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II. — INTERACTIONS GRANITATIONELLES EN ASTROPHYSIQUE

GRAVITATIONAL ENERGY YIELDS

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Résumé. — On met en opposition l'effondrement dynamique d'une structure sphérique sans pression interne suffisante avec la contraction lente, contrôlée par le rayonnement, qui prend place lorsque la pression existe. On montre qu'en relativité générale la contraction se transforme inévitablement en effondrement dynamique lorsque le potentiel gravitationnel correspond à une fraction importante de la masse au repos. On en conclut que la gravitation ne peut guère convertir plus de 1 % de la masse au repos en rayonnement.

Abstract. — The phase of slow gravitational contraction of a spherical body, with its control by radiation from the body, is contrasted with the dynamical collapse that occurs when gravitation is not balanced by pressure forces. It is shown that in general relativity contraction inevitably changes to collapse when the gravitational potential energy becomes a significant fraction of rest mass. Therefore gravitation can only convert at most about 1 % of rest mass into radiation.

The question to be discussed concerns the amount of energy that can be obtained from gravitation. This question is an old one. It has been asked frequently in the past fifty years but it became of particular interest when the quasars were discovered. The problem of finding large sources of energy became acute : the best source we know is the stellar nuclear reaction which cannot put into radiated energy more than barely 1 % of the rest mass. So the question is : can one use gravitation to turn a larger fraction of rest mass into radiated energy ?

I would like to start by discussing quite briefly the Newtonian situation, because I think that it is necessary to understand it in depth. Consider a spherical body in equilibrium so that the pressure gradient everywhere exactly balances the gravitational force. We make no assumption about structure other than that it is a spherically symmetrical body. If the distance from the centre is now slightly reduced, by the same factor for each particle, two things will evidently happen : the density of each portion of matter will increase, and the gravitational force on each such portion of matter will also increase. The crucial question therefore is whether the material is such

that the increased density results in an increase in pressure greater than, equal to or smaller than that required to counterbalance the increase in gravitational force. In the first case the body will re-expand (so that the original situation was stable), in the last case the body will collapse, while the intermediate case corresponds to neutral equilibrium. Suppose now that, for each piece of matter, the pressure varies with the same power γ of the density. Then it turns out that the required increase in pressure for neutral equilibrium occurs if $\gamma = \gamma_c (= 4/3)$. Thus, if $\gamma > \gamma_c$ the body is stable, if $\gamma < \gamma_c$ the body is unstable (*). If γ is greater than $4/3$ and the pressure depends on the temperature as well as on the density, the body can contract slowly provided it cools (e. g. by radiating) and, conversely, if it radiates it will contract. We can imagine the body in a state of contraction which is quasi-stationary (the accelerations due to the motion are negligible compared with the acceleration due to gravitation). We can evaluate the total amount of energy the body has radiated between two different states : this must be exactly equal to the diminution

(*) For most gases $\gamma > 4/3$.

in the total energy content (material energy plus gravitational potential energy) of the body. Note that the opacity of the body (which controls the *rate* of radiation) in no way affects the total amount radiated between the two states, but controls only the rate of contraction from one state to the other.

If γ is smaller than $4/3$, the body is strictly dynamically unstable. It will collapse with an acceleration of the order of the free fall. The motion is self-accelerating. This collapse is due to a lack of balance between gravitation and pressure, and has nothing to do with loss of heat by radiation. If the body were totally opaque it would nevertheless collapse. There is hence not the slightest reason why any energy must emerge from a star when it is collapsing.

We can say in a condensed way : contraction occurs only because of radiation and its speed is controlled by it ; in collapse, radiation is an incidental phenomenon.

How does General Relativity come into this long established Newtonian picture ? Relativity differs in its results from Newtonian theory only in a limited number of cases :

1. when relative velocities are comparable with the velocity of light ;
2. when the gravitational potential becomes comparable to the rest mass ; and
3. when the pressure energy becomes comparable with the rest mass.

It is the second of these cases which is relevant to our problem.

One can easily talk about this in relation to the dimensionless number GM/Rc^2 , where G is the Newtonian constant of gravitation, M the mass of the body and R the radius of the body. This is in fact a multiple of the ratio between the potential energy (a fraction of GM^2/R) and the rest mass Mc^2 .

Incidentally, there is no logically necessary connection between the gravitational potential becoming large and superdense bodies, because one can have $GM/Rc^2 \sim 1$ when the density is not at all large, provided M is large. After all, the density varies like MR^{-3} . If the mean density is that of air, $GM/Rc^2 = \frac{1}{2}$ provided $M = 5 \times 10^9 M_\odot$. (M_\odot , the mass of the Sun, equals 2×10^{33} g.)

General Relativity is a non-linear theory and so the critical value γ_c depends on GM/Rc^2 . When GM/Rc^2 is small, relativity is virtually identical with Newtonian theory, and so $\gamma_c = 4/3$, but for larger values, the non-linearity shows itself by gravitation becoming stronger than according to Newtonian theory. Thus γ_c becomes larger than $4/3$, particularly in the central

part of the body. Indeed, for otherwise permissible values of GM/Rc^2 (depending on the structure and for example equal to $4/9$ for a sphere of uniform density) the central value of γ_c goes to infinity. Any given material has some perfectly finite value of γ , and thus no material is strong enough not to collapse ($\gamma < \gamma_c$) at the centre of a body with even a modestly large GM/Rc^2 . Thus for any material, contraction can only continue to a certain point, and then collapse will occur. You might think that one could get a lot of energy by having a body of a material with $\gamma > 4/3$ and letting it contract slowly all the way until it becomes very small ; but in fact just when it comes down to a size where GM/Rc^2 begins to be significant, contraction will change to collapse because γ_c will have risen above γ . When it collapses we cannot expect that much energy will emerge : the collapse goes on very fast with a dynamical speed corresponding to free fall and, as has been said, in contrast to contraction, radiation now plays no part in permitting collapse.

Although there is nothing to prevent radiation from a collapsing body, it seems unlikely that much energy can emerge. Not only is time rather short, but there is a characteristic relativistic effect reducing the radiation, namely the substantial red shift at the surface, partly due to gravitation, partly due to Doppler effect. The radiated power is diminished by the square of this red shift factor which becomes infinite when, at the surface, $GM/Rc^2 = \frac{1}{2}$. And so at that stage the energy output certainly stops.

The essential energy yields can only occur in the stage of contraction and not during collapse. When contraction goes over into collapse is a question that depends on the material but, roughly speaking, one can say that not more than about 1 % of the rest mass can be radiated out of the body. This estimate arises because the potential energy per unit mass is only a fraction of GM/Rc^2 which at the onset of collapse is likely to be about $\frac{1}{4}$ or less. Moreover, during contraction a substantial portion of this gravitational energy is used up to heat the body, and only the remainder escapes as radiation. Hence gravitation can in our model produce an external energy yield not exceeding in order of magnitude the possible nuclear yield.

DISCUSSION

Question : What is the order of magnitude of GM/Rc^2 for the whole of the universe ?

Answer : Theoreticians would like to think that this number, or rather the associated and more immediately significant number $4 \pi G \rho v^2$ [where ρ is the mean

density of matter and ν is the time constant of the velocity-distance relation (distance divided by velocity)] is of order unity. What is known observationally about matter suggests a rather lower figure, perhaps lower by a factor 30 or so, but even a minute density of matter difficult to detect in intergalactic space could bring the figure up to order one.

Question : I don't quite get why γ goes to infinity. The critical γ , I thought, is of finite value larger than $4/3$, but not infinite.

Answer : For a Schwarzschild sphere (used as an example) the central pressure goes to infinity for a finite value of radius and density, and this implies an infinite γ . The same is true of more general models.

Question : Should we consider this as a definite and without appeal condemnation of a theory which wants to explain quasars by gravitation ?

Answer : Only within the framework of general relativity. Any modification of the theory (e. g. that proposed

by Hoyle and Narlikar) could lead to a different answer.

Question : This is a slightly different question not concerned so much with the energy but with the state of matter in such a gravitational collapse. What happens when the density goes to such high values ?

Answer : I know very little about this, but Wheeler and his colleagues have studied this question.

Question : Is there any alternative guessed about the source of energy in quasars ?

Answer : If the mass is sufficiently large, ordinary sources of energy will do. There is a particularly attractive suggestion that a quasar is a globular cluster of stars in the final stages of an evolution due to the evaporation of stars carrying positive energy away and thus leaving the total energy of the remainder more negative than before. Collisions of stars in such a very dense cluster at perhaps one tenth of the speed of light would give a plausible explanation of the radiation.