II. THÉORIE / THEORY $\gamma\gamma \rightarrow$ HADRONS:
ASYMPTOTIC BEHAVIOR AND DEEP INELASTIC SCATTERING
T. Walsh

To cite this version:
T. Walsh. II. THÉORIE / THEORY $\gamma\gamma \rightarrow$ HADRONS: ASYMPTOTIC BEHAVIOR AND DEEP INELASTIC SCATTERING. Journal de Physique Colloques, 1974, 35 (C2), pp.C2-77-C2-85. 10.1051/jphyscol:1974211 . jpa-00215521

HAL Id: jpa-00215521
https://hal.science/jpa-00215521
Submitted on 1 Jan 1974

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
\[ \gamma \gamma \rightarrow \text{HADRONS} : \text{ASYMPTOTIC BEHAVIOR AND DEEP INELASTIC SCATTERING} \]

T. F. WALSH

Theory Group, Deutsches Elektronen Synchrotron, Hamburg, Germany

Résumé. — Nous divisons notre discussion sur les collisions photon-photon en trois rubriques :

a) Deux photons réels ou presque réels (le processus est alors dominé par le caractère « hadronique » de ces photons). b) Un photon réel ou presque réel, et l’autre photon très virtuel (diffusion profondément inélastique sur un photon réel). c) Deux photons très virtuels (dans ce cas, la réaction présente une certaine similitude avec le processus \[ e^+ e^- \rightarrow \gamma^* \rightarrow \text{hadrons} \]). Divers modèles (en particulier celui de Regge et celui des partons) sont considérés.

Abstract. — We discuss photon-photon collisions under three rubrics: a) Two real or nearly real photons (the process is then dominated by the « hadronlike » character of these photons). b) One real or nearly real photon and one far off-shell photon (deep inelastic scattering on a real photon). c) Two far off-shell photons (the reaction then has some similarity with \[ e^+ e^- \rightarrow \gamma^* \rightarrow \text{hadrons} \]). Various models (in particular Regge and partons) are considered.

1. Introduction. — One can ask whether the physics of photon-photon annihilation into hadrons is of really fundamental interest. An experimental program to study this process will require considerable effort and ingenuity. Can we expect to gain physical insight commensurate with the effort expended? I think that the answer is yes, particularly where the asymptotic behavior in energy or photon mass is concerned. This is a current reaction with two variable masses, and it is becoming clear that current-induced processes have a great simplicity and importance as contrasted to pure hadron reactions. One may hope to learn something about the common constituents of the currents and the hadrons, or at least about the algebraic and dynamical structure the constituents leave behind before their exit from theoretical considerations.

It is convenient to discuss photon-photon reactions under three rubrics: two real or nearly real photons \( q_1^2 \sim -m_e^2 \) one real photon and one far off-shell photon, and two far off-shell photons. The first is dominated by the « hadronlike » character of a \( q^2 \approx 0 \) photon: \[ \gamma \gamma \rightarrow \text{hadrons} \] is like \( \rho^0 \rho^0 \rightarrow \text{hadrons} \). The second is at least in part like deep inelastic scattering with the target a real photon (mostly a vector meson). The last reaction has — in the proper kinematic region — a similarity to \[ e^+ e^- \rightarrow \gamma^* \rightarrow \text{hadrons} \]. In all these cases one can consider inclusive, semi-inclusive or exclusive reactions — for example in \[ \gamma^* + \gamma^* \rightarrow \pi^+ \pi^- \] (1) one can study the Compton amplitude, and not just its absorptive part. It is this possibility of continuously moving from one kind of physics to another in one reaction that provides the interest and the challenge where \( \gamma \gamma \rightarrow \text{hadrons} \) is concerned.

This talk is concerned with the asymptotic behavior in one or more of the three variables of the process, \( q_1^2, q_2^2, s \). See figure 1. In presenting a picture of this behavior, one has to lean on a model or models. Most of what I have to say is based on the parton model, but only in the very general sense embodied in the idea of a field theory with soft off-shell behavior or a transverse momentum cutoff. This is in most cases a physical realization of the algebra of currents on the light cone, and is of reasonable generality.

\( \gamma^* + \gamma^* \rightarrow \pi^+ \pi^- \) (1)

(1) We denote by \( \gamma^* \) photons with \( q^2 \) possibly far from zero.

1. FIG. 1. — The forward photon-photon amplitude.

The picture to be presented may be partly false, but it is simple and testable. It can be altered if this is made necessary by experimental developments.

A work about the literature. There is a lot of it. For an extensive bibliography, we refer the reader to the review by Terazawa [1], and an earlier one...
by Brodsky [2]. I will refer to work relevant to the material of the talk and probably incompletely at that.

2. Two real photons. — This case has, according to current prejudice, the least model dependence, so we discuss it first. The reader may feel, with justice, that we also do this so as to get the subject out of the way.

A simple estimate for the total $\gamma\gamma$ cross section follows by using the vector-dominance and quark models. Namely, we put [3]

$$\sigma_{\gamma\gamma}^{\text{tot}} \approx \left( \alpha \right)^2 \sigma_{\rho \rho}^{\text{tot}} + \text{isoscalar pieces}$$

$$\approx \left( \alpha \right)^2 \left( 1 + \frac{1}{3} \right)^2 \sigma_{\rho \rho}^{\text{tot}} \approx 0.25 \mu \text{b}$$

where

$$\sigma_{\rho \rho}^{\text{tot}} = \sigma_{\rho \rho}^{\text{tot}} = \frac{3}{2} \left( \sigma_{s}^{\text{tot}} + \sigma_{s}^{\text{tot}} \right)$$

via the additive quark model. This should work at high energy, and one can ask about the size of non-leading terms. In the same way one can get an estimate of about $0.015 \mu \text{b}$ for $\sigma_{\gamma\gamma \rho \rho}$.

$\gamma\gamma$ scattering is convenient in that the non-leading $(f - A_2)$ $t$-channel exchanges correspond to resonances which can be produced in the $s$-channel (e.g. $\gamma\gamma \to f^0 \to \pi^+ \pi^-$). We can thus relate the radiative width of these mesons to the size of the non-leading Regge term. Doing this very crudely by integrating a Regge term over a broad $s$-range and equating the result to the integral over a narrow $f$-meson, we get a non-leading term of the same order as the leading one (namely),

$$\sigma_{\gamma\gamma}(s) - \sigma_{\gamma\gamma}^{\text{tot}}(\infty) \sim 0.3 \mu \text{b}.s^{-1/2}$$

where $s$ is in GeV²) for radiative widths

$$\Gamma(f^0 \to \gamma\gamma) \sim 2 \text{ keV}$$

This agrees roughly with the size of non-leading trajectory contributions to, e.g., $\sigma_{\gamma\gamma}$; see figure 2.

An amusing aside is possible here. Consider the helicity structure of

$$\gamma_{L}(q_1) + \gamma_{L}(q_2) \to \gamma_{L}(q_1) + \gamma_{L}(q_2)$$

there are three forward amplitudes for two real photons ($W_{\gamma\gamma}^{+}$): $W_{\gamma\gamma}^{++}$, $W_{\gamma\gamma}^{+-}$ and $W_{\gamma\gamma}^{-+}$ [5] (2).

The first two are proportional to cross sections for photons of total $J_c$ along the collision axis $|J_c| = 0.2$ to go to hadrons. The last is an asymmetry. Let's consider factorization of the $t$-channel exchanges. Then up to terms with $x_R \approx 0$,

$$W_{\gamma\gamma}^{++} = W_{\gamma\gamma}^{+-} = \sum_R [\beta_R^2 \left( \frac{s}{\lambda_0} \right)^{2R}]$$

$$W_{\gamma\gamma}^{-+} \approx \text{constant}$$

i.e., $W_{\gamma\gamma}^{-+}/W_{\gamma\gamma}^{+}$ vanishes at least as fast as $1/s$. These are consequences of factorization (2a) and a result of Fox and Leader (2b) [6], [7]. Now suppose that we try to use local duality between $f$-exchange in the $t$-channel and the $f$-meson resonance in the $s$-channel. That is, we apply all available relations from both $t$-channel and $s$-channel to the $f^0$ $\gamma\gamma$ couplings. We will ignore the $A_2$ in all this. For a narrow $s$-channel resonance,

$$W_{\gamma\gamma}^{++} = n_R W_{\gamma\gamma}^{++}$$

where $n_R$ is the normality of the resonance $(n_f = +1)$. This follows from parity and $s$-channel factorization at a resonance. Putting all this together, we get the result that the $f$-trajectory decouples from $\gamma\gamma$ !

The culprit here is local duality: the $f$-meson can be produced in some definite helicity state and duality considerations need not apply locally to the unpolarized amplitude. For example, some calculations [8] give the result that the $s$-channel contribution with $J_c = 0$ ($W_{\gamma\gamma}^{++}$ or $W_{\gamma\gamma}^{-+}$) is small at $s \approx m_T^2$ relative to $W_{\gamma\gamma}^{-+}$ — i.e. the $f^0$ is produced with $|J_c| = 2$. One can check this by looking at the $f^0 \to \pi^+ \pi^-$ decay distribution [8]. A nice test of Regge ideas can be carried out by looking to see if $W_{\gamma\gamma}^{++} \ll W_{\gamma\gamma}^{++}$.

The foregoing estimate for $\sigma_{\gamma\gamma}^{\text{tot}}$ at high energy assumes that the Pomeron couples in $\gamma\gamma$ scattering. There is a well-known argument that it should not in $\gamma N \to \gamma N$ at $t = 0$ [9]. Since this is experimentally not the case, we are probably safe in assuming that no such decoupling occurs in $\gamma\gamma \to \gamma\gamma$ at $t = 0$.

Little is known about Regge cuts, so it is perhaps best to go on the assumption that we are dealing with factorizable poles. There is, however, a rule due to R. Worden which can indicate when certain Regge cuts approximately decouple [10]. Unfortunately,
this indicates that the cuts at $x_R \approx 0$ ($J/f$, etc.) reinforce one another in $\gamma N \to \gamma N$ and $\gamma\gamma \to \gamma\gamma$. This may make difficult the unambiguous test of ideas which involve singularities with $x_R \approx 0$.

The semi-inclusive distributions in $\gamma\gamma \to h + \text{anything}$ should look pretty much like $\gamma + p \to h + \text{anything}$, apart from possible parton terms. We expect a bounded transverse momentum and weak dependences on the azimuthal angle of $h$ relative to the photon polarization direction apart from leading particle ($p^0$) effects. The dependence on the Feynman scaling variable $x = p_{z,h}/p_{z,\text{max}}$ should be as in the projectile fragmentation region of $\gamma + p \to h + \text{anything}$.

3. Off-shell asymptotic limits. -- The physics of $\gamma\gamma \to \text{hadrons}$ depends on three variables: $q_2^1$, $q_2^2$ and $s = (q_1^1 + q_2^1)^2$. We have just seen what one can expect for $q_2^1 = q_2^2 = 0$ and $s \to \infty$. What about other asymptotic limits? One can imagine many, but for our purposes the following will be enough [11].

(i) Let $|q_2^1|$, $|q_2^2|$ and $s \to \infty$ but also demand that $s/q_2^1 q_2^2$ approach infinity [12], [13]. If $q_2^1 q_2^2$ is fixed, this means that $q_2^1 \propto s^\beta$ with $\beta < \frac{1}{2}$. We'll call this the scaling Regge (SR) limit. The Regge limit proper is $q_2^1, q_2^2$ fixed and $s \to \infty$ (R limit).

(ii) Let $|q_2^1|$, $|q_2^2|$ and $s \to \infty$ with the fixed ratio

$$\omega = 1 - \frac{s}{q_2^1 + q_2^2}$$

$$\xi = \frac{q_2^2 - q_1^1}{q_2^1 + q_1^1}.$$  

We shall call this the scaling limit (S limit) [11]. Limits where $q_2^1$ is fixed or diverges slowly correspond to $|\xi| = 1$, and then $\omega$ is a familiar Bjorken scaling variable.

(iii) Finally, consider the limit where $|q_2^1|$, $|q_2^2| \to \infty$ with the preceding $\xi$ fixed but

$$\frac{s}{q_2^1 + q_2^2} \to 0$$

--- i.e. $s$ diverges less rapidly than $q_2^1$ or $q_2^2$. This is the boundary $\omega = 1$ in (ii). We call this the double limit (DL limit) [14]-[17] since it can also be represented by

$$\lim_{s \to \infty} \lim_{|q_2^1|,|q_2^2| \to \infty} \equiv \lim_{m \to \infty}.$$

We shall see that in the parton model it is unimportant precisely how one defines this limit. As a boundary here we have the interesting case when $|q_2^1|$, $|q_2^2| \to \infty$ ($\xi$ fixed) and $s = \text{constant}$. This is the light cone limit (L limit) [15].

These are all shown schematically on figure 3.

In the rest of the talk, I would like to present the picture of the asymptotic behavior which emerges from the parton model [11]. This overlaps to a great extent what one expects from light cone considerations (for the L and DL limits). The field theory version of the parton model is due to Drell, Levy and Yan [18] and has been discussed in covariant form by Landshoff, Polkinghorne and Short [19] and Brodsky, Close and Gunion [20]. Why study this model rather than, say, QED? There are two reasons; (i) The model exhibits the physical basis of the light cone algebra results — the oft-mentioned finiteness of certain matrix elements of bilocals arises from the transverse momentum cutoff in the model and (ii) the disconnected pieces of the light cone algebra (giving rise to e.g.

$$\sigma(e^+ e^- \to \text{hadrons}) = \text{Re}(e^+ e^- \to \mu^+ \mu^-))$$

are explicitly contained. Thus the model is a clear formulation of possibly important physics. Almost everything that follows depends only on the reality of the cutoff and the separation of amplitudes into connected and disconnected ones, and not on further details. To be precise, we shall consider only fermion partons.

The elementary coupling in the model is of a photon to a parton pair. The basic assumption is that a connected parton-hadron or parton-parton amplitude vanishes rapidly whenever the squared four-momentum carried by a parton line diverges. This is the transverse momentum cutoff of the model (usually formulated in the $P = \infty$ frame), expressed in covariant form. As a consequence of this assumption, any diagram where both partons in a current channel couple to a connected amplitude vanish [19]. This last gives rise to the asymptotic relation of the parton model that

$$\sigma(e^+ e^- \to \text{hadrons}) = \sum e_t^2 \sigma(e^+ e^- \to \mu^+ \mu^-)$$

when applied to the electromagnetic current vacuum polarization tensor.
Provided the $|q^2|$ are asymptotic, $\gamma\gamma$ scattering can be written as the sum of the diagrams of figure 4. We are here only interested in discontinuities, which one measures in the fully inclusive reaction $\gamma + \gamma \to anything. We will discuss some exclusive reactions later.

\[
\begin{align*}
\text{(a)} & \quad \includegraphics[width=0.2\textwidth]{fig4a.png} \\
\text{(b)} & \quad \includegraphics[width=0.2\textwidth]{fig4b.png} \\
\text{(c)} & \quad \includegraphics[width=0.2\textwidth]{fig4c.png}
\end{align*}
\]

**FIG. 4.** — The parton model diagrams for the photon-photon amplitude.

Off-mass shell we have, besides the three absorptive parts of the forward $\gamma\gamma$ scattering amplitude mentioned already, five more [5]: $W^{++}_{00}, W^{++}_{0+}, W^{00}_0, W^{00}_0, W^{++}_0$. The first three are cross sections and the last two are asymmetries. The $\gamma^* \gamma^*$ cross sections and asymmetries are (in the $\gamma^* \gamma^*$ CM)

\[
\begin{align*}
\sigma_{\lambda_1\lambda_2}^{ij} & = \frac{e^4}{2[(s - q^2_1 - q^2_2)^2 - 4q^2_1 q^2_2]^{1/2}} W_{\lambda_1\lambda_2}^{ij} \\
& = \frac{e^4}{2q_1^2 + q_2^2} [\omega^2 + \tilde{e}^2 - 1]^{1/2} W_{\lambda_1\lambda_2}^{ij} \\
\end{align*}
\]

This fixes the normalization of the $W'$ s; the total on-shell cross section averaged over $\gamma\gamma$ polarizations is

\[
\sigma_{\gamma\gamma}^{\text{tot}} = \frac{e^4}{4s} (W^{++} + W^{++})
\]

4. The scaling regge limit. — Diagrams 4b and 4c are expected on dimensional grounds to correspond to $\sigma_{\gamma\gamma}, \propto 1/s$ and should be negligible in this limit. So we consider only 4a (see Kingsley [13]). As we might expect in the parton model with $j = \frac{1}{2}$ partons, the longitudinal amplitudes are asymptotically negligible [13]

\[
\lim_{s \to \infty} \left[ \begin{array}{c} W^{00}_{00} \ W^{0+}_{0+} \ W^{0+}_{0+} \ W^{9+}_{0+} \ W^{90}_{0+} \ W^{90}_{0+} \\
W^{+0}_{T}, & W^{+0}_{T}, & W^{+0}_{T}, & W^{+0}_{T}, & W^{+0}_{T}, & W^{+0}_{T} \end{array} \right] = 0
\]

where $W_T$ is any of the transverse amplitudes $W^{++}, W^{++}, W^{++}$. These longitudinal amplitudes, however, approach zero differently depending on the number of $\lambda = 0$ indices. In addition, there is one further relation

\[
W^{++} = W^{++} + W^{++}
\]

which is already familiar to us from the Regge analysis of real $\gamma\gamma$ scattering. It is not clear from the analysis of reference [13] whether one expects $W^{++}$ to vanish relative to $W^{++}$. Probably it does only in special cases (corresponding to no Regge cuts in the connected blob).

We can go a step further by assuming the dominance of factorizable Regge poles in off-shell compton scattering on nucleons and in $\gamma^* \gamma^*$ scattering. Then from this, scaling, and the smallness of $\sigma_\lambda/\sigma_T$ we can get (see also ref. [12])

\[
W^{++} = W^{++} \approx \sum_{\lambda} \left[ \gamma_+^\lambda(R) \right]^2 \left[ \frac{s\lambda_0}{(q^2_1 - m_0^2) (q^2_2 - m_0^2)} \right]^{2\lambda} \approx \frac{1}{1 + q^2}
\]

where $\gamma_+^\lambda(R)$ is supposed to be $q^2$ independent provided $s\lambda_0/(q^2_1,q^2_2) \gg 1$; $m_0^2$ is a mass parameter describing the approach to scaling, $m_0^2 \sim 0.3$ GeV$^2$, and $s_0 = 1$ GeV$^2$. This all follows from the relation for the Regge residues necessary to get scaling in

\[
\gamma^* N \to \gamma^* N, \quad \beta_+^\lambda(R) = \gamma_+^\lambda(R) \left( \frac{s\lambda_0}{|q^2|} \right)^{2\lambda} \approx \frac{1}{1 + q^2}
\]

For the pomeron contribution we can estimate

\[
\gamma_+^\lambda(R) \sim 0.13
\]

Provided, of course, that $\nu W^{(\lambda)}_{\gamma\gamma}$ is really dominated by a factorizable pomeron at large $\omega = 2 M vq^2$.

In the event that $s, q^2_1, q^2_2$ are all large but $s\lambda_0/(q^2_1,q^2_2)$ is not, it has been conjectured that the $W$'s are functions of this variable [12] — something one cannot conclude simply from the Regge form. It might be interesting to investigate the functional dependences of the scaling regge limit.

An unexplored point of some importance is what happens to the diagrams 4a as $s/|q^2|$ decreases from large values. One thing can be said, and that is that these diagrams should vanish in the $S$-limit when $s, |q^2_1|$ and $|q^2_2|$ all diverge with constant ratios, provided that we demand that all connected amplitudes in the model vanish as a squared parton four-momentum leading into a connected blob diverges. This has to include connected parton-parton amplitudes. The reason why the amplitudes of figure 4a vanish in this limit is that the momentum transfer into a connected blob must be proportional to the dimensional quantities in the problem and these all diverge in the $S$-limit. As a contrast, the momentum transfer along a parton line in the Bjorken limit of inelastic ep scattering is proportional to the squared nucleon mass, and only diverges (as $t_{\text{min}} \propto 1/(\omega - 1)$) for $\omega \to 1$. It is clear that the diagrams of figure 4a must have a kind of threshold behavior as $s\lambda_0/(q^2_1,q^2_2)$ decreases from the
large values where the Regge expansion might hold. This behavior depends of the unknown off-shell behavior of the connected parton-parton amplitudes, and it is not so directly accessible as the threshold behavior of \( W_{2}^{(3)} \) in inelastic ep scattering.

5. **The S-limit.** We have just seen that the leading diagrams in the scaling Regge limit don’t contribute to this limit. That Regge-type behavior is not to be expected is already clear when one notes that

\[
cos \theta_t = \omega [1 - \xi^2]^{-1/2}
\]

and only diverges when \( |\xi| \to 1 \) or \( \omega \to \infty \) (i.e. when one goes to inelastic scattering off a real photon or to the SR limit). The diagram 4c does not contribute for the same reason that the connected diagram in \( e^+ e^- \) annihilation does not (Fig. 5); one is left with a fully disconnected piece, and it can be calculated by noting that it is the box graph in QED. The helicity amplitudes are dimensionless (we constructed them that way) and it is perhaps reasonable that they become functions of \( s \) and \( \omega \) and not of \( q_1^2, q_2^2 \) and \( s \) separately [11, 17]

\[
\lim_{s \to \infty} W_{2s}^{(3)}(s, q_1^2, q_2^2) = W_{2s}^{(3)}(\xi, \omega).
\]

Fig. 5. — The connected and disconnected amplitudes for \( e^+ e^- \) annihilation.

From (5) we can see that the \( \gamma^0 \gamma^0 \) cross section is \( \propto 1/s \) in this limit, as we might expect from dimensional analysis, whereas it goes to a constant in the \( R \) limit. We expect that some of the amplitudes (11) appear in the \( SR \) and \( R \) limits in the form of Regge singularities with intercepts at or below \( \alpha_R = 0 \).

This fully disconnected piece has its peculiarities:

(i) The longitudinal amplitudes do not vanish, because that is a consequence of a transverse momentum cutoff which is not present in this term. However, the longitudinal amplitudes aren’t too important; they vanish as either \( \omega \to \infty \) or \( \omega \to 1 \). The former is the \( SR \) limit and the latter the \( DL \) limit which we shall discuss in a moment.

(ii) The transverse amplitudes are the most interesting (for the others, see [11]). These are

\[
W_{2s}^{\pm}(s, \xi) = \frac{1}{2\pi} \int_{-1}^{1} \frac{dz}{(1 - z^2)} \left[ (\xi^2 + \omega - 1)^2 z^2 + \xi^2 \omega^2 \right] \left[ \omega^2 - (\omega^2 + \xi^2 - 1) z^2 \right] \frac{dz}{z^2 - (\omega^2 - z^2 - 1) z^2}.
\]

and they diverge logarithmically as \( |\xi| \to 1 \), for any \( \omega \). Only \( W_{2s}^{+} \) survives as \( \omega \to \infty \) and only \( W_{2s}^{+} \) and \( W_{2s}^{-} \) survive as \( \omega \to 1 \). In this latter limit, we see that only states of total \( J_z = 0 \) (but any total \( J \)) are produced. Exactly the reverse happens in the Regge or \( SR \) limits for this term. It is interesting that non-parton calculations indicate a predominant \( W_{2s}^{+} \) term for the \( f^0 \) meson region in real \( \gamma \gamma \) interactions [8].

The significant feature of this disconnected term is its similarity to the parton diagram for \( e^+ e^- \) annihilation. This means that in the asymptotic \( S \) limit the hadron distributions should look qualitatively like those in \( e^+ e^- \) annihilation — only the familiar single-particle \( 1 + \cos^2 \theta \) angular distribution (for \( j = \frac{1}{2} \) partons) relative to the \( e^+ e^- \) axis is missing. That is a consequence of one-photon exchange in the \( s \)-channel and here we have parton exchange in the \( t \)- and \( u \)-channels. The events in \( \gamma^0 \gamma^0 \) annihilation should involve large \( p_T \) secondaries just as in \( e^+ e^- \) and other features (\( k/T \) ratios, etc.) should also be similar.

The similarity to \( e^+ e^- \) annihilation can be expressed in a way similar to the statement

\[
\sigma(e^+ e^- \rightarrow \text{hadrons}) = R\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)
\]

with \( R = \sum e_i^2 \) as \( q^2 \to \infty \). Namely, we take the two photon exchange part of \( e^+ e^- \rightarrow e^- + e_+ + \text{hadrons} \) in the \( S \)-limit and then [22]

\[
\lim_{s \to \infty} \sigma_{\gamma\gamma}(ee \rightarrow ee + \text{hadrons}) = \frac{T\sigma_{\gamma\gamma}(ee \rightarrow ee + \mu^+ \mu^-)}{s}
\]

where \( T = \sum e_i^4 \) in simple parton models. In some cases \( T \) and \( R \) are not simply related to parton charges. For example, in the so-called colored quark model [23] \( R = 2 \) and \( T = \frac{3}{2} \). In the Han-Nambu model (three integrally charged triplets [24]) one can require that the produced hadrons are singlets under the group of transformations of the triplet indices (charm). Then \( R = 2 \) and \( T = 2 \) [22]. If one had treated the Han-Nambu model as an ordinary parton model (all hadrons, including those with charm, being produced) then \( T = R = \sum e_i^4 = 4 \).

The relation between \( T \) and \( R \) can be expressed in a possibly more general way in terms of the Wilson expansions [25], [26] except that instead of simply relating two numbers (like \( R \) and the coefficient \( S \) of the \( n^0 \rightarrow \gamma \gamma \) decay [25]) one has to calculate here a function \( W_{2s}^{(3)}(\xi) \) (all this probably only works at \( \omega = 1 \)). For this one needs the entire light cone expansion [26]. The amplitude \( \gamma^0 \gamma^0 \rightarrow \gamma^0 \gamma^0 \) also offers a nice example of a conformal invariant four-point function and is perhaps interesting from that point of view [27].

Actually, this parton term should be present in real \( \gamma \gamma \) reactions and would lead to a class of events with unbounded transverse momentum [28]. It would only contribute appreciably to the total \( \gamma \gamma \) cross section at low \( s \) in the event that \( T \gtrsim 1 \), however.
6. The fixed-\(s\) behavior (\(L\) and \(DL\) limits). —

The last asymptotic limit we need to discuss is the \(L\) limit where \(s\) is fixed and \(|\vec{q}_1^2|\) and \(|\vec{q}_2^2| \to \infty\) with \(\xi\) fixed [29], [30]. Afterwards one can reach the threshold \(\omega = 1\) of the \(S\) limit by taking \(s \to \infty\) (the \(DL\) limit).

In the \(L\) limit only diagrams \(4b\) and \(4c\) survive; \(4b\) has no resonant behavior, so if we are interested in the \(\gamma^* \gamma^*\) form factors of resonances (Fig. 6a) or perhaps threshold \(\gamma^0 \gamma^0 \to \pi^+ \pi^-\) (Fig. 6b) we should only consider figure 4c. Since we have seen that large \(q^2\) photons (with \(s \ll |\vec{q}_1^2|, |\vec{q}_2^2|\)) couple only to transverse photons and only then in states of total \(J_p = 0\), we expect only one surviving helicity \(\text{amplitude}\) for all such states, namely \(T_{+\mp}\). Moreover, this should be a function of \(\xi\) alone — \textit{independent} of \(q^2\) at fixed \(\xi\),

\[
\lim_{L} T_{a(\pm)}(q^2_1, q^2_2) = T_{a(\pm)}(\xi)
\]

\[
\lim_{L} T_{a,00} = 0 \quad \text{otherwise}
\]

\[\text{Fig. 6. — The parton diagrams for } \gamma \gamma \to \text{resonance and } \gamma \gamma \to \pi^+ \pi^-\]

\((T_{++}\) and \(T_{--}\) are related by parity). To summarize, we note two essential points: first, the fact that if the final state is a pseudoscalar resonance the coupling is of the form

\[T_{a,i} \propto e^a_{i} e^*_{i} e_{\mu \nu i 0}(q_2 + q_1)^\mu (q_2 - q_1)^\nu\]

introduces a factor \(\xi\) in \(T_{++}\), as one can easily check.

For scalar resonances (normal parity states) we get a factor \(\xi^2\), so [11]

\[
\lim_{L} T_{++}\(\xi\) =
\begin{cases}
\xi f(\xi) & \text{abnormal parity resonance} \\
\xi^2 g(\xi) & \text{normal parity resonance}
\end{cases}
\]

This is similar to what happens when we form the combinations \(W_{\mp\mp}^{\pm\mp} \mp W_{\mp\mp}^{\pm\pm}\) in (12) for the limit \(\omega \to 1\). The above is a consequence of \(j = 1\) partons. We should not be surprised to see the latter behavior in (15) for reactions like \(\gamma^0 \gamma^0 \to \pi^+ \pi^-\) near \(s \approx 4 m^2\).

What happens when, at large but fixed \(|\vec{q}_1^2|, |\vec{q}_2^2|\), we increase \(s\)? This is an instructive exercise. We encounter first a set of resonances (\(\pi^0, \rho^0,\ldots\)) of all \(J\) but \(J_p = 0\). At larger \(s\) the resonance structure should disappear along with two-body channels like \(\pi^+ \pi^-\), whose asymptotic \(s\)-behavior should be like that in \(e^+ e^-\) annihilation. As \(s\) increases further we reach the Regge region for \(s \sim \text{a few times larger than } |\vec{q}^2|\), after which \(W_{\mp\mp}^{\pm\mp}\) (for example) rises linearly with \(s\). For a fanciful picture, see figure 7, where we compare \(W_{\mp\mp}^{\pm\mp}\) and the spectral function of the photon propagator \(\rho(s)\).

\[\text{Fig. 7. — Sketches of the two-photon and one-photon absorptive parts relative to those for a } \mu^+ \mu^- \text{ final state.}\]

The precise relation between connected and disconnected pieces (Fig. 4b, 4c) in the \(L\) limit is not specified by the model. A fascinating possibility, however, is that the leading light cone contribution at large \(s\) (Fig. 4c) \textit{averages} the resonance pieces at low \(s\); this would be a kind of light cone duality, and ought to hold in both \(e^+ e^-\) annihilation and the \(L\) or \(DL\) limits of \(\gamma^0 \gamma^0\) scattering [11], [32], [33].

7. Inelastic electron-photon scattering. —

The limit where one photon has fixed \(q_1^2 = 0\) and the other large \(|\vec{q}_2^2|\) at fixed \(\omega\) is related to the inelastic scattering of an electron on a photon target [34]. We can see here (where there is some similarity to inelastic eN scattering) the operation of all the preceding terms. Suppose, to enhance the familiarity, we consider \(\nu W_{\pm}^{(\nu)}\) for a photon target. The connected diagrams as in 4a now look different because \(q_1^2\) is not asymptotic; the photon connects directly to the blob and there is

\[\text{(*) The value of studying } \gamma^* \gamma^* \to \pi^+ \pi^- \text{ in the parton model has been emphasized by Brodsky et al. [31].}\]
no parton line running between the incident and outgoing photon with momentum \( q_1 \). In fact, this set of diagrams should look mostly like \( g^* p^0 \to g^* p^0 \).

We expect these connected diagrams scale and give

\[
vW_2^{(p)}(\infty) = \text{constant since } vW_2^{(q)}(\infty) = \text{constant}
\]

where \( \omega = \infty \) here means \( \omega \gtrsim 10 \); as a guess from the quark model and the assumption of pomeron dominance of \( vW_2^{(q)} \).

\[
vW_2^{(p)}(\omega \gg 1) \approx \frac{\alpha}{f_p^2/4\pi} \cdot \frac{2}{3} \left( 1 + \frac{1}{3} \right) vW_2^{(q)}(\omega \gg 1).
\]

What happens as \( \omega \to 1 \)? For the corrected diagrams, the squared momentum transfer along a parton line has a minimum value \( t_{\text{min}} \approx 1/(\omega - 1) \) and diverges as \( \omega \to 1 \); so these diagrams vanish near threshold. The rate at which they vanish is related to the \( g^* p^0 \)-meson form factors by a Drell-Yan-West relation [35]. For the proton with a dipole form factor, \( vW_2^{(p)} \propto (\omega - 1)^3 \). For a meson target with a less rapid decrease of the form factor, \( vW_2^{(p)} \) should decrease more slowly as \( \omega \to 1 \). It would be wrong to conclude from this that the \( g^* \gamma^0 \)-meson helicity form factors vanish as \( |q_2^2| \to \infty \). All this indicates that the \( g^* \gamma^0 \)-meson vertices should vanish and then the \( g^* \gamma^0 \)-meson vertices are entirely given by parton or light cone considerations (namely, the term \( T_{++}(1) \)) [29], [36]. Similarly, although the connected diagrams analogous to \( 4a \) vanish near threshold, the disconnected diagrams do not. In fact they give [37] *(4)*

\[
vW_2^{(p)}|_{\text{disc}} \propto \frac{(\omega - 1)^2 + 1}{\omega^2} \ln \frac{s}{m_{\text{parton}}}.
\]

for any fixed \( \omega \); this term is large near threshold and vanishes as \( \omega^{-1} \) for large \( \omega \), apart from the log. So there is present in inelastic electron-photon scattering a term which breaks Bjorken scaling and a simple minded threshold relation like that for the connected parton diagrams in inelastic e-N scattering. If there is any kind of duality between resonances and continuum near threshold in e-\( \gamma \) scattering, it involves the duality of resonances and this disconnected term, not a connected term as in e-N scattering.

Finally, we sketch \( vW_2^{(p)}(\omega, q^2) \) at a value of \( q^2 \) where the disconnected term at small \( \omega \) is roughly of the same magnitude as the connected term at large \( \omega \) (Fig. 8).

8. Conclusion. — What I have sketched out here is a theoretical picture of a reaction which represents a considerable experimental program for the future. How reliable is this picture? Can one do experiments? As to the picture: the details may be wrong in places, but in broad outline it is probably near the truth, granted that we have learned anything about deep inelastic processes from e-N and v-N experiments, and that e+ e- annihilation is not too drastically different from parton or light cone expectations. Some of the details which rest on the behavior of e+ e- annihilation may have to be modified if that turns out to have unexpected properties.

As to the feasibility of experiments, these are clearly most interesting with high energy e+ e- and e+ e- machines (\( E = 10-15 \) GeV). But one can still do a lot at lower energies. For example, the \( \pi^+ \pi^- \) final state in e-\( \gamma \) scattering contains a lot of interesting information and one can study this (as well as perhaps \( \gamma^0 \gamma \to \pi^0 \gamma \)) in some detail in the reaction

\[
e^- + e^- \to e^- + e^- + \pi^+ + \pi^-.
\]

where one detects both pions (or two photons from \( \pi^0 \to \gamma \gamma \)) and only the large angle scattered electron [40], integrating over the remaining small-angle scattered electron. The helicity structure of the \( f^0 \) meson in the \( \pi^+ \pi^- \) decay channel can be studied in this way — and the possible background

\[
e^- + e^- \to e^- + e^- + (C = -1 \text{ hadrons})
\]

cannot contribute to \( f^0 \) production. The reaction \( \gamma^0 \gamma \to \pi^+ \pi^- \) near threshold should also be interesting and can probably be separated from background. In general, there is no reason why one cannot try to study inelastic electron-photon scattering with e- e- beams by detecting the hadrons and large-angle electron, integrating over the small angle electron. The background of \( C = -1 \) hadrons can be calculated and subtracted.

So one can at least start studying deep inelastic \( \gamma^0 \gamma \) processes with the present generation of machines, and I think that this field offers exciting possibilities for both the near and distant future.
A. Zichichi. — If I understand what you have said, you propose to extend the new duality of Sakurai to the process \( e^+ e^- \rightarrow e^+ e^- + \) hadrons. However, in the single-photon case, there is no problem with the reference cross section. But in the two-photon case, for fixed beam energy, the comparison between \( \sigma(e^+ e^- \rightarrow e^+ e^- + \text{hadrons}) \) and \( \sigma(e^+ e^- \rightarrow e^+ e^- + \mu^+ \mu^-) \) must be done for equal values of the invariant masses of the hadron and the (\( \mu^+ \mu^- \)) systems respectively. This reduces the rates and makes the comparison quite difficult.

I also want to make a comment on your final conclusion. We all agree that a basic problem in hadron physics is the structure of the elementary particles. It seems to me that the most direct and simple way to study the constituents of the hadrons is to measure \( e^+ e^- \rightarrow \) hadrons), without any need to complicate our hard life with 2-photon processes. Do you agree?

T. F. Walsh. — If you detect two electrons at non-zero angle in ee \( \rightarrow e^+e^- + \text{hadrons} \) (2-photon part), then the reference variables \( q_1^2, q_2^2, s \) are fixed for the comparison process \( ee \rightarrow e\gamma \mu^+ \mu^- \) (2-photon part): it is true that the cross sections are small for large \( q_1^2 \) and \( q_2^2 \), but they rise with beam energy. At \( E = 15 \text{ GeV} \), the double deep inelastic process has a cross section of the order of \( 10^{-35} \text{ cm}^2 \), and should be a good candidate for study by the machines which will come after SPEAR and DORIS. I hope that on will try.
however, to look at these processes even with the current machines.

Concerning your second statement: The total cross section $\sigma(e^+ e^- \rightarrow \text{hadrons})$ is interesting, but only delivers one number ($R$). Fully inclusive $\gamma^* \gamma^* \rightarrow \text{hadrons}$ has a much richer structure, and I guess that makes life more complicated.

H. Terazawa. — I think Prof. Zichichi’s question was «where will the asymptotic behaviour start in the two-photon process?». This question is theoretically very difficult to answer. However, according to the SLAC-MIT experiment on deep inelastic electroproduction, I guess that it is fairly easy to reach the asymptotic region. I would believe that, if you have two virtual photons with masses larger than 1 GeV and if $s$ (the total invariant mass squared of the hadrons produced) is beyond the $f$ resonance, you will already find the asymptotic behaviour.

S. Brodsky. — I believe Prof. Zichichi is concerned about the choice of the value of $s_{\text{min}}$ for $ee \rightarrow ee$ hadrons in the quasi-real case when one computes the disconnected parton pair contribution. As an absurd example, we could take $s_{\text{min}} = 4 m^2_e$. In Preparata's model, the quark mass is infinite, although the connected diagrams reduce it to an effective hadron mass.

T. F. Walsh. — Yes, you have trouble in the case where one $q^2$ is zero because there is a log and one doesn’t know that mass to put in for the comparison process $ee \rightarrow ee\gamma \gamma$. This log doesn’t necessarily mess-up the duality of connected and disconnected contributions to $vW_2^{(y)}$ at low $s$ because it is really $\ln(s/\mu^2_{\text{parton}})$. When both $q_i^2$ are zero (e. g. when one estimates the parton contribution to $a_{\text{tot}}(ee \rightarrow ee + \text{all hadrons})$, then the $s_{\text{min}}$ problem you mentioned also turns up.