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Short Communication

Ising cellular automata : universality and critical exponents

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Résumé. — Des simulations numériques sont effectuées sur des réseaux d'automates bidimensionnels de type Ising en champ extérieur nul pour des tailles allant de 24^2 à 1600^2 . Au seuil d'apparition du chaos (instabilité d'étalement d'erreur) la dimension fractale d_f est 1,9 et celle associée au temps d_t est 1,3. Ces valeurs sont indiscernables de celles mesurées récemment pour les automates cellulaires de Kauffman, et proches de celles observées dans la percolation.

Abstract. — Two-dimensional Ising-like cellular automata are simulated in zero field for 24×24 to 1600×1600 systems. We find that the mass fractal dimensionality at the onset of "chaos" (instability of "damage spreading") is 1.9, and $d_t = 1.3$ for the time. These values are indistinguishable from those recently reported for the Kauffman cellular automata and close to those observed for percolation.

Cellular automata are fast becoming a high powered computational technique for simulating complex systems, for example genetic switching cycles, hydrodynamics, immunology, neural networks and chemical reactions[1, 2]. Ising-like cellular automata which incorporated the Boltzmann probability distribution of the Ising model but with quenched random numbers were recently introduced by Mac Isaac *et al.* [3, 4]. Consider the two-dimensional square Ising lattice in a magnetic field, h . The energy at site i is

$$E_i = -J\sigma_i \sum \sigma_\delta + h\sigma_i$$

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and the probability of finding the spin at site i up is

$$P(\uparrow) = \exp(-E(\uparrow)/kT) / (\exp(-E(\uparrow)/kT) + \exp(E(\uparrow)/kT))$$

where k is the Boltzmann constant, T the temperature and $E_i(\uparrow) = -E_i(\downarrow)$. In a usual heat bath simulation an independent random number is considered at the attempted transition at each and every lattice site but if the random numbers are quenched initially for the sixteen possible configurations for the nearest neighbours around site i then we recover an Ising cellular automata (ICA) where now deterministic rules govern the time evolution of the system. The ICA is constructed as follows : a temperature is stipulated and the Boltzmann probabilities calculated for the 5 distinct energy values for the 16 possible neighbour configurations ; a random number is compared with the Boltzmann probability for a given configuration and if it is less than this probability then the rule is that the spin will be up at that site at the next time step or otherwise the spin will be down at the next time step ; the procedure is continued for the 15 other configurations at the site in question ; these procedures are then repeated for all the other sites of the system. We have defined deterministic "rules" which govern completely the subsequent evolution of the system, and we now start the simulation keeping these rules fixed.

This ICA at high temperatures in a magnetic field is the Kauffman [5] model which was introduced to study the switching properties of genes. A phase diagram comparable to the Ising model and percolation is expected for the ICA and the Kauffman model also. At low temperatures a local perturbation will remain localised while at high temperatures the perturbation will propagate throughout the system. We shall refer to the former case as "frozen" and the latter case as "chaotic". At low temperatures the system is non-ergodic in the sense that the ensemble average for example leads to the Boltzmann distribution of energies whilst the time average is quite different, especially at low temperatures [3, 4].

In this note we investigate the critical properties at the onset of "chaos" for $L \times L$ square lattices in the range $24 < L < 1600$. Equilibrium configurations are generated [6] with the standard heat bath algorithm, a replica of the system made and a line of spins along the central row is kept permanently in the opposite state to the spins in the equivalent row of the original system. Both systems are subjected to the same set of rules and should evolve in the same manner apart from the effects introduced by the perturbation. Are these effects localised or do they propagate throughout the system ? A measure of separation of the two systems is the Hamming distance - the number of spins in opposite states. These sites are referred to as damaged. The large lattices considered i.e. $L > 64$ were simulated on a CRAY YMP using the da Silva Herrmann [7] algorithm which allowed 190 spin flips per microsecond per processor.

We now consider the spatial and temporal evolution of the damage. For $T < T_k(L)$ the damage is localised to the vicinity of the permanently damaged line whilst for $T > T_k(L)$ the damage propagates freely to the edge of the system.

Figure 1 shows the variation of $T_k(L)$ vs. $1/L$, the asymptotic value appears to be $T_k(\infty) = 5.6$ in units of J/kT , more than twice the Ising Curie temperature T_c ! This behaviour is similar but more dramatic than that observed for some Ising model studies [8] where the transition to chaos does not coincide with T_c .

We have monitored the variation of the magnetisation with temperature of the ICA and this goes to zero within a few percent of the Curie temperature. Figure 2 shows the variation of the damage vs. L at the time when the damage reaches the boundary for the first time and figure 3 shows the average time taken vs. system size, L . These results are both made at $T_k(L)$. The

expected scaling behaviour

$$M \propto L^{d_f}$$

$$\langle t \rangle \propto L^{d_t}$$

is observed with a fractal dimensionality d_f of 1.9 and d_t of 1.3.

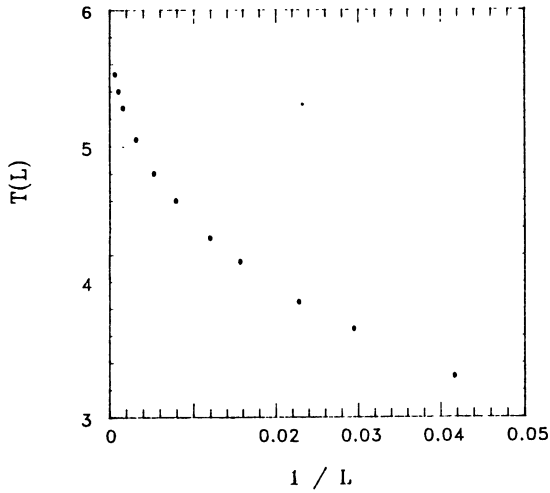


Fig. 1. — Variation of $T_k(L)$ vs. $1/L$. $T_k(\infty)$ is 5.6.

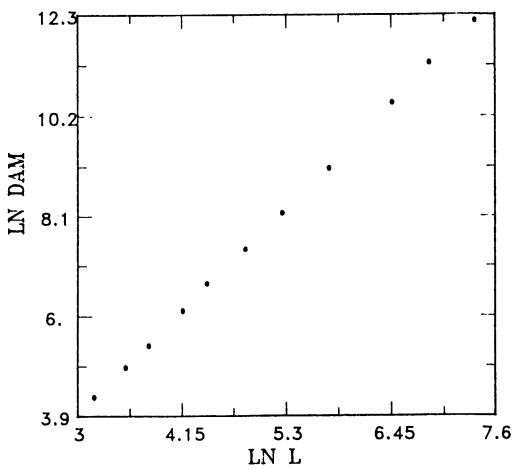


Fig. 2

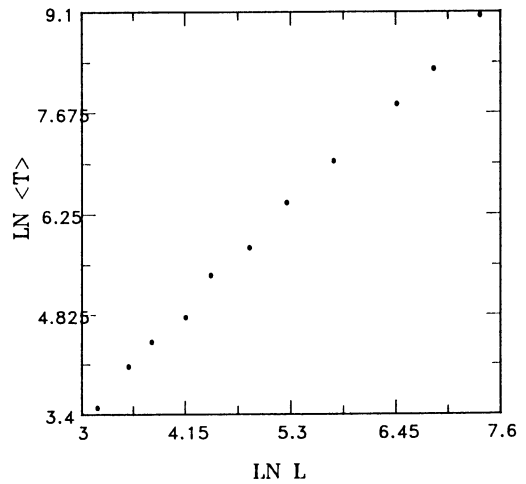


Fig. 3

Fig. 2. — ln Damage vs. ln L at $T_k(L)$. A least squares fit gives a d_f of 1.9.

Fig. 3. — ln $\langle t \rangle$ vs. ln L at $T_k(L)$. A least squares fit of the data yields a value of d_t of 1.3.

The equivalent exponents for the Kauffman model remained elusive until the recent careful investigation by Stauffer [9] who resolved earlier inconsistencies and reported $1.8 < d_f < 1.9$ and $1.3 < d_t < 1.4$. These values are compatible with our data and there exists a line of critical points between this zero-field critical point and the critical point at infinite temperature and non-zero field. This latter point is the critical threshold of the Kauffman model. It is most likely that the critical exponents are unchanged along this line and hence these ICA all belong to the same universality class. There remains the tantalizing prospect that the critical exponents measured by damage spreading for Kauffman and ICA may also belong to the same universality class as percolation.

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