



**HAL**  
open science

# Non-universal roughening of kinetic self-affine interfaces

Yi-Cheng Zhang

► **To cite this version:**

Yi-Cheng Zhang. Non-universal roughening of kinetic self-affine interfaces. *Journal de Physique*, 1990, 51 (19), pp.2129-2134. 10.1051/jphys:0199000510190212900 . jpa-00212516

**HAL Id: jpa-00212516**

**<https://hal.science/jpa-00212516>**

Submitted on 4 Feb 2008

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

---



---

# LE JOURNAL DE PHYSIQUE

---



---

*J. Phys. France* 51 (1990) 2129–21341<sup>er</sup> OCTOBRE 1990, PAGE 2129

Classification

*Physics Abstracts*

05.70Ln — 61.50Cj — 68.35Fx — 05.40+j

**Short Communication****Non-universal roughening of kinetic self-affine interfaces**

Yi-Cheng Zhang

INFN and Dipartimento di Fisica, Università di Roma, Piazzale A. Moro, 00185 Roma, Italy

*(Received on July 24, 1990, accepted on July 26, 1990)*

**Abstract.** — Recent experiments of wetting immiscible displacement in porous media and bacteria colony growth are considered as realizations of the Eden interfaces. I propose an explanation of the discrepancy in the scaling behavior between the experiments and the theory. Due to an instability inherent in the growth processes, the presence of unbounded non-Gaussian noise can violate the naive scaling universality.

In many physical situations an interface separates two distinct phases. When one phase advances into the other under a driving force a kinetic interface is created. Kinetic interfaces cannot generally be studied in the fashion of equilibrium physics. One simple model is the Eden model [1] which was originally motivated to describe biological growth.

On a strip geometry an Eden interface can be described by a stochastic differential equation [2]. The system is in a  $d + 1$  dimensional space, with the direction of growth singled out. Denote by  $h(\mathbf{x}, t)$  the height of the interface at time  $t$  and position  $\mathbf{x}$  which is a vector in the  $d$ -dimensional base plane. We have the following evolution equation:

$$\dot{h} = \nabla^2 h + \frac{1}{2}(\nabla h)^2 + \eta(\mathbf{x}, t), \quad (1)$$

where the coefficient of the surface tension and the coupling constant are assumed to be 1, since they can be absorbed by a rescaling of  $h$ ,  $\mathbf{x}$  and  $t$ .  $\eta(\mathbf{x}, t)$  is stochastic noise with a two-point correlation:

$$\langle \eta(\mathbf{x}, t)\eta(\mathbf{x}', t') \rangle = 2\delta(\mathbf{x} - \mathbf{x}')\delta(t - t'), \quad (2)$$

where  $\langle \cdot \rangle$  denotes the ensemble average over realizations of noise, for many practical purpose it can be replaced by a temporal average.

The key ingredients of equations (1) and (2) are the surface tension term  $\Delta h$  which tends to locally smoothen out fluctuations; a non-linear term  $(\nabla h)^2$  from  $\sqrt{1 + (\nabla h)^2} = 1 + (\nabla h)^2/2 + \dots$ , which represents the uniformly lateral growth; and stochastic noise that is believed to be ubiquitous in realistic systems.

There is considerable theoretical [3] (analytical and computational) research activity on problems related to equation (1). It is believed by many workers in the field that equation (1) provides a valid hydrodynamic description of a broad class of interface models involving the local growth mechanism. The most interesting observable is the average width [2]  $w(L, t)$ , which is expected to be a scaling function of the sample size  $L$  and the growth time  $t$ .  $w(L, t) \sim L^\chi f(t/L^z)$ ,  $f(u \rightarrow \infty) \rightarrow \text{const}$ ,  $f(u \rightarrow 0) \rightarrow u^{\chi/z}$ . Surfaces with  $\chi < 1$  are called self-affine fractals, they are anisotropic and globally flat ( $w/L \rightarrow 0$ ). Theoretical analysis gives the scaling exponents exactly in 1+1 dimensions:  $\chi = 1/2$  and  $z = 3/2$ . Higher dimensional exponents so far are only known numerically [3].

Very recently there are some experimental realizations of the interface growth, which should inject new enthusiasm into the field. Rubio *et al.* [4] and Horváth *et al.* [5] have performed experiments on wetting immiscible displacement in a porous medium. Another experiment by Vicsek *et al.* [6] studies bacteria (*Escherichia coli* or *Bacillus subtilis*) colony development on a nutrient (agar) plate. This is very close to the original spirit of Eden.

Despite of their apparently dissimilar microscopic origins, the above growth mechanisms are local and approximately homogeneous. I attempt to view that the above interfaces are effectively described by the simple equations (1) and (2).

Rubio *et al.* reported that their self-affine interface has the roughening exponent  $\chi = 0.73 \pm 0.03$ . Horváth *et al.* found that for their experiment  $\chi \approx 0.81$ , they also measured the time exponent  $\chi/z \approx 0.65$ . For the bacteria expansion problem, Vicsek *et al.* estimated that the bacteria colony has a self-affine border with the roughening exponent  $\chi = 0.78 \pm 0.06$ . While these findings are very interesting, the values of the roughening and time exponents are certainly in contradiction with the current theoretical prediction [2, 3]  $\chi = 1/2$ . Elsewhere I shall show that long range correlated noise cannot account for the larger roughening exponent.

In this work I propose that the discrepancy between the experiments and theory is due to the fact that in nature there is no *Gaussian* noise, as it is usually assumed in theoretical models. In most of statistical physics microscopic details are not important on large scales, this goes under the name of *universality*. For the present growth problem I shall show that microscopic details can indeed influence large scale behavior in a substantial way, thus violating the naive universality concept. I rely on the hypothesis that equations (1), (2) are still a valid hydrodynamic description of the above experiments, but abandon the requirement that  $\eta$  is a Gaussian or truncated noise. To be precise I assume noise is independently distributed on each site (a lattice regularization is understood) according to the following distribution density:

$$P(\eta) \sim \frac{1}{\eta^{1+\mu}}, \quad \eta > 1; \quad P(\eta) = 0 \quad \text{otherwise.} \quad (3)$$

This distribution does not have an absolute cutoff, singularly large values of  $\eta$  can appear. This does not imply however, that infinitely large value will appear. In any finite sample of  $N$  degrees of freedom there is a typical cutoff on the largest  $\eta$  value,  $\eta_{\max}$ . We have  $N/\eta_{\max}^{1+\mu} \sim O(1)$ , which implies

$$\eta_{\max} \sim N^{1/(1+\mu)}. \quad (4)$$

In this work I restrict myself to cases when  $\infty > \mu > 2$  so that the above distribution has a well-defined mean and variance. In a continuum approximation, equation (2) can still be retained.

I simulate a discrete version of equation (1) on a square lattice

$$h(i, t + 1) = \max(h(i - 1, t) + \eta(i - 1, t), h(i + 1, t) + \eta(i + 1, t)), \quad (5)$$

where  $i$  and  $t$  are integers,  $i$  runs over only even indices if  $t$  is even, odd if  $t$  odd.  $h(i, t)$  and  $\eta(i, t)$  are continuous variables. The initial condition is  $h = 0$ , periodic boundary conditions are used in the transverse direction  $i$ , the transverse size is  $L$ . Equation (5) implies a checkerboard updating rule. It can be considered as a simultaneous ballistic deposition model [7]. Equation (5) can be derived in the directed polymer representation [8] in the zero temperature limit. Elsewhere I will show that the particular choice of parameters of equation (1) does not influence the scaling behavior for the same  $\mu$ . I conjecture, on the basis of the following simulations, that there is a (sub-) universality for each  $\mu$ . I have taken this liberty to choose the much simpler equation (5) to carry out most of the simulations.

An interface is said to be in a stationary state when its average width reaches a constant. In figure 1 snapshots of an interface at successive time steps are shown. The transverse size is  $L = 1000$ . The interface has evolved for 50000 time steps after starting from a straight line geometry. It is for  $\mu = 3$  and the interface's roughening exponent is  $\chi = 0.75 \pm 1$ . The growth direction is upwards. The longer time interval between the snapshots is 30 time steps, the shorter interval at the top is 5 time steps. One can notice that occasionally there are some abnormally large thrusts on the growth front.

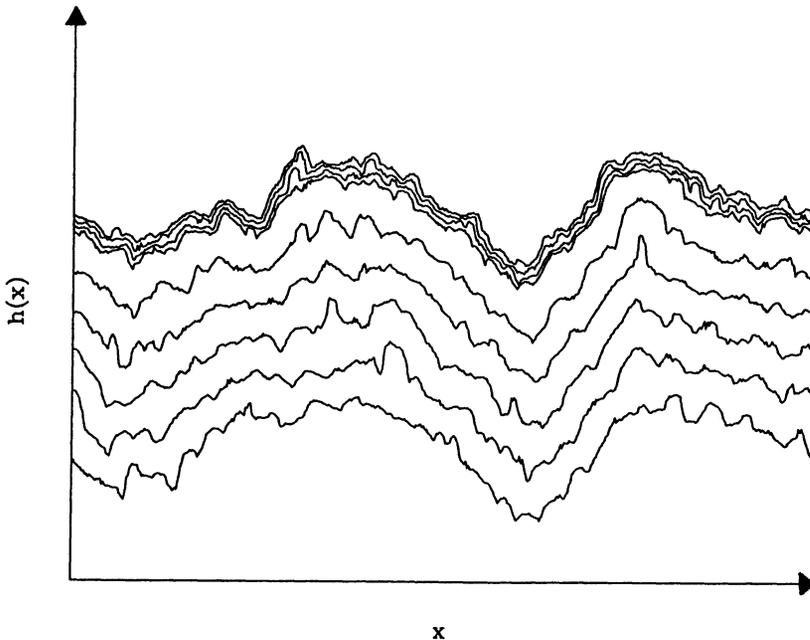


Fig. 1. — Successive snapshots of an interface evolved according to equation (5). The longer time intervals are 30 time steps, the shorter ones are 5 time steps.

Figure 2 presents two examples of the scaling behavior of the interface's width against system size on a log-log scale. The data are for two different values of  $\mu$  (2.5 and 3). The statistics is obtained by averaging the width over time. The initial relaxational data are abandoned.

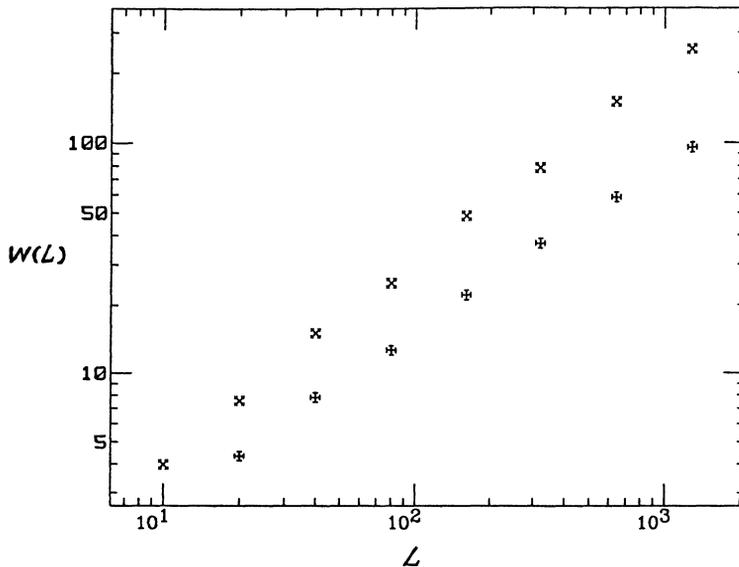


Fig. 2. — The interface width  $w(L)$  (vertical) against the size  $L$  for two values of  $\mu$  (2.5, 3). The data appear to scale with the exponent  $w(L) \sim L^\chi$ ,  $\chi \approx 0.82, 0.75$ , respectively.

In figure 3 I show the systematic assessment of the simulation results. The estimates of the roughening exponent  $\chi$  is plotted against  $\mu$ . For  $\mu \rightarrow 2$  it appears that the interface has the maximally attainable self-affine roughening exponent  $\chi = 1$ . Elsewhere [9] I will address what can happen to an interface if  $\mu \leq 2$  (Lévy distribution). In the inset I plot the sum of the two exponents  $\chi + z$ , where the temporal scaling exponent  $z$  has been calculated independently [10] in the directed polymer representation. We see that the sum is approximately 2, thus confirming the exponent identity which can be derived on the basis of a Galilean invariance [11].

We may conclude that the growth equation (1) combined with unbounded noise (3), can account for the observed larger roughening exponent. Even though the simulations are made on rather small lattices, I expect that larger lattices would not change at least qualitatively the present conclusion. It is worthy noting that the experiments are also performed on limited sizes. However, there are many questions to be answered. I should emphasize that I do not have solid microscopic justifications as to why in the experiments power-law noise distributions are preferred over Gaussian or truncated ones. I want nevertheless to argue that it is equally hard to ascertain that they are not there. After all, this has to do with microscopic details to which traditionally not much attention has been paid. It is plausible that unbounded noise in real systems comes from rare combinations of all contributing factors, the larger the system, the rarer a combination can be.

Is there a way to check if the present approach is applicable to the experiments, besides the fact that roughening exponent  $\chi$  can be made to agree? I believe that the effective noise distribution can be revealed: examine the snapshots on the top of figure 1, if the time interval is short enough between successive snapshots the effective *microscopic* noise could be picked up and analyzed. This should be possible for the experiments as well.

It may be surprising that microscopic details do influence the large scale behavior for the growth processes described by (1) and (2). A clue to an intuitively understanding of this apparant puzzle is the following: while an interface is growing, like that shown in figure 1, it encounters some rare

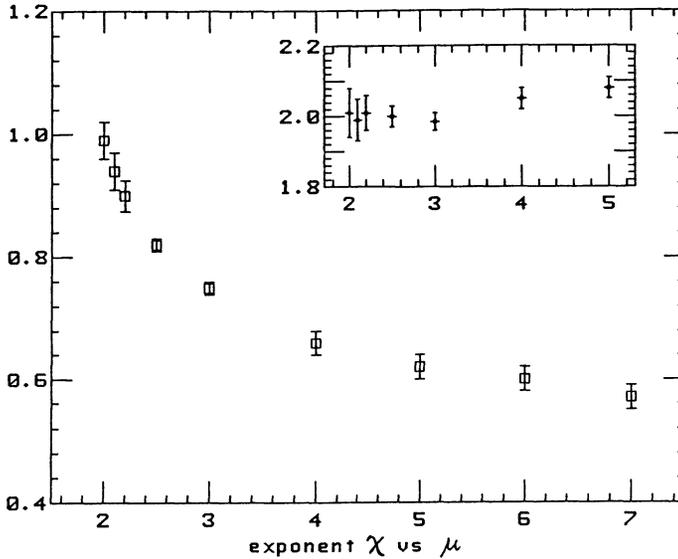


Fig. 3. — Estimates of  $\chi$  (vertical) vs. various values of  $\mu$ . Error bars are from a subjective assessment. In the inset the sum  $\chi + z$  is compared to the expected value 2.

thrusts with respect to average places. These rare thrusts have much more influence than their small statistical weight would suggest to have. This is so by two kinds of mechanism : 1) Amplification: during the lateral growth a rare large thrust will expand laterally to cover its neighboring area as if there were many simultaneous thrusts; 2) Memory: a large thrust probably will result in a hill on the interface, this geometry will be remembered for a long time. If during this long time there appears another rare thrust they can have combined effect to roughen the interface.

### Summary.

In this work I point out an instability inherent (or hidden) in the growth model (1) and (2). This instability can be attributed to the above amplification and memory mechanisms of rare, abnormally large noise values. I suggest that the experimentally observed larger roughening exponent is a manifestation of this instability.

### Acknowledgements.

I thank J. Amar, H. Herrmann, V. Horváth and D. Stauffer for helpful discussions. This work took about 1hr CPU on a Cray-YMP.

### References

- [1] EDEN M., Symposium on Information Theory in Biology, Ed. H.P. Yockey (New York, Pergamon Press) 1958, p.359.

- [2] KARDAR M., PARISI G. and ZHANG Y.-C., *Phys. Rev. Lett.* **56** (1986) 889.
- [3] For recent reviews see KRUG J. and SPOHN H., *Solid Far From Equilibrium: Growth, Morphology and Defects*, Ed. C. Godrèche (1990) to be published;  
VICSEK T., *Fractal Growth Phenomena* (World Scientific, Singapore) 1989;  
FAMILY F., *Physica A*, to appear.
- [4] RUBIO M.A., EDWARDS C.A., DOUGHERTY A. and GOLLUB J.P., *Phys. Rev. Lett.* **63** (1989) 1685.
- [5] HORVÁTH V.K., FAMILY F. and VICSEK T., Emory preprint 1990, to be published.
- [6] VICSEK T., CSERZŐ M. and HORVÁTH V.K., *Self-Affine Growth of Bacterial Colonies*, to appear in *Physica A* (1990).
- [7] FAMILY F. and VICSEK T., *J. Phys.* **A18** (1985) L75.
- [8] KARDAR and ZHANG Y.-C., *Phys. Rev. Lett.* **58** (1987) 2087.
- [9] ZHANG Y.-C., to be published (1989).
- [10] MARINI-BETTOLO-MARCONI U. and ZHANG Y.-C., to be published.
- [11] MEDINA E., HWA T., KARDAR M. and ZHANG Y.-C., *Phys. Rev.* **A39** (1989) 3053.

Cet article a été imprimé avec le Macro Package "Editions de Physique Avril 1990".