + c_{11} k_y^2 + c_{44} k_z^2 \end{cases} \quad (12)$$

where c_{11} is the uniaxial compressional modulus, c_{66} the shear modulus, and c_{44} the tilt modulus of the FLL.

Equation (12) applies to a FL continuum. For a discrete lattice with nearest neighbor interactions, the parabolae k_x^2 and k_y^2 are replaced by periodic functions of the type $[(2/d)\sin(kd/2)]^2$, and the local approximation (12) still applies if $k \ll k_B \approx \pi/d$, i.e., if the strain varies slowly over the lattice spacing d . For the FLL with moderate or large b or κ the interaction is of long range, and the picture changes completely as shown in figure 3. The correct GL-result for $\tilde{\phi}_{\alpha\beta}$ - a simple sum (for $b \ll 1$) or a double sum (for $b \gtrsim 1$) over reciprocal FLL vectors - may be approximated for $k < 0.7 k_B$ by an expression similar to equation (12) but with c_{11}

and c_{44} replaced by functions of $k = |\underline{k}|$:

$$c_{11}(k) = B^2 \frac{dH_a}{dB} \frac{1}{(1 + k^2/k_h^2)(1 + k^2/k_\psi^2)} \quad (13)$$

$$c_{44}(k) = \frac{B^2}{\mu_0} \frac{1}{1 + k^2/k_h^2} + B (H_a - B/\mu_0) \quad (14)$$

and with unchanged shear modulus $c_{66} \approx B^2(dH_a/dB)(1-b)^2/10bk^2$. Equations (13) and (14) and the modified equation (12) apply to arbitrary b and κ and are called the "continuum" or "isotropic" approximation.

Our results show that for the FLL local elasticity theory applies only if $k \ll k_h$, i.e. if the strain varies slowly over the length $\lambda/(1-b)^{1/2}$, which may be much larger than $d = 2.7 \lambda/\kappa b^{1/2}$. As k goes from 0 to k_B , $c_{11}(k)$ and $c_{44}(k)$ decrease by factors $k_h^2 k_\psi^2/k_B^2 = (1-b)^2/2b^2 \kappa^2$ and $2k_h^2 k_B^2 = (1-b)/bk^2$, respectively. The correct lattice sums for $\tilde{\phi}_{\alpha\beta}(\underline{k})$ yield an even larger decrease of the FLL stiffness and also exhibit a pronounced anisotropy close to the BZ boundary.

The compliance $1/\tilde{\phi}_{xx}(k,0,0)$ is sharply peaked at the BZ boundary. As a consequence, planar pinning forces yield a weakly damped oscillatory displacement field, with amplitude usually much larger than the constant strain superimposed to it /30/. This surprising result follows even from the London model. It means that, close to a planar pinning center, neighboring FLL planes are shifted in opposite direction.

The dispersion of the compressional and the tilt moduli has a large effect on the maximum value of the elastic FL displacements caused by local pinning forces. This is shown in table II for planar forces, line forces parallel and perpendicular to the FLs, and for point forces. The results of the previous (S_L) and the present (S_{NL}) theories may differ by several orders of magnitude; they coincide only for line forces parallel (more precisely: at an angle less than $(c_{66}/c_{44})^{1/2} \approx (1-b)/3\kappa$) to the FLs.

The linear elastic behavior of a lattice of straight FLs can be reproduced to good accuracy by the induction-dependent potential

$$V(r) = \frac{\phi_0 k_h^2}{\mu_0 2\pi} [K_0(k_h r) - K_0(k_\psi r)] \quad (15)$$

between FLs, where $r = |\underline{r}_V - \underline{r}_\mu|$ and K_0 is the Hankel function. This means that for small strain the FLL behaves as if the FLs interact by the two-body potential (15) composed of a magnetic repulsion of range $\lambda/(1-b)^{1/2}/32/$ and a "core attraction" of

Table II

TYPE OF PINNING FORCE	MAXIMUM FL DISPLACEMENT		CORRECTION FACTOR S_{NL}/S_L		
	S_L	S_{NL}	GENERAL EXPRESSION	EXAMPLE $b = 0.8$ $\kappa = 1.5$	$\kappa = 30$
PLANAR FORCE PAIR $\delta(x-D) - \delta(x+D)$	$\sim \frac{1}{c_{11}} \sim \frac{1}{b^2}$	$\sim \frac{\kappa^2}{(1-b)^2}$	$1 + \frac{d}{D} \kappa^2 \left(\frac{b}{1-b}\right)^2$	100	40000
LINE FORCE PARALLEL $\delta(x) \delta(y)$	$\sim \frac{1}{c_{66}} \sim \frac{\kappa^2}{b(1-b)^2}$	$\sim \frac{\kappa^2}{b(1-b)^2}$	1	1	1
LINE FORCE PERPENDICULAR $\delta(x) \delta(z)$	$\sim \frac{1}{\sqrt{c_{11} c_{44}}} \sim \frac{1}{b^2}$	$\sim \frac{\kappa^2}{\sqrt{b} (1-b)^{3/2}}$	$1 + \frac{2\kappa^2}{3 \ln(D/d)} \left(\frac{b}{1-b}\right)^{3/2}$	10	4000
POINT FORCE $\delta(x) \delta(y) \delta(z)$	$\sim \frac{1}{\sqrt{c_{66} c_{44}}} \sim \frac{\kappa}{b^{3/2} (1-b)}$	$\sim \frac{\kappa^2}{b(1-b)^{3/2}}$	$1 + \frac{\kappa}{\sqrt{2}} \left(\frac{b}{1-b}\right)^{1/2}$	3	50

range $0.7 \xi/(1-b)^{1/2}$. This potential could be used to calculate, at least approximately, some nonlinear elastic properties of the FLL.

5. PINNING FORCES.- As an example, we illustrate the pinning forces derived from the above theory for the case of a point pinning center with diameter $\lesssim \xi$ and with position $\underline{a} = (a_x; a_y; a_z)$. A general expression for the perturbation energy of such a defect is

$$F_1 = \alpha_0 |\Psi|^2 + \frac{1}{2} \beta_0 |\Psi|^4 + \gamma_0 \left| \left(\frac{\nabla}{i\kappa} - \underline{A} \right) \Psi \right|^2 + \underline{\mu}_0 \underline{H} + \frac{1}{2} \chi_0 \underline{H}^2 \quad (16)$$

where $\Psi = \Psi(\underline{a})$, $\underline{A} = \underline{A}(\underline{a})$, and $\alpha_0, \beta_0, \gamma_0, \underline{\mu}_0, \chi_0$ are constants. For small inductions we find (a) that $\underline{P}_V(z)$ depends only on the distance $|\underline{r}_V(z) - \underline{a}|$ between FL element and defect and (b) that all types of pinning forces can be derived from potentials which depend on this distance. These properties were expected and have been used in summation theories. At larger inductions $b > 0.3$ the character of pinning forces changes: The $\underline{P}_V(z)$ in general depend on $\underline{r}_V(z)$ and on \underline{a} separately. The forces exhibit different behavior depending on the interaction type. This is shown in figure 4: Magnetic forces ($\underline{\mu}_0 \neq 0$ or $\chi_0 \neq 0$) still decrease monotonically with $|\underline{r}_V(z) - \underline{a}|$ and still derive from a potential. But defects coupling to Ψ ($\alpha_0 \neq 0, \beta_0 \neq 0, \text{ or } \gamma_0 \neq 0$) are oscillatory functions of $\underline{r}_V(z)$ and do not derive from a potential. In particular, the entire force field vanishes identically when the pinning center coincides with the

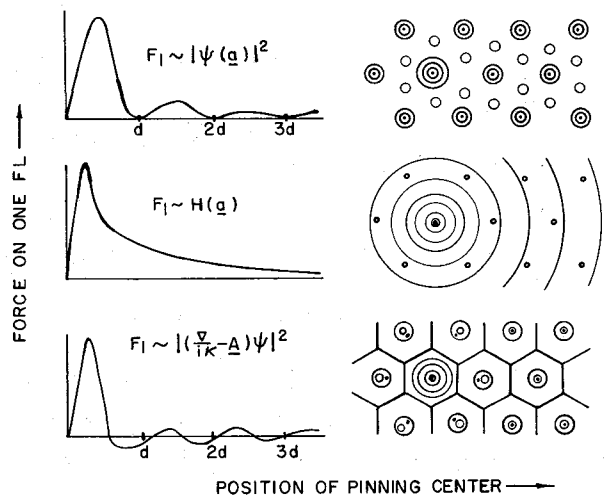


Fig. 4 : The force exerted by a point pinning center situated at $\underline{r} = \underline{a}$ on the FL at $\underline{r}_V = 0$, plotted for three different types of interaction, equation (18). L.h.s. : The x-component of the force plotted versus a_x for $a_y = 0$
R.h.s. : Lines of equal force in the \underline{a} -plane

axis of any FL. This means that a pinning center located at a FL center does not exert a force on any FL if it couples to Ψ , but it repels or attracts all FLs (except the one it is sitting at) if it couples to \underline{H} . This result is due to the fact that the general expression for $|\Psi|$ is a product (7) of factors each of which has a zero along one FL, whereas \underline{H} is the sum (9) of modified isolated vortex fields which decrease monotonically.

These results show that our concept of pinning forces, and of the elastic response of the FLL

to them, has to be modified at medium and large inductions.

This work was supported by the U.S. Department of Energy, Office of Basic Energy Sciences, Materials Sciences Division and by the Deutsche Forschungsgemeinschaft.

References

- /1/ Gorkov, L.P., Sov. Phys. JETP 7 (1958) 505
- /2/ Eilenberger, S., Z. Phys. 214 (1968) 195
- /3/ Kramer, L., and Pesch, W., J. Low Temp. Phys. 14 (1974) 29
- /4/ Pesch, W., and Kramer, L., J. Low Temp. Phys. 15 (1974) 367
- /5/ Watts-Tobin, R., Kramer, L., Pesch, W., J. Low Temp. Phys. 17 (1974) 71
- /6/ Pesch, W., Watts-Tobin, R., Kramer, L., Z. Phys. 269 (1974) 253
- /7/ Kramer, L., Pesch, W., Watts-Tobin, R., Solid State Commun., 14 (1974) 1251
- /8/ Kramer, L., Pesch, W., von Seggern, R., Berichte der KFA Jülich n° 1149, Zentralbibliothek der KFA, Jülich, Germany, 1974
- /9/ Brandt, U., Pesch, W., Tewordt, L., Z. Phys. 201 (1967) 209
- /10/ Brandt, E.H., Phys. Status Solidi (b) 64 (1974) 257
- /11/ Brandt, E.H., J. Low Temp. Phys. 24 (1976) 409 and 427
- /12/ Brandt, E.H., Phys. Status Solidi (b) 77 (1976) 105
- /13/ Delrieu, J.M., J. Low Temp. Phys. 6 (1972) 197
- /14/ Delrieu, J.M., Thesis, Université Paris-Sud (1974)
- /15/ Kramer, L., and Pesch, W., Z. Phys. 269 (1974) 59
- /16/ Kramer, L., in Low Temp. Physics-LT 14, vol. 2, ed. by M. Krusius and M. Vuorio (North Holland, Amsterdam) 1975, p. 281
- /17/ Christen, D.K., Kercher, H.R., Sekula, S.T., Thorel, P., to be submitted to Phys. Rev.
- /18/ Obst, B., and Brandt, E.H., Phys. Lett. 64A (1978) 460
- /19/ Campbell, A.M., Evetts, T.E., Critical Currents in Superconductors (Taylor and Francis, London) 1972
- /20/ Haasen, P., and Freyhardt, H.C., ed., Int. Discussion Meeting on Flux Pinning in Superconductors (Goltze, Göttingen) 1975
- /21/ Küpfer, H., and Gey, W., Philos. Mag. 36 (1977) 859
- /22/ Kramer, E.J., J. Appl. Phys. 49 (1978) 742
- /23/ Kramer, E.J., J. Nucl. Mater. (1978)
- /24/ Campbell, A.M., Philos. Mag. (1978)
- /25/ Kramer, L., Phys. Rev. 118 (1971) 3821
- /26/ Brandt, E.H., Phys. Status Solidi (b) 51 (1972) 345
- /27/ Brandt, E.H., J. Low Temp. Phys. 26 (1977) 709, 735
- /28/ Brandt, E.H., J. Low Temp. Phys. 28 (1977) 263, 291
- /29/ Schmucker, R., and Brandt, E.H., Phys. Status Solidi (b) 79 (1977) 479
- /30/ Brandt, E.H., Phys. Status Solidi (b) 84 (1977) 269, 237
- /31/ Brandt, E.H., to be published in Philos. Mag.
- /32/ Clem, J.R., in Low Temp. Physics - LT 14, Vol. 2, ed. by M. Krusius and M. Vuorio, (North Holland, Amsterdam) 1975, p. 285

POST DEADLINE PAPERS ACCEPTED FOR CONTRIBUTION

- Study of superconducting Al₅ compounds of the Nb-Ge-Si system prepared by CVD .
S. Païdassi
- Superconductivity of disordered palladium .
B. Stritzker
- Superconductivity in a new high pressure phase of scandium .
J. Wittig, C. Probst, F.A. Schmidt, K.A. Gschneider
- Theoretical calculation of T_c for lead .
A.D. Zdetsis, D.A. Papaconstantopoulos, E.N. Economou
- Influence of effective phonon spectrum on T_c of superconductors .
Li Hung-Cheng
- Theory of superconducting critical temperature .
Wu Hang-Sheng, Chi Kuang-Ta
- Non equilibrium superconductivity in electromagnetic radiation field .
V.M. Dmitriev, E.V. Khristenko
- Cross-like film structures . Microwave irradiation stimulated superconductivity and Josephson effect .
Yu.G. Bevza, V.I. Karamushko, I.M. Dmitrenko
- Enhancement of the energy gap in superconducting Aluminum by tunneling extraction of quasiparticles .
C.C. Chi, J. Clarke
- Analytical solution of resistively shunted Josephson junctions .
Yao Shih-Sheng
- The effect of the magnetic field on the height of microwave-induced step of a Josephson tunneling junction .
Wu Hang-Sheng, Chi Kuang-Ta, Liu Fu-Sui
- AC Josephson effect in Nb₃Sn thin film bridges .
A.I. Golovashkin, B.G. Zhurkin, A.N. Lykoy, V.I. Novikov
- Flux flow measurements in irreversible type II superconductors .
J.M. Lumley, J.E. Evetts, C.H. Chao
- Anisotropic transition from type I to type II superconductivity in Tantalum .
H.W. Weber, J.F. Sporna, E. Seidl