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FLUXON-BREATHER-PLASMA OSCILLATION DECAY IN LONG JOSEPHSON JUNCTIONS

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Résumé.- Les solutions exactes de l'équation de sine-Gordon, qui décrivent les oscillations sur une jonction Josephson longue et uni-dimensionnelle avec conditions au bord de circuit ouvert, suggèrent que les tourbillons qui ralentissent jusqu'à une vitesse critique de propagation se transforment en "breathers", qui à leur tour, lorsque leur amplitude diminue jusqu'à la valeur critique, se transforment en oscillations du plasma.

Abstract.- Exact solutions of the sine-Gordon equation describing oscillations on a long, one-dimensional Josephson junction with open circuit boundary conditions suggest that fluxons that slow to a critical propagation velocity decay into breathers, which in turn, when their amplitude diminishes to a critical value, decay into plasma oscillations.

The solutions of the sine-Gordon equation with finite boundary conditions recently reported by Costabile et al. /1/, which furnish exact analytical expressions for the three fundamental types of oscillations on long Josephson junctions, viz., fluxons, breathers, and plasma oscillations, suggest the existence of excitation/decay mechanisms between these modes of oscillation. The analysis of such mechanisms is of considerable importance in the study of applications of the Josephson junction as a generator of high frequency radiation/2/ and as an element for digital computation/3/.

Neglecting dissipative effects, a long, one-dimensional Josephson junction is described by the equation $\phi_{xx} - \phi_{tt} = \sin \phi$ (1) where ϕ is the magnetic flux normalized respect to $h/4\pi e$, x is distance normalized with respect to the Josephson penetration length λ_J , and t is time normalized with respect to the inverse of the Josephson plasma frequency ω_J . The boundary conditions $\phi_x(0,t) = 0 = \phi_x(L,t)$ (2) correspond to imposing open-circuit terminations at the two ends of such a junction having normalized length L . Physically, such boundary conditions can be closely approximated by using an "overlap" geometry /4/. Fulton /5/, in a series of remarkably detailed observations on a mechanical analog, has recently reviewed the qualitative nature of the solutions of (1) under the boundary conditions (2). We repeat here, for convenience, the analytic solutions obtained by Costabile, et al. /1/.

1) Plasma oscillation (see figure 20 of Fulton /5/) :

$$\phi = 4 \tan^{-1} [A \operatorname{cn}(\beta x; k_f) \operatorname{cn}(\Omega t; k_g)] \quad (3)$$

where

$$k_f^2 = \frac{A^2 [\beta^2 (1+A^2) + 1]}{\beta^2 (1+A^2)^2}; k_g^2 = \frac{A^2 [\Omega^2 (1+A^2) - 1]}{\Omega^2 (1+A^2)^2} \quad (4a,b)$$

and Ω , β and A are related by the nonlinear dispersion equation

$$\Omega^2 - \beta^2 = \frac{1-A^2}{1+A^2} \quad (5)$$

Imposition of the boundary conditions (2) fixes the spatial periodicity as

$$\beta_n = \frac{2n}{L} K(k_f) \quad (6)$$

where $n = 1, 2, \dots$ is the number of nodes in the standing wave and $K(k)$ is the complete elliptic integral of the first kind. For $n = 0$, the entire length of the line oscillates in phase as

$$\phi = 4 \tan^{-1} [A \operatorname{sn}(\Omega t; k)]; k = A^2 \text{ and } \Omega = 1/(1+A^2) \quad (7)$$

2) Breather oscillation (see figure 28 of Fulton /5/) :

$$\phi = 4 \tan^{-1} \{A \operatorname{dn}[\beta(x-x_0); k_f] \operatorname{sn}(\Omega t; k_g)\} \quad (8)$$

where

$$k_f^2 = 1 - \frac{1-\beta^2(1+A^2)/A^2}{\beta^2(1+A^2)}; k_g^2 = \frac{A^2 [1-\Omega^2(1+A^2)]}{\Omega^2(1+A^2)} \quad (9a,b)$$

and the nonlinear dispersion equation is

$$\beta = \Omega A. \quad (10)$$

The boundary conditions (2) now require

$$\beta_n = \frac{n}{L} K(k_f) \quad (11)$$

with two possible values for x_0 : a) $\beta x_0 = K(k_f)$, and b) $x_0 = 0$. For n even, a) corresponds to breathers located near the center of the line, and b) corresponds to fluxons bound to virtual antfluxons at both ends of the line. For n odd, a) and b) are equivalent.

3) Fluxon oscillation (see figures 25 and 26 of Fulton /5/ :

$$\phi = 4 \tan^{-1} \left[A \operatorname{dn}(\beta x; k_f) \operatorname{tn}(\Omega t; k_g) \right] \quad (12)$$

where
$$\frac{\beta^2(A^2-1)-1}{\beta^2(A^2-1)} \quad (13a, b)$$

$$k_f^2 = 1 - \frac{A^2}{\beta^2(A^2-1)} ; k_g^2 = 1 - \frac{A^2[\Omega^2(A-1)-1]}{\Omega^2(A-1)}$$

The nonlinear dispersion equation is again as in (10) and the boundary conditions (2) again require (11).

The existence of excitation/decay mechanisms between these fundamental modes of oscillation is suggested by the following observations.

1) Breather-plasma oscillation excitation/decay :

Setting $\beta < A/(1+A^2)$ in (8) and (9) yields solutions that map onto those with $\beta > A/(1+A^2)$. Setting $\beta = A/(1+A^2)$ in (9) yields $k_f^2 = 0$ and $k_g^2 = A^4$. Recalling that $\operatorname{dn}(X;0) = 1$, and assuming $A < 1$, this substitution reduces (8) to (7). Setting $\beta = A/(1+A^2)$ in (11) and recalling that $K(0) = \pi/2$ thus yields a minimum value for A in (8) as

$$A_{\min} = \frac{L}{n\pi} - \left[\left(\frac{L}{n\pi} \right)^2 - 1 \right]^{1/2} \quad (14)$$

The $n = 0$ plasma oscillation, on the other hand, can exist for all A in the range $0 < A < 1$. These facts suggest that for $A < A_{\min}$, a breather must necessarily decay into an $n = 0$ plasma oscillation provided that, from (14), $L/n \geq \pi$, and that an $n = 0$ plasma oscillation with $A \geq A_{\min}$ can excite a breather.

2) Fluxon-breather excitation/decay :

Setting $\beta > A^2/A^2-1$ in (12) and (13) yields solutions that map onto those with $\beta < A^2/A^2-1$. Thus, setting $\beta = A^2/A^2-1$ in (11) and (13a) yields a maximum value for A in (12) as

$$\frac{A_{\max, f}^2}{A_{\max, f}^2 - 1} = \frac{n}{L} K(\{1 - (1/A_{\max, f}^4)\}^{1/2}) \quad (15)$$

Setting $\beta = A/(A^2-1)^{1/2}$ in (13) yields $k_f^2 = 1$ and $k_g^2 = 1$. Recalling that $\operatorname{dn}(X;1) = \operatorname{sech}(X)$ and $\operatorname{tn}(X;1) = \sinh(X)$, this substitution, with the identification $A \equiv l/u$, where u is the velocity in the center of mass reference frame, reduces (12) to the form of the fluxon-antifluxon collision on the infinite line reported by Scott, et al. /6/. Recalling that $K(1) \rightarrow \infty$, these results imply that solutions of the type (12) can be found on the finite line for all A such that solutions of the type (12) can be found on the finite line for all A such that $1 < A \leq A_{\max, f}$.

Setting $\beta < A^2/(1+A^2)$ in (8) and (9) yields solutions that map onto those with $\beta > A^2/(1+A^2)$. Setting $\beta = A^2/(1+A^2)$ in (11) and (9a) and assuming $A > 1$ thus yields a maximum value for A in (8) as

$$\frac{A_{\max, b}^2}{A_{\max, b}^2 + 1} = \frac{n}{L} K(\{1 - (1/A_{\max, b}^4)\}^{1/2}) \quad (16)$$

Breathers, accordingly, can exist for $A_{\min} \leq A \leq A_{\max, b}$.

From (15) and (16), $A_{\max, b} < A_{\max, f}$; however, for $L/n \gg 1$, the two maxima tend toward equality. Recalling that $\operatorname{sn}(iX; k) = i \operatorname{tn}(X; k')$, the transformation $A \rightarrow -iA$, $\Omega \rightarrow i\Omega$, $\beta \rightarrow \beta$, which preserves (10), transforms (8) and (9) into (12) and (13). Recalling that a breather represents a bound state of a fluxon and an antifluxon, these facts suggest that a fluxon that slows to near the critical propagation velocity ($A \rightarrow A_{\max, f}$) while encountering an antifluxon can decay into a breather, and that a breather that increases in amplitude to near $A_{\max, b}$ can excite a fluxon-antifluxon pair.

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