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Mechanical size effects in 2d granular media

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Résumé. — Nous présentons ici une analyse critique d'expériences portant sur la déformation uniaxiale d'empilements réguliers de cylindres à 2d. Cette analyse est basée sur une étude des effets de taille finie. Le rôle du désordre de contact est souligné. L'influence des parois, en particulier en ce qui concerne les frottements, est discutée ; le rapport entre force transmise et force appliquée a été mesuré. Nous avons étudié l'influence de la taille des échantillons en faisant varier leur hauteur, leur largeur, ou les deux simultanément. Dans la mesure du possible, les analyses sont confortées par des observations en photoélasticité. Nous avons mis en évidence le rôle important joué par une longueur caractéristique supérieure à la taille des grains.

Abstract. — We present a critical analysis of experiments on 2d regular packings of cylinders submitted to a uniaxial vertical pressure, by studying finite size effects. The role of the disorder of contacts is emphasized. The influence of the walls, especially the frictions, is discussed ; the ratio of the transmitted to the applied force has been measured. We have tested the influence of the size of the samples by increasing width, height or both simultaneously. Conclusions are partly supported by photoelasticity measurements. The role of a characteristic length larger than the grain size is put into evidence.

1. Introduction.

The weak compression of 2d-granular media (i.e. when the local deformations at the contact between two grains remain *elastic*) has been studied in some recent papers [1-3]. The main conclusions are consistent with previous results in 3 dimensions [4-7]. Two points are essential :

(i) the most important stresses are transmitted by a network. Chains of stable contacts form rigid arms of aligned grains ; these *arms*, or links, trap large forces because of their relative stiffness compared to that of the other groups of grains in the packing. So, this network which is more and more connected when the macroscopic stress increases, uses only a fraction of the contacts between the grains : most of the contacts do not *feel* the macroscopic force applied. These results are clearly shown by photo-

elastic studies [8-10] and some numerical simulations [2] ;

(ii) when the *applied* stress is *uniaxial* ⁽¹⁾, the relation between the macroscopic force F and the deformation Δh is non linear and can be fitted by a power law [3]

$$F \sim (\Delta h)^m$$

where m is a *macroscopic* exponent ; it is in general different from the *microscopic* exponent μ (~ 1.5) which appears in the microscopic law for the compression between two grains (Hertz law).

This behaviour is mainly related to the grain size distribution in mixtures or, in the monosize case, to

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⁽¹⁾ The correct term should be *oedometric* (and not uniaxial) as grains are submitted to horizontal stresses at the walls ; but with K_0 constant, the deformation can be considered as uniaxial. We prefer to keep the word *uniaxial* which was already used in previous papers.

the local heterogeneities due to geometrical defects, which are responsible for a disordered distribution of the contacts (in particular some contacts between grains are missing). These heterogeneities are of crucial importance because of the small scale of the deformations which were observed [3] in the experimental force range.

Thus, *good* and *bad* contacts coexist in granular media. The good contacts generate a connected network and then are not randomly distributed in space. This is very important for the properties of granular media under compression, for the mechanical properties first, but also for all properties for which contacts between grains are essential: transport properties (electrical or thermal), solid-solid reaction, sintering, etc. For example, the macroscopic electrical resistivity of a packing of conducting grains is probably mainly that of the strongest stresses network as the current chooses preferentially the good conducting contacts.

However, in geometrically homogeneous media (i.e. where the size of the sample is hopefully greater than any physically relevant length), the exponent is not constant on the whole force range. One may expect 3 regimes as the force increases:

(i) at *small force*, and particularly during the first application of the stress, the most important part of the deformation is due to local displacements like slidings and rotations. Large stresses may be carried by *rigid arms* which are (correlated) stable bonds between grains, as can be seen in photoelastic observations. In that regime, these rigid arms are not — or poorly — connected. The range of forces where this behaviour may be observed depends on the geometry of the packing, the mechanical characteristics of the grains and the frictions between grains;

(ii) at *intermediate force*, which is often the experimental case, especially when the system has been compressed in a preliminary step, the behaviour is that described at the beginning of the introduction. Most of the deformation is due to the Hertz compression at the contacts between grains (even if the contact force is not really radial), but all the possible contacts are not yet active (= stressed). The strong stresses network and its extension through the medium play an essential role and the concept of *good* (or *active*) contacts is then fundamental;

(iii) at *very large force*, all contacts are active or, at least, their number does not vary anymore: the macroscopic behaviour is given by Hertz law. This was seen on a 2d packing of rubber cylinders [3a] which are very soft, but such a situation is quite exceptional, for it usually implies so strong macroscopic forces that both grains individually and the whole packing cannot have an elastic response (plasticity domain, rupture, ... see Ref. [6]).

Situations (ii) and (iii) have been simulated on a regular lattice of *symmetrical Zener diodes*, with a random distribution for the threshold voltages of the diodes [11]. The analogy is the following: the fluctuations of the threshold voltage correspond to the fluctuations of the number of active contacts, the microscopic current-voltage characteristics above threshold corresponds to the Hertz microscopic law when the contact is active. The exponent calculated in the intermediate zone where diodes are not all active, is different from the microscopic exponent, like in our mechanical studies.

Here, we are mainly interested in the behaviour (ii). The characteristic length scale of the deformation mechanism is larger than the grain size. So, the finite size effects on the mechanical properties are to be studied. By varying the size of the samples, we can also study the influence, on the mechanical properties of the packing, of

— the wall effects: the limits of the samples are hard walls, the effects of which on the geometrical structure of the packing can be important;

— the friction effects between grains and the container, which can give rise to arching.

The relative influence of these mechanisms seems to be difficult to know.

We have studied the behaviour of 2d packings of cylinders (short cylinders with parallel horizontal axes) when we change their macroscopic size by varying their height, their width or both simultaneously. We have chosen to work with *ordered* packings of *equal* cylinders: the geometry of such a system is simple. But the cylinders present geometrical irregularities which are weak (compared to their diameter) but sufficient (compared to local deformations) to generate a disorder in the distribution of the contacts in the force range that we have used.

In section 2, we describe our experimental system. In section 3, we give geometrical considerations related to our experiment. The main results on size effects — analysed by four different methods — are gathered in the section 4. They are discussed in section 5.

2. Experimental procedure.

The experimental system is made of plexiglass cylinders, of length 2.5 cm, with horizontal parallel axes, which are placed in an ordered manner (Schneebeli model [12]). These cylinders have geometrical defects (diameter fluctuations, ellipticity, bending...) so that their diameter can be estimated as 4.0 ± 0.1 mm. They are not really in contact all along their common generator (they are slightly bended and crossed); thus, we may consider that the microscopic compression law is of the Hertz type, with a microscopic exponent of the order of 1.5; we measured $\mu = 1.8 \pm 0.1$.

The packing, at its maximal size, is made with 48 rows of alternately 44 and 45 cylinders forming a triangular lattice ; it is put into a rigid frame, then placed into an *INSTRON 1175 universal testing machine*, as schematized in figure 1. A vertical displacement Δh is imposed on the upper plate and we measure the corresponding compressive force F . The experiments are made at low speed (0.5 mm min^{-1}) so that the observed phenomena are speed independent. Several pressure cycles are performed until the results are stabilized and reproducible (3 cycles for our regular lattice, about 40 for a disordered packing). The force range we have considered (paragraph 4.4 excepted) is 0 to 2 000 N.

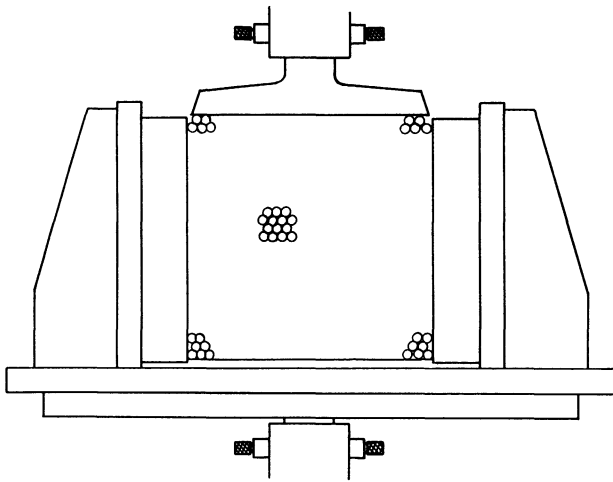


Fig. 1. — Experimental apparatus scheme.

Let us comment on the experimental procedure indicated above. In the first cycle, many macroscopic reorganizations occur which lead to irreversible geometrical modifications (most geometrical arches are eliminated, for example). This is quite clear in disordered systems where blocks made of several grains may slide collectively ; it remains true in our ordered samples though the slidings are then less important. The measured quantities crucially depend on the geometrical arrangement of the grains at rest and would be different in another sample. Thus, the first cycle is totally atypical for our purpose and must not be taken into account for the stress-deformation measurements, even if the geometrical deformation during this first cycle may be important for the later behaviour. On the other hand, in further cycles, the relative locations of the grains are nearly fixed. At low force, they can still rearrange a little ; some contacts disappear and others are created, but the changes are local and partly reversible when the stress is relaxed. This corresponds to zone (i) in the introduction and it may be considered as a *consolidation phase*. When the vertical pressure increases, grains do not move anymore and the modifications are mainly due to the Hertz defor-

mation at the contacts (zone (ii) of the introduction or *consolidated phase*).

We had to face two technical problems that we shall discuss now. First, the upper surface of the samples is rough, because of the irregularities ; so, it is difficult to define the zero of the displacement : is it defined from the first grain which is in contact with the piston or from an average on the grains of the first layer ? As the deformation is generally very weak, we must answer this question to get a precise value for the macroscopic exponent m . Experimentally, it appears that it is more convenient to define the *zero* from the stress ; in our case (with a 6 1/2 digits voltmeter) the force digit which determines this experimental zero corresponds to 0.2 N. The origin defined in this manner is reproducible from the second pressure cycle.

The second problem deals with the strain-stress plots which are not the same at increasing or decreasing pressure. These hystereses stay, even after many pressure cycles (but they are quickly stabilized and reproducible). Actually, the measurements are relevant in the increasing part of the cycle only, because then, assuming that the coefficient of lateral pressure K_0 is constant, we may consider [5] that the deformation is uniaxial, as was already stated in previous papers [3].

3. Geometrical analysis.

Let us first assume that the sample is very large. When no macroscopic force is applied, its initial structure has some characteristic features which partially condition its further behaviour. In our case, it is built in a non collective manner (close to a *grain by grain* procedure) and under *gravity*. Each new grain is in equilibrium if it has two contacts with the packing (one may assume that the size fluctuations are such that the probability to have a third contact is 0). The mean coordination number (average number of contacts per grain) is then equal to 4. But the contacts are not equally distributed along the 3 possible directions. Using an argument close to that by Roux *et al.* [1b], we may consider that each grain has one contact along direction Y (or Z) (cf. Fig. 2),

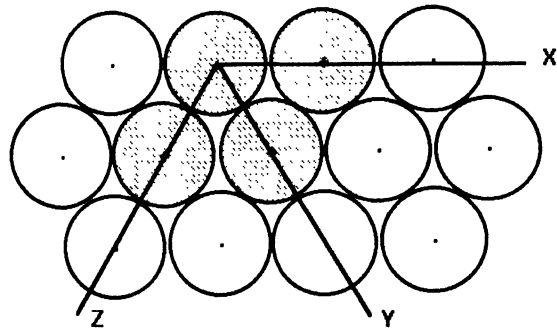


Fig. 2. — 2d ordered packings : directions of the contacts.

then, with equal probability a second contact along Z (or Y) or X . The probability to have a contact along the Y or Z directions is 0.75 and it is only 0.5 along the X direction. The spatial distribution of the contacts is thus initially non isotropic (the same is true in 3d disordered packings [13]).

If now, a uniaxial vertical strain is applied, some slidings and rotations (local or collective) occur, essentially during the first cycle. So this deformation leads to an increase of the number of contacts in the Y and Z directions, and to a decrease of that number in the horizontal X direction. The initial anisotropy will be accentuated. Our photoelastic views confirm that few horizontal contacts transmit stresses (paragraph 4.1). By ignoring the X contacts, the triangular lattice can be replaced, in a first step, by a rhombus lattice (with angle $\pi/3$); this implies that long range correlations take place, as can be seen below. Let us point out that the above analysis is true on average only: the situation is very different from one sample to another as very small changes in the position of the grains at rest may considerably modify the distribution of the stresses when the force is turned on.

The horizontal and vertical walls of the vessel may play an important role in the geometrical definition of the packing: a hard wall generates geometrical changes in the packing. In a disordered system, the modifications — on the packing fraction and coordination number for instance — may be seen on distances up to several diameters of grains [14]. In our regular system, we may consider that they are negligible along the vertical walls, at least beyond the first column of grains. For the horizontal bottom wall (the role of the upper moving one can be very different as we shall see later) the grains are all in contact with the lower plate because of gravity: we assume that this will not much alter the mechanical behaviour of the system, except for small samples.

The first cycle of pressure is interesting for the geometrical modifications it implies, and because the packing keeps memory of these modifications, during the other cycles of pressure, *for the same maximum pressure* [13]. The grains of the upper layer form first a *rough surface*, because their centers are certainly not at the same level, even after several cycles. When the plane piston is working, the geometrical structure is modified at the scale of several rows. Actually, the restructuration is less important in an ordered packing, but may be seen clearly in a disordered case. To study that problem, we have built a disordered monosize packing by placing some larger cylinders of diameter $\phi = 1$ cm along the vertical and bottom walls of the rigid frame [15]. Before the pressure is applied, horizontal lines have been drawn on the circular sections of the cylinders, from one vertical boundary to the other. During the first cycle of pressure, the deformations

of these lines show noticeable geometrical rearrangements (slidings, rotations) in the four or five first layers. Beyond these layers, the lines are less deformed, the deformation is more homogeneous and does not seem to depend on the distance to the upper plate. Thus, one may think that, from a geometrical point of view, the effect of the *moving wall* is similar to the above mentioned classical *geometrical wall effect*. Even in our *ordered system*, that effect may induce important modifications in the distribution of contacts in the first layers, although the displacements are not perceptible. However, these geometrical modifications in the upper layers may induce at large distances important *mechanical correlations* by favouring *rigid arms* which should be created from the top [16].

4. Uniaxial compression study.

The above analysis shows the necessity of testing both the influence of the walls and of the size of the samples. We present here four different approaches. They agree rather well: there exist a small and a large size behaviours, separated by a transitional regime. The asymptotic situation seems to be reached already for 30×30 samples i.e. far before our 44×48 maximal packing.

4.1 PHOTOELASTIC STUDY. — The material (plexiglass) used here was chosen because of its good photoelastic response. The sample is placed between two crossed polarizers [8, 9], the spatial distribution of the intergranular forces is not homogeneous at the grain scale: the lines of strongest stresses form a network which is more and more connected when the pressure increases; however many grains remain excluded. The network of active bonds is not isotropic; for our ordered packings, there is only a small number of horizontal bonds carrying the stress (as discussed above). The observation of this network is a good test of the macroscopic homogeneity of the sample under compression (photo Fig. 3).

An important question is to know whether one of the walls plays an important role when the force is increased. When a grain at the upper layer is higher than the others, it is the first to be stressed as seen above and it may generate lines of strongest stress within an angle $\pi/3$, which may propagate far away in the packing, with sometimes very few stressed grains inside the cone. This *cone effect* is easy to understand. As the force is vectorial, if we apply a vertical force on a grain, it can *propagate* only inside the Z - Y angle (a cone in 3d) of the grain, and it will do so along a network organized in such a way that this network is stable with respect to this force.

But the strongest stresses network is not carried by the grains close to the upper plate only. Except in some specific cases, the first bright bond appears in a

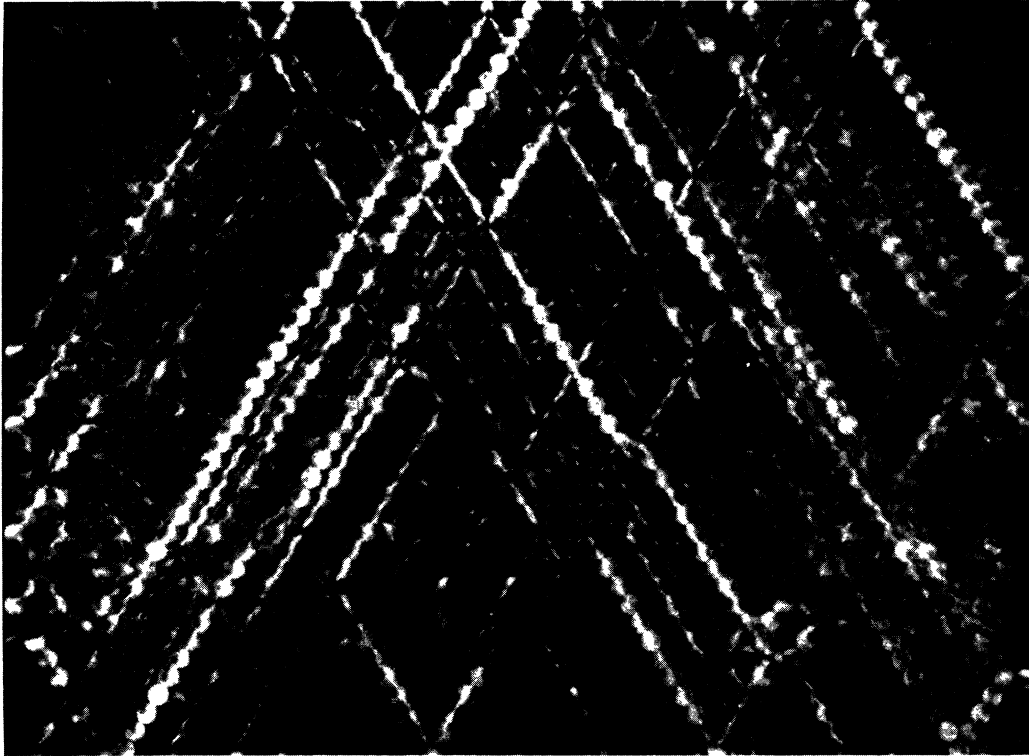


Fig. 3. — Photoelastic view of our regular packing ($n = 42$, $L = 44$ or 45) at $F = 2\,000$ N: the strongest stresses network appears clearly.

random position in the system : this observation is a test for the homogeneity of the sample. Moreover, we have observed some pronounced *cone effects* starting from grains which are at first sight randomly situated in the bulk of the packing ; like above, the bonds inside the cone are organized so that the network is stable with respect to the force applied to the grain. Because of the vectorial character of the force, bright bonds rearrange along straight lines which are as long as permitted by the geometrical defects in the packing. These straight lines are often the generators of the cones and thus, we can consider that the observed network is the superposition of several individual *cones*.

So, two types of disorder (or order) can play an important role in these long range correlations in the transmission of the force :

— The disorder of contacts. The absence of contacts between two grains forbids the transmission of the force.

— The *orientational* order. In our *ordered* packings, the force can be transmitted along one of the preferential directions of the lattice, if no contacts are lacking. In more disordered systems, the force may be transmitted at some distance, even if the translational order is destroyed by dislocation. Indeed, the orientational order may be preserved at the scale of the sample if there are not too many

defects in the lattice [17] and, according to our observations, this orientational order seems to be of great importance for the mechanical properties of granular materials (this is presently under study).

We have studied the photoelastic response of the packings as a function of their height n ($n =$ number of rows), for a constant width of $L = 44$ or (alternately) 45 cylinders. When the packing has less than 10 layers, one observes isolated arms, strongly stressed, going from the upper to the lower edge of the sample. When there are more than 10 layers, the system branches out, and a network begins to appear. When $n > 20-24$ layers, the network looks macroscopically homogeneous and as shown above. So, the photoelastic study displays two different behaviours, for small and large height respectively, separated by a rather wide transition zone ($10 < n < 22$) (see the photos in Fig. 4).

4.2 INFLUENCE OF THE FRICTION ON THE VERTICAL WALLS. — One problem of the uniaxial deformation of granular systems is the estimation of the role of friction along the vertical walls which may modify the mechanical behaviour. In particular, Dantu [4] has shown that when the width is very small compared to the height, the lower part of the sample can *ignore* the stress which is applied at the top : the vertical force on a row at height z ($z = 0$ at the top of

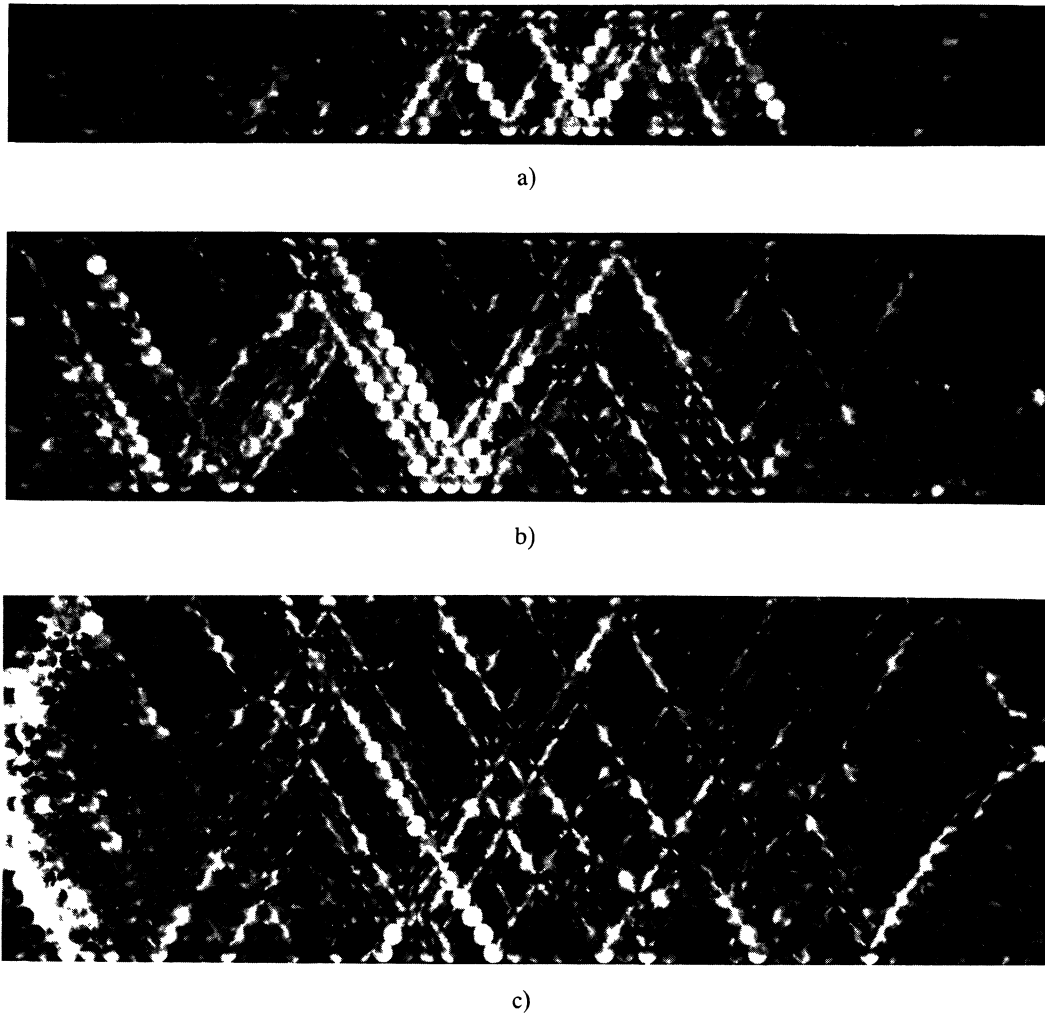


Fig. 4. — Strongest stresses network at $F = 2\,000\text{ N}$ for regular packings of plexiglass cylinders of different heights (a) $n = 7$ (below the transition) (b) $n = 14$ (transitional case) (c) $n = 20$ (just above the transition).

the packing) varies as $F(z) = F(0) e^{-\alpha z}$ which is generally correct in high containers, assuming that the friction coefficient along the wall is constant; these results have been weakened by the beautiful experiments of Ito and Kuno [18] which clearly show that frictions are less important at the top of the container.

We have performed a comparable study for our situation, i.e. when the aspect ratio (height/width = n/L) of the sample is less than 1. In all these experiments, the width is constant ($L = 44$ cylinders).

For technical reasons — the presence of strength gauges would modify the geometry of the packing — we have not measured the stress normal to the wall at different heights. We were only able to measure the total resulting force when the packing is made of 48 rows; it varies linearly when the vertical stress is increased, the ratio of the normal to the vertical force remaining close to $1/4$: so, we can consider

that the lateral pressure coefficient K_0 is constant according to reference [5].

We have measured — with the help of previously calibrated deformation gauges — the force which is transmitted to the bottom of the sample as a function of the applied force, and this was done for several heights of the packings. The transmitted force grows linearly with the applied force for all the heights we have studied. On the figure 5, the slope of this characteristic is plotted as a function of the height. It is close to 1 when $n < 10$, then it changes rapidly as n increases and becomes slowly varying when $n > 20$ and of the order of 0.92, which means that in all cases frictions on vertical walls is small. This behaviour is compatible with what was observed in photoelasticity (Sect. 4.1) and with the experiments at small height ($n \leq L$) of Ito and Kuno [18].

4.3 STRAIN-STRESS LAW AND HEIGHT EFFECTS. — The experimental values given below are obtained

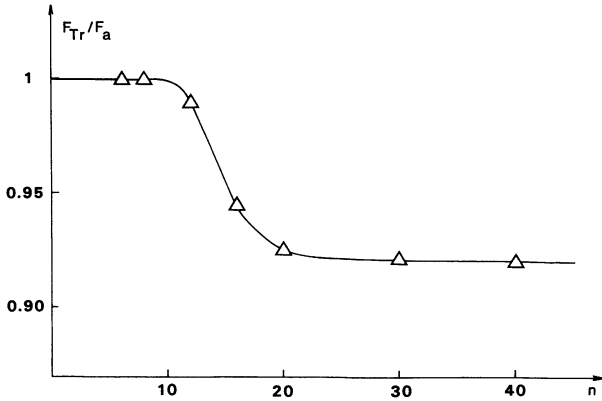
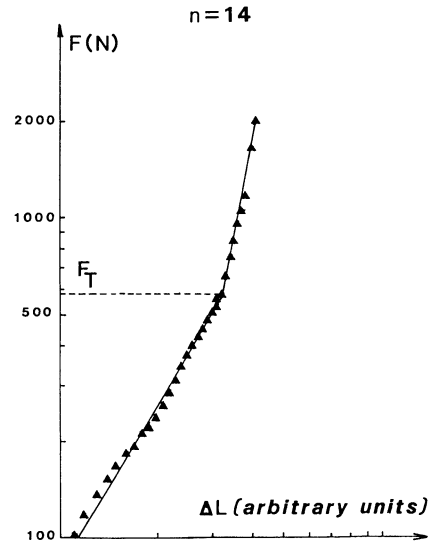


Fig. 5. — Variations of F_{Tr}/F_a for $F_a = 2\,000\text{ N}$ and different heights n ($F_a =$ applied force, $F_{Tr} =$ transmitted force).

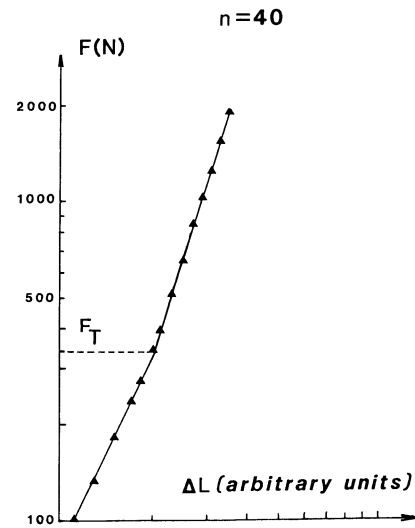
for increasing stress, during the third cycle of pressure and are the averages over three different samples with the same dimensions. We have studied packings with a number of horizontal layers going from $n = 6$ up to $n = 44$ with alternately 44 and 45 cylinders. The force range is 0-2 000 N. We measured the deformation Δh of the packing as a function of the applied force F .

Once the cycles are reproducible, the plots $F = g(\Delta h)$ in log-log scale consist in two linear parts and the discrepancy between the two slopes decreases when n increases. There is a *small force* behaviour and a *larger force* behaviour, both of them may be described by a power law. As said in section 2, we consider that the behaviour at small force corresponds to a *consolidation phase*, the other one to the *consolidated phase*. The reproducibility in the consolidation phase after the first cycle is due to a quasi reversible disappearance of the geometrical arches [6]. The transition force F_T , at the cross-over between the two regimes is roughly constant and close to 600 N when $n < 20$, then changes rapidly to the new value 300 N when $n > 20$, as shown on figure 6.

For a given height, we have two macroscopic exponents, let us say m_l in the *small force* range and m_h in the consolidated zone ; m_l is constant and close to 1.5 when $n < 10$, grows for $10 < n < 20$, then remains constant and close to 2-2.5 for $n > 20$; m_h , which is very high for small n , of the order of 10 (is it still a power law?), decreases and for $n > 24$, it remains nearly constant and equal to 3.4 ± 0.4 ; it is not possible to decide whether it is the asymptotic or still a slowly varying transitional value (see Fig. 7). In the same experiments, we also studied the deformation rate normalized to the number of rows. It is approximately constant for $n > 20$ (see Fig. 8). This is compatible with a theoretical description where all layers should behave in the same way [7, 19].



a)



b)

Fig. 6. — Stress-strain law in log-log scale (a) for small height ($n = 14$) and (b) for large height ($n = 40$).

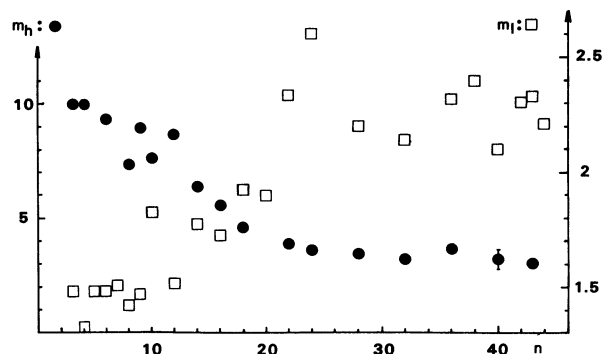


Fig. 7. — Exponents m_l and m_h as a function of the height n of the packing. The two regimes appear clearly as n increases.

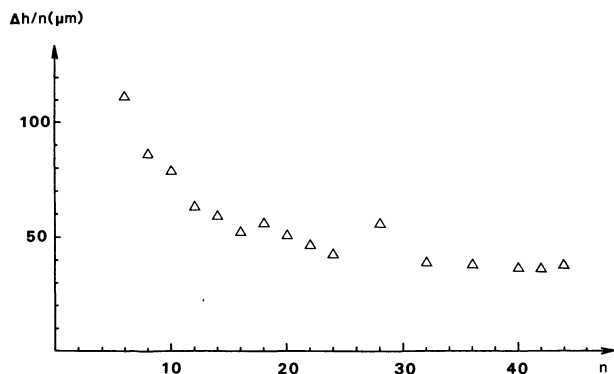


Fig. 8. — Deformation rate $\Delta h/n$ as a function of height n .

4.4 INFLUENCE OF THE WIDTH. — We performed the same study for a constant height $n = 46$. The width L is modified by adding rectified blocks. A small incommensurability between the width of the sample and the size of the frame does not alter the exponents in the power laws we get though it may modify the height of the packing through a better lateral restructuration. On the other hand, we worked here in a constant pressure range — the maximum pressure being that which corresponds to the maximum force in the previous experiments. We observed again the two regimes: one at low pressure, the other at high pressure. Up to the experimental errors, which may be rather important here, the transition pressure P_T — defined in the same way as F_T in 4.3 — does not vary for $n = 46$ when the width L changes from $L = 6$ to $L = 46$.

Exponents m_1 and m_h are shown on figure 9. A transitional regime for $10 < L < 20$ clearly appears. The exponent m_1 is close to 1.5 for $L < 10$, it increases in the range $10 < L < 20$, then remains constant and close to 2 for $L > 20$. The upper exponent m_h is constant and close to 1.8 when $L < 10$, then it increases in the range $10 < L < 20$, and becomes equal to the asymptotic value $m_h = 3.4$ when $L > 20$.

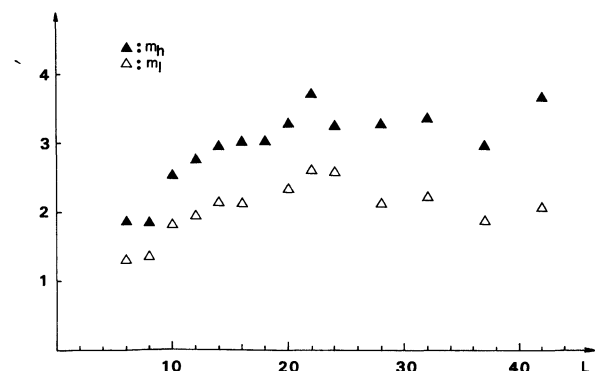


Fig. 9. — Exponents m_1 and m_h as a function of the width L of the packing for a constant height $n = 46$.

A similar study was made for the opposite situation i.e. for small height $n = 8$; the fluctuations were so important even for large L that we cannot give more than a trend: it seems that m_h for large L is more or less constant, of the order of 5, while m_1 like in the first case is close to 2-2.5.

This study gives very similar results to those of the preceding subsection: two regimes according to the width of the sample. The transitional ranges $10 < L < 20$ and $10 < n < 20$ are quite comparable. The limit of these regimes ($L \sim 10$; $L \sim 20$) are compatible with a length defined from a connected path as we shall discuss below.

To complete the study, we have performed some measurements when both height n and width L vary, their aspect ratio n/L being kept constant. We find the same conclusions: the exponent m_h decreases from $m_h \approx 5$ for $n = L = 20$ to its asymptotic value 3.4 when we have a 40×40 packing.

5. Discussion.

The above results, showing two different behaviours for our packings — one for $n < 10$, the other for $n > 20$ — can be understood from different mechanisms. Schematically, these mechanisms have three sources:

(i) Influence of the horizontal walls: both the geometrical wall effect at the bottom of the container and chiefly the geometrical reorganization in the packing at the first cycle of pressure (effect of the upper active wall) may be responsible for a different compressibility of the packing over about 10 layers. As the height increases, the role played by these 10 layers becomes less and less important. Then, large height measurements correspond to the asymptotic behaviour.

(ii) Influence of the vertical walls: we already noticed that the pure geometrical effect is certainly negligible in ordered samples. But the vertical walls are responsible for frictions which may be very important, as emphasized above (subsect. 4.2). One may consider that the grain-wall friction coefficient is constant whatever the height is. This is the reason why, in experiments comparable to ours, Dantu [4] proposes an exponential decrement of the ratio of the applied to the transmitted force with increasing height, when the height of the packing is large as compared to its width (large aspect ratio n/L). Actually, the friction coefficient may vary between its dynamical and static values [6]: under compression, the friction coefficient changes with the height as grains of the upper layers move together with the piston while those at the bottom do not.

Dantu [4] argued that frictions at the walls and frictions between grains may provide an explanation

to the *hammerhardening* or arching effects in samples with large aspect ratio: the bottom part of the packing sees only a small part of the stress applied at the top wall. In our case, the same argument leads to a variation of the ratio [applied stress/transmitted stress] as described in paragraph 4.2: grains at the bottom are anchoring points for arches. This could explain the different behaviour of our samples at small n (no arches) and large n (presence of arches).

(iii) *Finite size effects*: our samples, before the first cycle, present a disorder in the repartition of the contacts and the anisotropy is still emphasized after several pressure cycles (see § 3). So, the geometrical probability p for one bond to exist is very high, even before the third cycle and is much greater than 0.75 which is its value before the first compression cycle. Along each one of the Y and Z directions (cf. Fig. 2), straight links of length s (i.e. $s + 1$ sites are in contact) — which have a certain stability though they are not rigid — exist with probability $p^s(1 - p)$, their average length being $\langle s \rangle = p/(1 - p)$. These geometrical links would be statistically connected to the top and bottom plates when $n < 10$ and would not be anymore connected for $n > 20$. This would modify the deformations of the system under compression: these continuous links should be the starting points for the rigid straight links seen in photoelasticity for $n < 10$ and should not be long enough to connect the two plates for $n > 20$, whence the existence of the network. The very high exponent ($m_h \sim 10$) which was measured for $n < 10$ would arise from the existence at increasing stress of more and more such rigid arms running from the bottom to the top.

However, the absence of some contacts and the geometrical defects of our cylinders (mainly, the generators are not straight), make the packing not rigid; it may be distorted because of quasi-reversible slidings and flexion of the cylinders. When the packing is compressed, active (or rigid) contacts are progressively created which in turn generate rigid clusters, which are only statistical as a small increase of the stress may greatly modify the network (see for example the film realized by L.C.P.C. [20]). These rigid clusters grow in order to form a stable structure going from the bottom to the top of the sample which is now consolidated. This is similar to a percolation process [21]. But the problem here is not a scalar one: the stresses and the deformations are vectorial, as underlined above. And moreover, because the transmission of the stresses is made through the contacts between grains, the model of *central-forces percolation* [22] seems to be the relevant one. So, the structure of this rigid cluster is different from that of the infinite cluster in scalar percolation theory: in particular, the number of *active* bonds has to be larger in our case. The force necessary to get this rigid cluster would then corre-

spond to the transition force F_T that we have observed between the two mechanical regimes (which are present in the grain packings in a general manner). This transitional force mainly depends on the packing structure, on the friction coefficient between grains and on the intrinsic mechanical characteristics and geometrical defects of the materials (the bending of our plexiglass cylinders, ...); for instance, in packings made of parallel steel cylinders where the geometry is ten times more precise — particularly the axis is less curved — we found that the transitional force is much weaker. As to the two values of F_T for small and large n , they could be due to a different geometrical structure at large and small heights, either because of the role of the piston (near the top plate, the disorder of the contacts is more important and the *threshold* higher) or because the frictions at the walls may be different in the two regimes, as said above.

Thus, the existence of two regimes at low and intermediate stress, similar to the regimes in numerical simulations with Zener diodes [11, 23], may be explained in a *percolation* framework, starting from the *active* contacts. Below and above the threshold F_T , the structure of the sample is not the same and the mechanical behaviour (whence the exponent m) may be very different: below F_T , the deformation mechanisms are very complex, and depend strongly on the frictions between grains; above F_T , the exponent m varies essentially with the disorder of contacts.

It appears that the influence of the height may be interpreted either from a mechanical and geometrical point of view (mostly because of the friction at the walls) or from a statistical manner. The study of the influence of the width clearly shows that the geometrical and mechanical (friction) roles of the walls are not the most important contributions to the size effects. Otherwise, we should have observed a monotonic evolution of our parameters instead of a *step*-like variation (see Fig. 5).

6. Conclusion.

Using different kinds of experiments, we have studied the size effects for uniaxial compression of 2d granular media. These experiments show two regimes according to the force which is applied: a consolidation regime, and a consolidated one; in each of them, the relation between the macroscopic force and the deformation Δh can be fitted by a power law.

Our experiments have underlined above all the role of a characteristic length, larger than the grain size and defined from the geometrical properties of the packing. This characteristic length is greatly sensitive to the heterogeneities, and presumably to the *orientational* order. The discrepancy between

microscopic and *macroscopic* laws is explained by the existence of this characteristic length.

It appears that new experiments have to be done to study how this characteristic length depends on the structure of the granular media, and particularly on the orientationnal order (even defined at short range). That question may be important for many processes in these media.

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