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## ANOMALOUS REFLECTION OF ULTRASONIC BEAMS AT THE INTERFACE OF A SOLID MEDIUM WITH ATTENUATION

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**Résumé.** — On examine la réflexion, sous incidence oblique, à la surface d'un milieu élastique solide présentant une atténuation, d'une onde sonore se propageant dans un liquide. On trouve des expressions générales du module et de la phase du coefficient de réflexion acoustique.

On calcule, par la méthode de l'intégrale de Fourier, un champ du faisceau ultrasonore réfléchi. On montre que, pour les angles d'incidence qui correspondent à l'excitation d'ondes de Rayleigh à l'interface solide-liquide, on observe, outre un déplacement latéral du faisceau, une variation de l'intensité du faisceau réfléchi. Ces deux effets dépendent fortement du taux de perte d'énergie acoustique dans le milieu. Pour une atténuation supérieure à une certaine valeur critique (lorsque la composante active de l'impédance d'un milieu est égale à l'impédance du liquide) la réflexion est anormale, le déplacement disparaît pratiquement et l'on observe, au lieu du déplacement, une brusque diminution de l'intensité du faisceau réfléchi.

On expose une comparaison des particularités de la réflexion d'un faisceau ultrasonore sur des milieux d'atténuations différentes, par la méthode de Toepler. Les données expérimentales sont en accord avec la théorie.

Ces particularités peuvent être utilisées pour étudier les caractéristiques des ondes et des pseudo-ondes élastiques de surface sur des cristaux anisotropes.

**Abstract.** — Reflection of an oblique incident sound wave in liquid at the interface of a solid elastic medium with attenuation is examined. General expressions for modulus and phase of acoustic reflection coefficient are found.

A field of the reflected ultrasonic beam is computed by the Fourier-integral method. It is shown that for incident angles which correspond to the Rayleigh wave excitation at solid-liquid interface a change in intensity of the reflected beam is to be observed in addition to a sideways displacement of the beam. Both mentioned effects depend strongly on rate of acoustic energy loss in the medium. For attenuation exceeding some critical value (when active impedance constituent of a medium is equal to the liquid impedance) the reflection is anomalous, displacement practically vanishes and instead of a displacement an abrupt decrease of a reflected beam intensity takes place.

Comparison of peculiarities of an ultrasonic beam reflection from media with different attenuation is demonstrated by Toepler shadow method. Experimental data are in accordance with the theory.

These peculiarities can be used for studying characteristics of elastic surface and pseudosurface waves on anisotropic crystals.

P. Langevin [1] was the first to suggest the study of objects in water medium and observation of ultrasonic beams reflected from them. These ideas were further developed in a number of works which investigated peculiarities of acoustic reflection from bodies of different materials and different surface shape. This paper reports on a new and, it seems to us, rather interesting effect connected with anomalous ultrasonic beam reflection at the interface of a medium with a high attenuation of a sound.

As it is well known [2] in case of a total internal reflection of an ultrasonic beam at the liquid-solid interface in the absence of attenuation and for angles of  $\Theta_R = \arcsin(q_R/q_0)$  a sideways displacement of the beam will be observed, which equals

$$\Delta^0 = \frac{4}{q_0} \frac{\rho}{\rho_0} \left( \frac{r(r-s)}{s(s-1)} \right)^{1/2} \times \frac{1 + 6s^2(1-g) - 2s(3-2g)}{s-g}, \quad (1)$$

where  $\rho, \rho_0$  are medium and liquid densities ;

$r = (q_0/q_l)^2, s = (q_R/q_l)^2, g = (q_e/q_l)^2, q_e, q_l, q_R$  wave numbers of longitudinal, shear and Rayleigh surface waves respectively ;  $q_0$  wave number of waves in liquid.

Let us consider a more general case of attenuation in a medium ; then for a plane monochromatic wave the modulus and phase of reflection coefficient will be expressed as [3]

$$|R| = \frac{[(M_1^2 + M_2^2 - Z_0^2)^2 + 4M_2^2 Z_0^2]^{1/2}}{(M_1 + Z_0)^2 + M_2^2} \quad (2)$$

$$\varphi = \arctg \frac{2Z_0 M_2}{M_1^2 + M_2^2 - Z_0^2}, \quad (3)$$

where  $M_1$  and  $M_2$  are real and imaginary components of the complex acoustic impedance of the medium respectively

$$\begin{aligned} M_1 &= \text{Re} [\bar{Z}_e \cos^2 2\bar{\beta}_l + \bar{Z}_l \sin^2 2\bar{\beta}_l], \\ M_2 &= \text{Im} \end{aligned} \quad (4)$$

$\bar{Z}_{e,t} = \rho\omega/\bar{q}_{e,t} \cos \bar{\beta}_{e,t}$  impedance for longitudinal or shear waves ;  $Z_0 = \rho_0 \omega/q_0 \cos \Theta$  impedance for liquid ;  $\bar{q}_{e,t} = q_{e,t}(1 + j\delta_{e,t})$  and  $\bar{\beta}_{e,t}$  complex wave numbers and angles of refraction for each of the waves ;  $\Theta$  angle of incidence ;  $\delta_{e,t} = \alpha_{e,t}/q_{e,t}$  attenuation in solid medium.

Angles  $\bar{\beta}_{e,t}$  can be determined from the following equality of tangential components of the wave vectors of the incident wave and each of the refracted waves

$$q_0 \sin \Theta = \bar{q}_{e,t} \sin \bar{\beta}_{e,t}. \quad (5)$$

For angles of incidence greater than the second critical angle  $\Theta > \Theta_{2cr}$ ,  $= \arcsin q_t/q_0$  and retaining linear  $\delta_{e,t}$  terms in eq. (5) we shall have :

$$\bar{\beta}_{e,t} = \left[ \frac{\pi}{2} - \delta_{e,t} \frac{n_{e,t}}{(n_{e,t}^2 - 1)^{1/2}} \right] - j \operatorname{Arsh} (n_{e,t}^2 - 1)^{1/2}, \quad (6)$$

here the following notation is introduced

$$n_{e,t} = \frac{q_0}{q_{e,t}} \sin \Theta, \quad n_{e,t} > 1.$$

Longitudinal and shear refracted waves are inhomogeneous, the effect of attenuation leading to the fact that an angle is formed between the plane of constant phase and the plane of constant amplitude slightly less than  $\pi/2$ .

The impedance of longitudinal or shear waves may be represented by

$$\bar{Z}_{e,t} = \frac{-j\rho\omega}{q_{e,t}(n_{e,t}^2 - 1)^{1/2}} \left( 1 + j \frac{\delta_{e,t}}{n_{e,t}^2 - 1} \right). \quad (7)$$

Inserting expressions (6) and (7) into (4) one can get impedance components for solid medium

$$M_1 = \delta_e \frac{\rho\omega(2n_t^2 - 1)^2}{q_e(n_e^2 - 1)^{3/2}} + \delta_t \frac{\rho\omega}{q_t} 4n_t^2 \left[ \frac{4n_t^2 - 3}{(n_t^2 - 1)^{1/2}} - \frac{2(2n_t^2 - 1)}{(n_t^2 - g)^{1/2}} \right], \quad (8)$$

$$M_2 = \frac{\rho\omega}{q_t} 4n_t^2(n_t^2 - 1)^{1/2} - \frac{\rho\omega}{q_e(n_e^2 - 1)^{1/2}} (2n_t^2 - 1)^2. \quad (9)$$

If the angle of incidence  $\Theta$  is equal to  $\Theta_R$  the formula (9) degenerates into an equation for Rayleigh waves on the free surface

$$M_2^{\Theta_R} = 4s[(s - 1)(s - g)]^{1/2} - (2s - 1)^2 = 0. \quad (10)$$

Component  $M_1^{\Theta_R}$  can be rewritten in this case as

$$M_1^{\Theta_R} = 4s \frac{\rho\omega}{q_t} \times \left[ \delta_e \frac{g(s - 1)^{1/2}}{s - g} + \delta_t \frac{3 - 2s}{(2s - 1)(s - 1)^{1/2}} \right], \quad (11)$$

the relationship of terms expressing attenuation of longitudinal and shear waves is

$$\frac{g(2s - 1)(s - 1)}{(3 - 2s)(s - g)} \frac{\delta_e}{\delta_t} = \xi \frac{\delta_e}{\delta_t}.$$

For most solid bodies Poisson's ration will be within the 0.2-0.4 range ;  $\xi$  varying from 0.22 to 0.038. Moreover, if one takes into account that  $\delta_e < \delta_t$  as a rule, it becomes evident that the attenuation of longitudinal waves may be neglected, and further we shall consider only  $\delta_t$ .

As is seen from formula (8), for angles of incidence  $\Theta > \Theta_{2cr}$  the attenuation causes the active impedance constituent of the medium. Modulus of the reflection coefficient is always less than unity ; while approaching angle  $\Theta = \Theta_R$  it goes through minimum, which equals

$$|R|^{\Theta_R} = \left| \frac{M_1 - Z_0}{M_1 + Z_0} \right|_{\Theta=\Theta_R}. \quad (12)$$

If  $\delta_t$  is small, then  $M_1 < Z_0$  and the solid medium will be equivalent to the medium with low acoustic impedance ; phases of incident and reflected waves differing by  $\pi$ .

When  $M_1^{\Theta_R} = Z_0$ ,  $|R|^{\Theta_R} = 0$ . The attenuation corresponding to the above critical case is

$$\delta_t^{cr} = \frac{\rho_0}{\rho} \frac{(2s - 1)}{4s(3 - 2s)} \left( \frac{s - 1}{r - s} \right)^{1/2}. \quad (13)$$

The value of  $\delta_t^{cr}$  is quite large and can be realized only for materials with a high attenuation. Assuming, for example  $C_t/C_0 = 2$ ,  $C_e/C_0 = 4$ ,  $\rho/\rho_0 = 2.5$ , we shall obtain  $\delta_t^{cr} = 0.0384$ , this equals approximately 2.1 dB per wavelength.

For the case of  $M_1^{\Theta_R} > Z_0$  there is no change o phase for the reflection.

Figure 1 shows the variation of values  $Z_0$ ,  $M_2$  and  $M_1$  for different  $\delta_t$  near point  $\Theta_R$ .

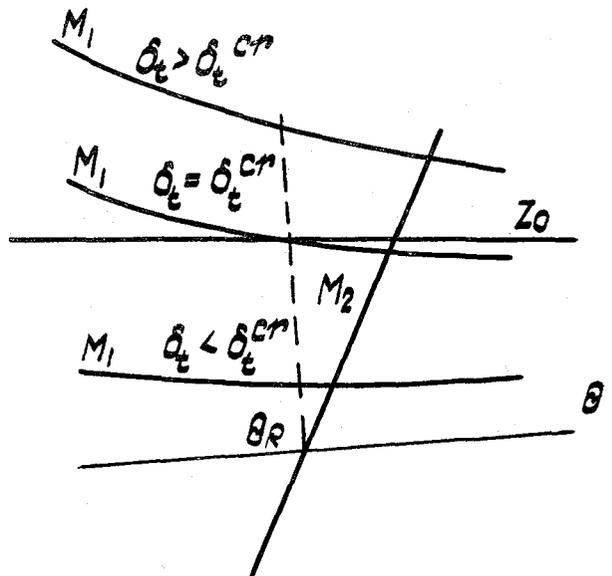


FIG. 1.

Let us consider the peculiarities of reflection of the sound beam. The beam is assumed for simplicity to be an infinite band in one direction. The geometrical axis of the beam and the normal to interface make angle  $\Theta_R$ . The width of trace formed by the beam on the interface will be  $2a$ . Neglecting the effect of diffractive spread of the beam and assuming the amplitude of potential to be constant in the beam cross-section, the exciting acoustic field at the solid-liquid interface can be written as

$$\psi_n(x) = \begin{cases} \Psi_0 e^{jq_R x}, & -a < x < a, \\ 0, & |x| > a. \end{cases} \quad (14)$$

Using Fourier transform,  $\psi_n(x)$  can be rewritten in the form of:

$$\begin{aligned} \psi_n(x) &= \frac{\Psi_0}{2\pi} \int_{-\infty}^{\infty} e^{jq_x x} \int_{-a}^a e^{j(q_R - q_x)\xi} d\xi dq_x = \\ &= \frac{\Psi_0 e^{jq_R x}}{\pi} \int_{-\infty}^{\infty} \frac{\sin a\Omega}{\Omega} e^{j\Omega x} d\Omega, \end{aligned} \quad (15)$$

where

$$q_x = q_0 \sin \theta, \quad \Omega = q_x - q_R.$$

The field of the reflected beam will be determined by

$$\psi_{\text{ref}}(x) = \frac{\Psi_0 e^{jq_R x}}{\pi} \int_{-\infty}^{\infty} |R(q_x)| e^{j\varphi(q_x)} \frac{\sin a\Omega}{\Omega} e^{j\Omega x} d\Omega. \quad (16)$$

The computation of this integral can be obtained by the approximation method, representing both the modulus and the phase of the reflection coefficient as an infinite  $\Omega$  power series. Close to  $q_x = q_R$  the modulus goes through a minimum and the rate of phase variation is maximum, therefore

$$R'(q_R) \approx 0, \quad \varphi''(q_R) \approx 0 \quad (*) \quad (17)$$

(primes mark derivatives with respect to  $q_x$ ).

Expressions of  $R(q_x)$  and  $\varphi(q_x)$  are accurate to second order terms

$$\left. \begin{aligned} R(q_x) &\approx R_0(q_R) + \frac{\Omega^2}{2} R''(q_R) + \dots \\ \varphi(q_x) &\approx \varphi_0(q_R) + \Omega \varphi'(q_R) + \dots \end{aligned} \right\} \quad (18)$$

Using these expansions we shall express the potential of the reflected beam as a sum

$$\psi_{\text{ref}}(x) = \psi_1(x) + \psi_2(x). \quad (19)$$

(\*) A more detailed analysis shows that  $\varphi''$  becomes zero for angle  $\Theta_R$  somewhat smaller than  $\Theta_R$  (when there is no attenuation). However the difference of angles  $\Theta'_R$  and  $\Theta_R$  is negligible in the case  $\rho C_{e,t} \gg \rho_0 C_0$ .

Functions  $\psi_1(x)$  and  $\psi_2(x)$  will be equal

$$\psi_1(x) = \frac{\Psi_0 R_0}{\pi} e^{j(\varphi_0 + q_R x)} \int_{-\infty}^{\infty} \frac{\sin a\Omega}{\Omega} e^{j\Omega(x + \varphi')} d\Omega, \quad (20)$$

$$\psi_2(x) = \frac{\Psi_0 R''}{2\pi} e^{j(\varphi_0 + q_R x)} \int_{-\Omega_1}^{\Omega_1} \Omega \sin a\Omega e^{j\Omega(x + \varphi')} d\Omega. \quad (21)$$

Comparing formulas (20) and (15) we shall see that the distribution of acoustic field which is given by expression (20) for  $\psi_1(x)$  is identical to the field of incident beam. However, the reflected beam is displaced along axis  $x$  by interval  $-\varphi'$  and has an amplitude which is smaller by a factor of  $R_0$

$$|\psi_1(x)| = |R_0 \psi_n(x + \varphi')|. \quad (22)$$

When computing the integral (21) the range of integration is to be confined to  $\pm \Omega_1$  values. In this interval the above expansion for modulus of reflection coefficient is true

$$\begin{aligned} \int_{-\Omega_1}^{\Omega_1} \Omega \sin a\Omega e^{j\Omega(x + \varphi')} d\Omega &= \\ &= \frac{1}{x_1^2} (\sin x_1 \Omega_1 - x_1 \Omega_1 \cos x_1 \Omega_1) + \\ &+ \frac{1}{x_2^2} (\sin x_2 \Omega_1 - x_2 \Omega_1 \cos x_2 \Omega_1), \end{aligned} \quad (23)$$

here the notations are introduced

$$x_1 = a + (x + \varphi'), \quad x_2 = a - (x + \varphi').$$

Therefore the component of potential  $\psi_2(x)$  characterizes the variation of amplitude in the cross-section of the reflected beam; the modulation depth of amplitude is proportional to the factor  $R''$ .

Let us consider the variation of modulus and of phase angle derivative of reflection coefficient depending on the attenuation in a medium. From formulas (12) and (13) one obtains

$$R_0(q_R) = \left| \frac{1 - \delta_t/\delta_t^{\text{cr}}}{1 + \delta_t/\delta_t^{\text{cr}}} \right|. \quad (24)$$

Using eq. (3) and (9) we shall have

$$\varphi'(q_R) = - \frac{1}{1 - (M_1^{\Theta_R}/Z_0)^2} \frac{2}{Z_0} \left( \frac{\partial M_2}{\partial q_x} \right)_{\Theta_R}, \quad (25)$$

where

$$\frac{2}{Z_0} \left( \frac{\partial M_2}{\partial q_x} \right)_{\Theta_R} = \Delta^0$$

displacement when there is no attenuation;  $\Delta^0$  can be computed from formula (1). Finally the variation of phase angle derivative can be expressed as follows

$$\varphi'(q_R) = \frac{-\Delta^0}{1 - (\delta_t/\delta_t^{\text{cr}})^2} \quad (26)$$

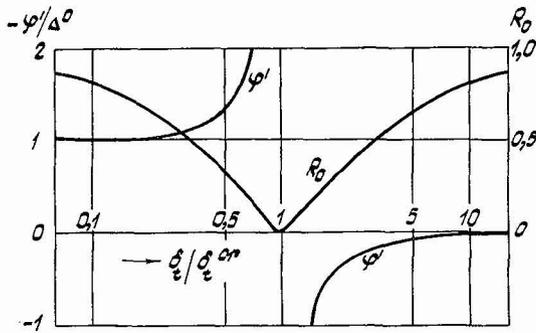


FIG. 2.

Figure 2 gives magnitudes  $R_0(q_R)$  and  $\phi'(q_R)$  as a function of relationship  $\delta_i/\delta_i^{cr}$  for

$$\delta_i \leq \delta_i^{cr} \quad \phi'(q_R) = -\Delta^0,$$

if

$$\delta_i > \delta_i^{cr} \quad \phi'(q_R) \rightarrow 0.$$

Second derivative  $R''(q_R)$  which determines amplitude  $\psi_2(x)$  can be obtained by digital differentiation of curves  $R(\theta)$  similar to those given in paper [3]

$$R''(q_R) = \frac{1}{q_0^2 \cos^2 \Theta_R} \left( \frac{\partial^2 R}{\partial \Theta^2} \right)_{\Theta_R}. \quad (27)$$

The computation shows that for small attenuation  $R''(q_R)$  grows approximately as the linear function of  $\delta_i$  goes through the maximum at  $\delta_i = \delta_i^{cr}$  and then begins to drop. Therefore from the relationships found above one can conclude that for quite a large attenuation the reflection of ultrasonic beam becomes anomalous at angles of incidence close to  $\Theta_R$ . The angular relationship of intensity has an abrupt decrease, the structure of reflected beam becomes sharply non-homogeneous. At the same time the sideways displacement changes appreciably; for attenuation exceeding the critical value the displacement practically vanishes.

The Toepler shadow method was used for experimental observation of peculiarities of ultrasonic beam reflection. Some samples of rocks with quite different attenuation were examined. Operating frequency was 4.7 Mc distilled water was used as emission liquid, the diameter of the transducer was 18 mm.

Figure 3 demonstrates the examples of photographs illustrating the most characteristic case of reflection close to the angle  $\theta = \Theta_R$ .

1) Single crystal quartz, xz plane ( $\delta_i/\delta_i^{cr} \leq 10^{-5}$ ). The intensity of reflected beam is constant in the cross-section, while displacement equals  $\Delta^0$ .

2) Quartzite ( $\delta_i/\delta_i^{cr} \approx 0.25$ ). Non-homogeneous structure of the reflected beam is visualized very well; the displacement will be approximately equal to  $\Delta^0$ .

3) Sandstone ( $\delta_i/\delta_i^{cr} \approx 3$ ). An abrupt decrease

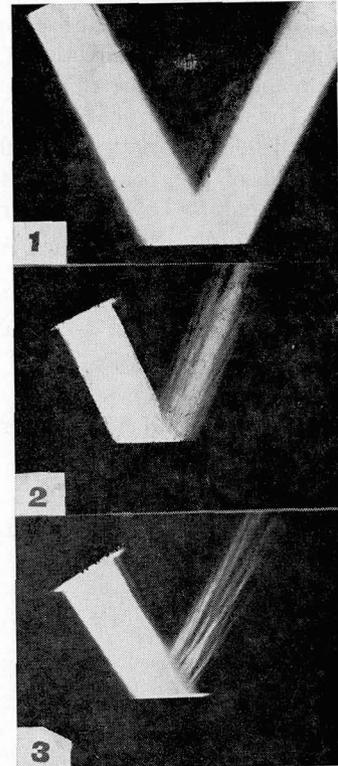


FIG. 3.

of intensity of the reflected beam occurs near the angle  $\Theta_R$ ; in this case the sideways displacement will not be observed.

These photographs confirm the theoretical conclusions presented above.

The peculiarities of beam reflection suggested here can be used for many problems of ultra-acoustics. One of the possible applications of the method considered above is the measuring of attenuation in solids possessing relatively high losses caused by scattering or dissipative mechanism, for example, in rocks where common methods for measuring of attenuation cannot be used.

Another very interesting application is connected with the study of elastic surface waves on single crystals. The recent work [4] shows that on the free surface of anisotropic crystal the so-called pseudo-surface waves can exist besides the normal surface Rayleigh waves. The pseudo-surface wave appears when the phase velocity exceeds one of the lowest quasi-transverse bulk wave in the same direction. An important peculiarity of these waves is in attenuation along the surface due to the energy radiating into the crystal (propagation vector tilted down into the solid). The optical method for the examination of such waves may be used. Measuring both the angular dependency of the reflected beam intensity and the sideways displacement in frequency range allows to determine the velocity and estimate the attenuation of pseudo-surface waves.

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