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Conductivity and rupture in crack-deteriorated systems

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Résumé. — La conductivité électrique et la force de rupture de systèmes modèles fissurés à deux dimensions sont étudiées expérimentalement. Deux cas sont considérés. Dans le cas 1, les fissures sont disposées aléatoirement sur les liens d'un réseau carré dessiné sur le continuum. La taille finie de nos systèmes permet à peine d'atteindre le régime critique de la percolation et on mesure plutôt un comportement décrit par des théories de milieux effectifs. Dans le cas 2, les positions et les orientations des fissures sont choisies aléatoirement sur le continuum (fromage « bleu d'Auvergne »). Dans ce cas, de forts effets d'écrantage permettent d'atteindre le régime critique et de mesurer les exposants critiques correspondants. Nous avons comparé ces mesures avec des résultats théoriques récents qui prennent en compte les différents modes de déformation à l'extrémité des fissures. En particulier, le flambement de feuilles minces joue un rôle important et peut changer les exposants critiques.

Abstract. — Experimental results on the conductivity and rupture properties of two-dimensional continuum crack deteriorated model systems are presented. Two different systems are considered. In case 1, cracks are randomly positioned on the bonds of a regular square lattice drawn on the continuous system. Our finite size systems barely reach the critical percolation regime. The results rather correspond to a mean field regime which may be described by effective medium theories. In case 2, crack positions and orientations are randomly chosen in the 2D continuum (« blue cheese »). In this case, strong screening effects allow us to obtain the critical regime and to measure the corresponding critical exponents. The measurements are compared with recent theoretical results which take account of the different modes of deformations occurring locally at the crack level. In particular, buckling in thin foils is found to be an important deformation mode which screens efficiently the stress enhancement at the tips of the cracks, thereby changing the critical exponents.

1. Introduction.

Understanding the transport properties of heteregeneous or random media constitutes a major challenge of present research activities and also receives strong incentive in the field of applied and engineering science due to the many real and potential applications. Among the heterogeneous media, systems containing crack play a fundamental role : cracks are the

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primary enemy to fight in order to prevent fatigue and eventually failure of mechanical systems. Damage occurs in a system by the apparition of micro-cracks which eventually coalesce and create a macro-crack which may destabilize and lead to global rupture. At another level, it is through arrays of cracks that water or oil permeate in soil and can be recovered. On the other hand, cracks or rather faults, as they are called in this context, control the brittleness of the earth crust and thus the occurrence of earthquakes. One could think of many other relevant examples where cracks play an important role.

Experimental and theoretical studies of crack deteriorated systems have essentially been developed in the field of theoretical and applied mechanics. The continuing refinement of these approaches is the subject of a very intense research effort in this field. However, focus has been put on single or few crack problems like periodic lattices of cracks by using exact analytical methods. For dilute crack concentration, homogeneization approaches have been used such as, for instance, effective medium theories [1-3] or continuous damage theory [4]. These approaches apply when certain symmetries exist or when the concentration of cracks is not too large. However the weak crack concentration limit does not always apply in many systems of physical and industrial interest. This is our main motivation for studying the other limit of strong heterogeneity within the frame of the percolation model. A second motivation is found from the field of statistical physics where it was realized that the critical behaviour of electrical conductivity, fluid permeability, elasticity modulus and failure threshold was different in discrete-lattice percolation and in a class of continuous percolation models (« Swiss cheese » model [5]) pointing out the existence of at least two universality classes of transport phenomena in percolation. Percolation of cracks (termed « blue cheese » model in [6] has been shown to yield still another critical behaviour worth studying for its own sake.

Many studies have been devoted to the theoretical prediction of the conductivity in disordered systems, but less work has been devoted to consideration of these predictions on the experimental side [7]. Benguigui [8] has measured the electrical conductivity G of Cu and Al foils punched with holes randomly distributed on a square lattice and on a continuum near the percolation threshold. He finds that $G \propto (1 - N/N_c)^t$ with $t = 1.1 \pm 0.2$ where N is the number of holes. Sofo *et al.* [9] have measured the conductance of randomly punched metallized mylar sheets and find the critical conductivity exponent $t = 1.4 \pm 0.2$. Lobb and Forester [10] find the critical exponent $t = 1.24 \pm 0.13$ for holes distributed on a continuum. All these results confirm the prediction of Halperin *et al.* [5] that t has the same value for discrete lattice percolation network or for disordered continuum system in two dimensions.

In the following, we report experimental results on the conductivity and rupture properties of two-dimensional continuum crack deteriorated model systems. In section 2, we define more precisely the two types of systems which have been studied : in case 1, cracks are randomly positioned on the bonds of a regular square lattice drawn on the continuous system. In case 2, crack positions and orientations have random values on the continuous system. This is the « blue cheese » model introduced in reference [6]. The geometrical properties of these systems which are reported in a separate paper [11] and the theoretical transport properties which have been previously analysed [6] are briefly summarized in section 2. In section 3, the measured electrical conductivity and mechanical rupture properties of the two models are described and compared to predictions. In the first case, the finite size of our system barely allows us to reach the critical percolation regime. Both electrical and rupture properties could be explained from effective medium approaches. In the second case, strong screening effects allow us to obtain the critical regime for system sizes of the same order as in the first case and to measure the corresponding critical exponents. The results of the rupture are compared with the theoretical predictions of the « blue cheese » model which takes account of the different modes of deformations occurring locally at the crack scale. In particular, buckling in thin foils

is found to be an important deformation mode which screens efficiently the stress enhancement at the tips of the cracks, thereby renormalizing the critical exponents to their value for discrete lattices.

2. Theoretical properties of crack-deteriorated systems.

2.1 GEOMETRICAL PROPERTIES OF THE PERCOLATION MODELS. — We restrict ourselves to the two-dimensional space displayed in figure 1. In model 1, empty rectangular holes of length « a » with negligible width are distributed along the bonds of a square lattice drawn on a uniform electric or elastic medium. In model 2, the cracks are randomly distributed in position and orientation in a surface of size $L \times L$. As the number $n = N/L^2$ of cracks per unit surface increases, a percolation threshold n_c is reached corresponding to the geometrical deconnection of the sample in multiple fragments. We denote $n_c(1)$ and $n_c(2)$ for models 1 and 2 respectively. The relevance of these blue-cheese models is suggested from the crack structure of many mechanical [12] (damaged crystals, solids, ceramics, rocks...) and natural systems [13] (arrays of faults in geology in relation to oil recovery, geothermics and earthquakes [14]...) which often consist in random arrays of micro-cracks of vanishing width.

The geometrical properties of model 1 are those of the discrete bond percolation model. In particular, it is known exactly that $n_c(1) = 1/2$. For model 2, previous numerical studies have shown that $n_c(2)$ decreases as the crack length a increase i.e. $n_c(2) \approx Ca^{-2}$ [15-17], resulting in the existence of a quasi-invariant $C = n_c a^2$ independent of a. The intuitive idea is that each crack defines an excluded surface of the order of a^2 around it. Hence, the critical percolation threshold $n_c(2)$ corresponds to the presence of a constant average number of cracks in each typical surface a^2. The value of the invariant C has been estimated by a number of authors. A recent estimate [11] obtained from a careful analysis of finite size effects gives $C = 5.640 \pm 0.005$ for the asymptotic limit $a \to 0$. Thus, near threshold, the number of cracks in a surface a^2 is of the order of the invariant $C \approx 5.6$ in contrast to the value « 1 » in the same surface for model 1. In other words, almost six times as many cracks are needed in model 2 compared to model 1 to reach the percolation threshold, all other values being equal.

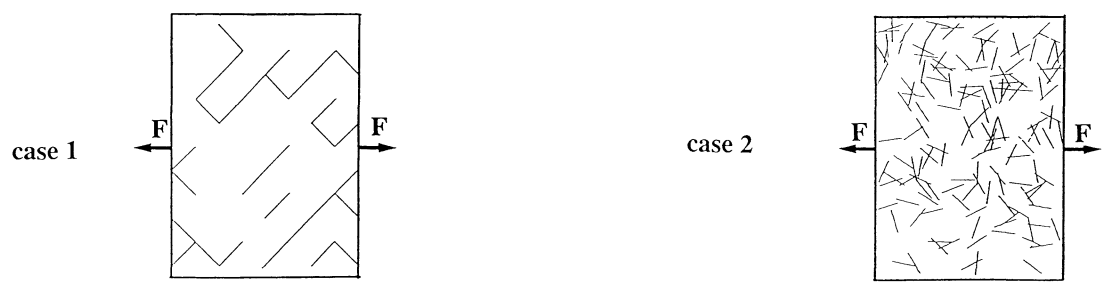

Fig. 1. — Different types of chosen crack distribution. a) case 1 : cracks are randomly positioned on the bonds of a regular square lattice. b) case 2 : crack positions and orientations are randomly chosen in the 2D continuum.

2.2 TRANSPORT PROPERTIES. — In the so called « Swiss cheese » model, Halperin *et al.* [5] used a scaling analysis to estimate various critical exponents of the transport properties (conductivity, permeability, Young modulus...) of systems punched with holes. In the « blue cheese » model [6], Sornette extended this analysis to the case for which the transport

medium is the space between randomly placed clefts. In both cases it was found that the critical behavior of the electrical conductivity is the same for model 1 (discrete lattice) and model 2 (continuum). The electrical conductivity obeys the power law $\Sigma/\Sigma_0 \propto (p - p_c)^t$ near the percolation threshold with the universal value $t = 1.300$ found numerically for two dimensional systems [18].

2.3 RUPTURE PROPERTIES. — In a system containing cracks, brittle rupture following the Griffith mechanism will occur under tensile stress (mode I [19]). This rupture scenario is expected to be prominent for cracks with vanishing width. In this failure mechanism, the stress concentration at the apex of a crack tends to open the two edges and to lengthen the crack, thus leading to failure. This mechanism must be considered when the tips of the cracks are sharp, i.e. without screening due for example to the presence of rounding or of a cavity at the tips of the cracks. The Griffith criterion states that a crack growth is governed by the balance between the mechanical energy released and the fracture surface energy spent as the crack propagates. In the blue cheese model, each crack is surrounded by many other cracks which interact with it, locally enhance or screen the stress concentration at the crack tip. It has been argued in reference [6] that the Griffith criterion could be extended in the presence of these interactions and the analysis could be reduced to the study of the generic configuration involving one crack in close proximity (to within a distance δ) to a border created by another crack (Fig. 2). The analysis of the stress enhancement in this configuration leads to the prediction of the critical behaviour of the rupture threshold. In general the critical exponent of the rupture is distinct from its discrete counterparts and crucially depends upon the deformation mode. However if the sheet is thin, buckling (out of plane deformation) occurs in order to relax more efficiently the in-plane elastic stresses [6]. In this case, it has been shown that buckling suppresses the predicted additive correction (equal to 1/2) to the failure critical exponent $f = d\nu$ due to stress fluctuations at the crack tips in this continuous crack percolation model (d is the dimensionality of the system and ν is the critical exponent of the correlation length). Hence, one recovers the discrete lattice exponent for the critical behaviour.

This value of the failure exponent $f = d\nu$ in the discrete case stems directly from the node-link-blob structure of the percolation latice near threshold. The force felt by a bond in a

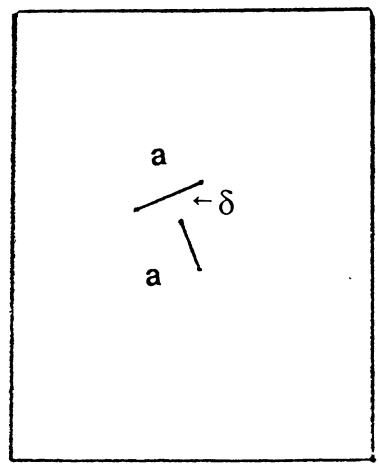

Fig. 2. — Typical configuration of two neighboring cracks of length a, showing the micro-bond topology of thickness δ.

macro-link is proportional to ξ^{d-1} due to enhancement coming from the bottleneck effect that the force is transmitted only by the macro-links separated from each other by the typical distance ξ. An additional factor ξ appears which stems from the lever arm effect which increases the local force acting on a microbond to a value proportional to $\xi^d \sim (N_c - N)^{d\nu}$. The failure is thus proportional to ξ^{-d} thereby yielding the rupture exponent $d\nu$ [20].

3. Experimental results.

We have performed electrical conductivity and rupture stress experiments [21] on aluminium sheets of thickness ~ 14 μm to 25 μm and typical size 30 × 30 cm. Each sheet is deteriorated by N identical craks of same length in the range $a = 1.5$ to 3 cm and distributed following to the rules of model 1 or 2. The cracks are cut with a thin blade and the width w of the cracks are negligible as compared to their length $a(w \leqslant a/100)$.

3.1 ELECTRICAL CONDUCTIVITY OF THE CRACK PERCOLATION MODELS. — To the best of our knowledge, no such experimental results have been reported previously in crack deteriorated systems. Cracks can represent for example the lines of fractures in rocks that must be avoided for an electrical current to pass through the system.

Figure 3a (resp. 4a) shows the variation of the normalized conductance $\Sigma_1(N)/\Sigma_1(0)$ (resp. $\Sigma_2(N)/\Sigma_2(0)$) as a function of the reduced crack number N/N_c for model 1 (resp. for model 2) where cracks are distributed on a lattice (resp. on a continuum). In case 1 (resp. 2), N_c is approximately equal to 340 (resp. 490). It is clear that $N_{c2} > N_{c1}$ since there is no « excluded surface » effects in model 2 in contrast to model 1 in which cracks are allowed to be in contact to others only by one extremity. This has already been discussed in section 2.1. Note that the two figures are not directly comparable since the sizes of the aluminium sheets were different for each system.

Note that the slopes at the origin of the two curves $\Sigma_1(N)/\Sigma_1(0)$ and $\Sigma_2(N)/\Sigma_2(0)$ are not equal, contrary to what one could expect naively on the basis that, in this « isolated crack » regime, the crack density is so small that interactions between different cracks can be neglected. The difference can be traced back to the fact that the decrease of the conductance, induced by the addition of a single crack oriented with an angle ϕ with respect to the average horizontal equipotentials, is a non-linear function of ϕ. We expect the conductance to depend on the first moments of ϕ which are different in the two models. For example, in model 1, the second moment is $\langle \phi^2 \rangle = (\pi/4)^2 \approx 0.617$ whereas in model 2, one must take the average value $\langle \phi^2 \rangle$ over all possible crack orientations between 0 and $\pi/2$ which gives $\langle \phi^2 \rangle = (\pi/2)^2/3 \approx 0.822$. Note that this problem is related to the Einstein correction for the viscosity in a suspension of rods as a function of the concentration of rods.

As the number of slits increases, the behaviour of $\Sigma_1(N)$ becomes even more different from that of $\Sigma_2(N)$. One can see clearly the upward curvature of $\Sigma_2(N)$ in figure 4a whereas this phenomenon can only be seen in figure 3a for $\Sigma_1(N)$ in the vicinity of N_c. We propose to explain this difference from the fact that transport properties of model 2 are controlled by strong screening effects between cracks [6, 21, 22] coming from the existence of overlapping. Indeed, near threshold, the number of cracks in a surface a^2 is of the order of the invariant $C \approx 5.6$ in model 2, in contrast to the value « 1 » in the same surface for model 1. We thus expect that, for a same value of a/L, ratio of the crack length over the system size, model 2 will sooner enter the critical region, as observed experimentally.

This explains why we have been able to reach the scaling regime for $\Sigma_1(N)$ in our finite samples only in the vicinity of N_c : an upward curvature can be observed near N_c in figure 3a, and increasing fluctuations are observed in this region. Trying to extract a critical exponent gives the curve shown in figure 3b which represents the logarithm of the conductivity

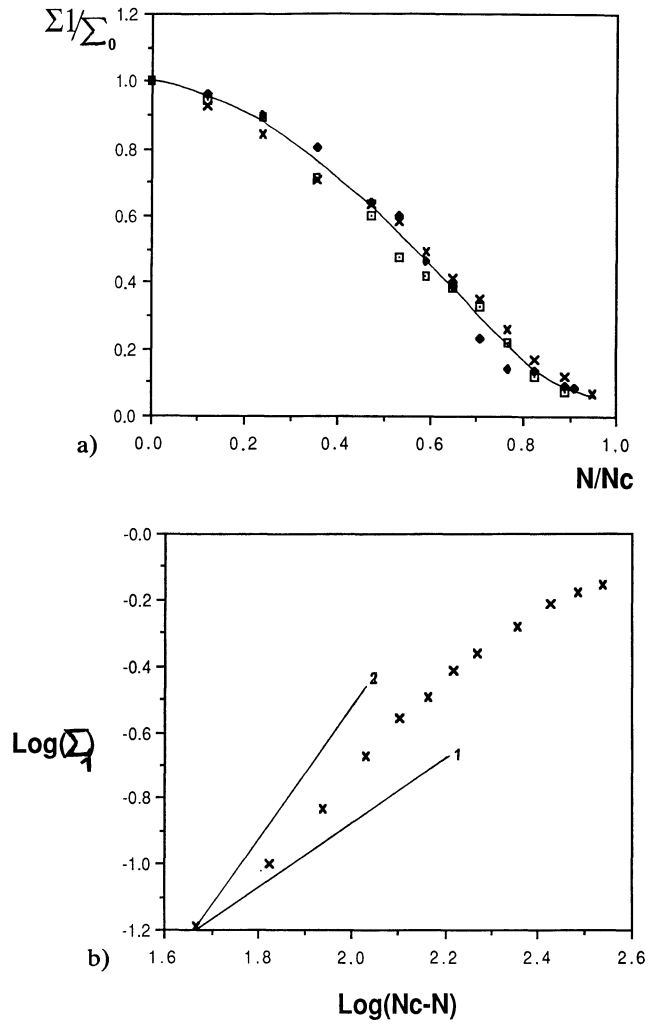

Fig. 3. — a) Normalized conductance $\Sigma_1(N)/\Sigma_1(0)$ of an aluminium sheet punched by cracks as a function of the number N of cracks for a distribution on a square lattice (model 1). Three experiments are reported corresponding to three different realizations of crack configurations all other parameters being fixed. In these examples, the system size is 30×30 cm, the length of the cracks is $a = 1.5$ cm and the percolation threshold is found near $N_c \approx 330$ for all three realizations. The superposition of the three curves gives an idea about the uncertainty of the measurements. The continuous line corresponds to the average values of the three measurements. b) Logarithm of the normalized conductance $\Sigma_1(N)/\Sigma_1(0)$ averaged over three measurements (continuous line in Fig. 3a) *versus* Log $(N_c - N)$ for a distribution of cracks on a square lattice. The straight lines 1 and 2 have a slope respectively equal to 1 and 2.

averaged over the three experiments presented in figure 3a as a function of Log $(N_c - N)$ with $N_c \approx 338$. Notwithstanding the limited range over which the scaling is expected to be valid, we find that $\Sigma_1(N)$ is well fitted, in the vicinity $N \approx N_c$ by the functional form

$$\Sigma_1(N) \sim (N_c - N)^t \qquad (1)$$

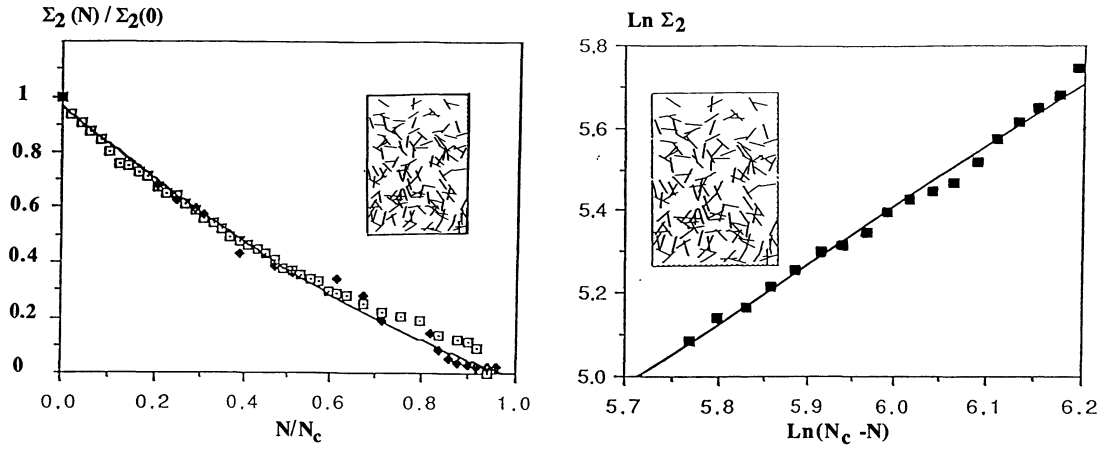

Fig. 4. — Normalized conductance $\Sigma_2(N)/\Sigma_2(0)$ of an aluminium sheet punched by cracks as a function of the normalized number of cracks for a continuum distribution : a) linear scale, b) log-log representation near the percolation threshold of the average over two experiments. In this example, the system size is $\approx 30 \times 30$ cm, the length of the cracks is $a = 3$ cm and the percolation threshold is $N_c = 490$.

with $t = 1.3 \pm 0.2$. A similar fit carried over for each of the three experiments gives values ranging from 1.2 to 1.4 confirming this result. Note that we have presented these fits for completeness but remain aware of their limited value coming from the smallness of the range over which they have been obtained.

In model 2, the scaling regime is clearly observed and a measure of the exponent t can be made with better confidence than for model 1. In figure 4b, the log-log plot version of figure 4a is presented from which we deduce the critical exponent $t = 1.4 \pm 0.1$ near the percolation threshold, also in agreement with the theoretical value 1.3 [5, 6]. Local spatial fluctuations of conductivity induced by the continuum do not appear to change the conductivity critical exponent from its discrete value in two dimensions.

3.2 MECHANICAL RUPTURE PROPERTIES OF THE CRACK PERCOLATION MODELS. — Scaling laws in rupture of percolation models have recently been much studied theoretically [23]. Yet experimental results are scarse. For example, Benguigui *et al.* [24] have studied the rupture stress of Cu and Al foils punched with holes distributed at random on a square lattice. They find that the rupture force F_r follows the behaviour $F_r \sim (1 - N/N_c)^f$ with $f = 2.5 \pm 0.2$ which compares well with prediction $f = 2\,\nu = 2.66$.

To our knowledge, no experimental results of rupture on crack deteriorated systems have been reported previously. The experimental set-up is represented in figure 5. One bar is fixed and an external force F is applied to the other mobile bar. Figure 6a represents the failure threshold stress F_r for model 2 as a function of the number of cracks. We see that a small number of cracks decreases markedly F_r. This decrease is stronger for the rupture stress than for the conductivity and can be understood from the fact that the rupture stress is sensitive to high order moments of the distribution of the stresses. At low concentrations, the failure threshold could, in principle, be obtained from an effective medium approach [2] which translates into the mechanical context the ideas developed, for example, in the determination of an effective dielectric constant in a composite medium. In this approach, the effective stress

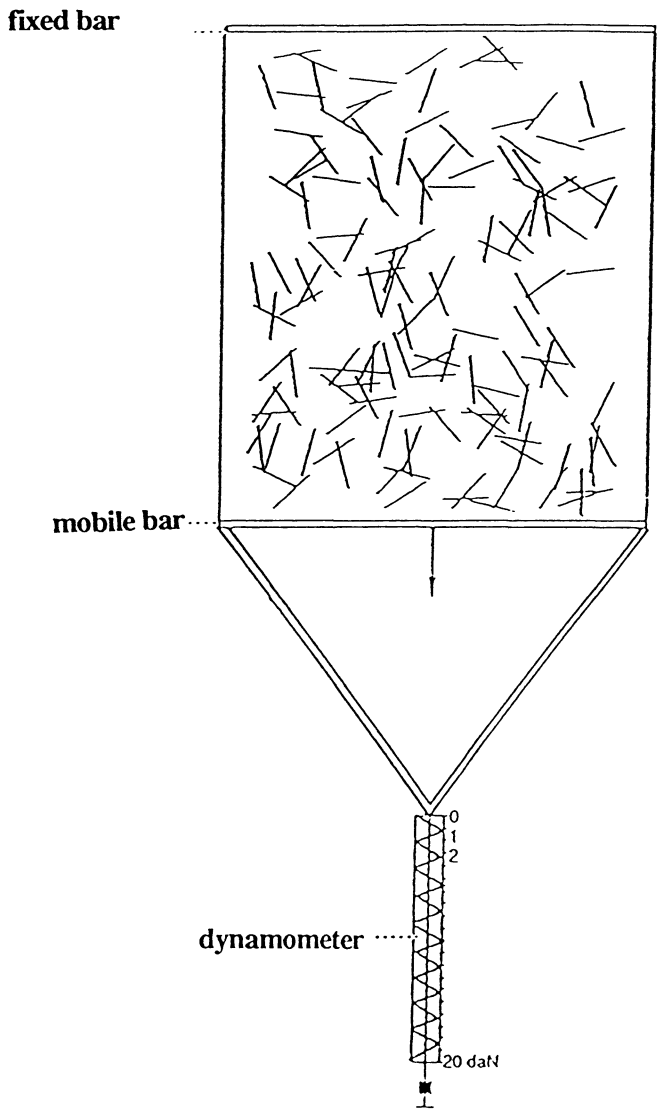

Fig. 5. — Experimental set up which has been used for the measurement of the rupture force threshold.

felt by a given crack is considered as the sum of the external applied stress plus the stress created by the presence of all the other cracks. Due to the slow decrease (r^{-2}) of the dipole stress created by a crack as a function of distance r, one must take into account the contribution of many cracks [2]. The method of reference [2] is based on the superposition technique and the ideas of self-consistency applied to the average traction on individual cracks. In principle, it yields approximate analytical solutions for the stress intensity factor accurate up to quite close distances between cracks. However, its implementation is quite cumbersome. We do not try to compare our results with this approach and rather focus on the critical behaviour near the percolation thresholds where all effective medium theories are bound to fail.

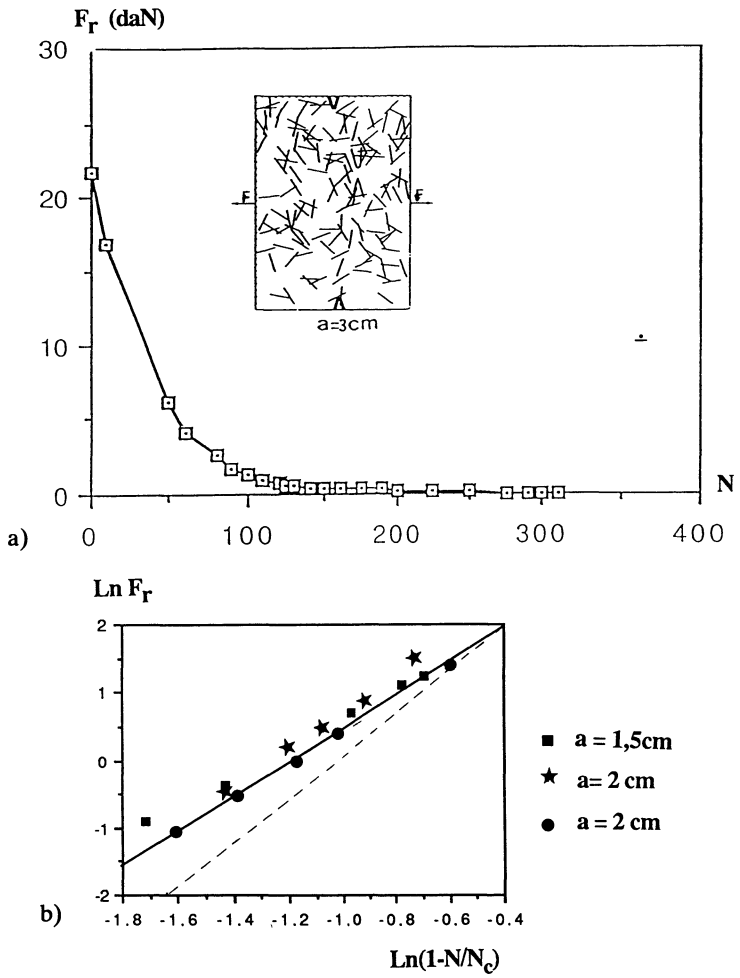

Fig. 6. — Rupture force threshold F_r as a function of the number of cracks randomly distributed on a continuum (model 2) : a) linear scale ; b) log-log representation near the percolation threshold for different crack lengths a showing an average slope of 2.6. The broken line corresponds to the slope 3.16.

The behaviour of the failure threshold in models 1 and 2 are different. This is due to the sensitivity of failure to the « weakest » part of the system or in other words to the largest and « more dangerous » defect occurring in the system [22, 25]. Dangerous defects are assemblies of neighboring cracks which interact constructively so as to create a strong tip enhancement factor. Due to the different topology of the two models, one expects [and we verify] that the failure threshold decreases, at first, more rapidly with N for model 1 than for model 2. This is due to the fact that all cracks weaken the system efficiently in model 1. Furthermore there is a larger probability for the occurrence of adjacent cracks which reinforce each other in model 1 due to its on-lattice construction than in the off-lattice model 2.

In the vicinity of the percolation threshold, we measure, on the log-log plot of figure 6b, a rupture exponent $F_m = 2.5 \pm 0.2$ in agreement with the expected result $2\,\nu = 2.66$ for model 2. It is remarkable that we are able to reach the critical regime in such small systems ($N < 1000$). This can be explained by the very efficient screening (also at work in the

conductivity of section 3) of the stress concentration by the numerous random cracks surrounding each crack. The result is the same for a variant of model 2 in which the positions of the cracks are random but their orientation can be $\pm \pi/4$ with respect to the direction of the applied stress.

Let us mention an interesting observation made on the system during its rupture. As we apply an increasing tensile stress as the two ends of the sheet, we observe a complex buckling and a marked tearing of the sheet. This is due to the fact that the macroscopic 2d-plate is not rigidly clamped, and because of its small thickness, its elastic curvature modulus is very small (proportional to ε^3 where ε is the thickness of the foil). It is preferable for the elastic plate to buckle out of the plane in order to relax the in-plane elastic stresses. The screening of the strain by buckling is so efficient that one recovers the discrete-lattice exponents $f_m = d\nu = 2.66$ [6]. Without buckling, the predicted exponent should be $f_m = 3.16$ [6] and would correspond to the dashed line in figure 6b, which is clearly not suitable for fitting the data.

The results of the failure threshold for model 1 are presented in figure 7. We measure an apparent failure exponent around 1.4 ± 0.2 which is very different from the expected value 2.66. We believe that this apparent exponent is the signature of the crossover from a mean field behaviour to the critical regime. As for the conductivity problem, stress concentration screening is much less efficient in this model than in model 2 and finite size effects are more stringent. We have not tried to use larger systems because of the prohibitive time needed to prepare them. Indeed, contrary to the case of the conductivity, one needs a different foil for each measurement point since each determination of the failure threshold destroys the sample.

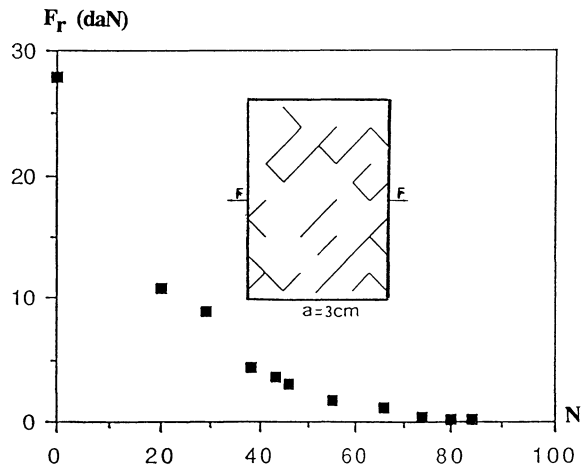

Fig. 7. — Rupture force threshold F_r as a function of the number of cracks randomly distributed on the bonds of a square lattice. The length of the cracks is $a = 3$ cm.

4. Conclusion and perspectives.

We have presented experimental results on the conductivity and rupture properties of crack deteriorated systems. We have observed some of the regimes predicted in the theoretical analysis developed in reference [6]. In the off-lattice model 2, we have been able to reach the critical percolation regime and measure critical exponents. On the contrary, the behavior of

the on-lattice models 1 is more akin to that of an effective medium clearly outside the critical regime because of the finite size of our systems, except in the close vicinity of the percolation threshold. The difference between the two models stems from different screening of the stress enhancement by neighboring cracks which change the sensitivity to finite size effects. In a sequel work, we intend to explore the regime of failure in the absence of buckling deformation and to verify the existence of a directed percolation threshold for the mechanical behaviour.

Acknowledgments.

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