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On the role of momentum tensions in micropolar ferrohydrodynamics

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Résumé. — Nous considérons des écoulements stationnaires rectilignes de ferrofluides micropolaires. Nous étudions plus particulièrement les discontinuités tangentielles qui y apparaissent dans une limite partiellement dissipative. Nous montrons l'importance des tensions de moment angulaire (contraintes en couple) dans l'analyse de ces discontinuités et dans la construction de solutions dans les régions présentant un fort gradient des paramètres de base. Nous obtenons et analysons la solution du problème de Poiseuille dans le cas de la ferrohydrodynamique micropolaire.

Abstract. — The steady flows with straight stream lines of micropolar ferrofluid are considered. The tangential discontinuities appearing in such flows of partially dissipative ferrofluid are studied. It is illustrated that momentum tensions (couple stress) are of primary importance in the investigation of those discontinuities and in constructing solutions in the regions of large gradients of constitutive parameters. The solution of Poiseuille's problem in micropolar ferrohydrodynamics is obtained and analysed.

1. Introduction.

The simplest mathematical models of ferrohydrodynamics are based on different assumptions about the binding energy of a single ferroparticle's magnetic moment with its body [1]. The binding energy can be characterized by a nondimensional parameter

$$\sigma = \frac{K_A V}{k_B T} \quad (K_A \text{ stands for the constant of magnetic}$$

anisotropy of ferroparticle material, V for the ferroparticle volume, k_B for the Boltzmann constant, and T for the temperature). $\sigma \ll 1$ corresponds to an approximation of « quasistationary ferrohydrodynamics » [2, 3]. In this case the orientation of magnetic moments of ferroparticles does not cause ferroparticle's rotation, and establishment of equilibrium magnetization occurs during a characteristic time of the order of the time of Larmor's precession of a single ferroparticle's magnetic moment. In such an approximation the density U for the internal energy of the closed thermodynamic system « medium + electromagnetic field » depends only on three constitutive parameters : the ferrofluid density ρ , the mass density s of entropy and the magnetic field strength \mathbf{H} .

The finite values of energy of the magnetic anisotropy ($\sigma \geq 1$) correspond to an approximation

of « micropolar ferrohydrodynamics » [1, 4]. In this case the ordering of the ferroparticles' magnetic moments is accompanied by the own rotation of the ferroparticles and, consequently, by the origin of the internal moment of momentum \mathbf{k} in the medium. The description of ferrofluid motion then needs the introduction of two more constitutive parameters : the mass density \mathbf{k} of internal moment of momentum and a magnetic moment \mathbf{M}/ρ of unity of mass (\mathbf{M} is the ferrofluid magnetization) [4, 5]. The change of magnetization and internal moment of momentum with time is accompanied simultaneously by their diffusion in the ferrofluid. The corresponding transport mechanisms on the phenomenological level can be described by introducing momentum and magnetic momentum tensions (couple stress) [5]. (About other possible approaches to ferrofluid description see, for example [6, 7].)

To this point the role of momentum tensions in ferrohydrodynamics have been insufficiently investigated. In the present paper such an investigation is carried out with an interesting example (from the point of view of physics) of micropolar ferrofluid flow with straight streamlines. It is shown that taking into account the momentum tensions in the equations of ferrohydrodynamics appears to be necessary in the regions of large gradients of constitutive

parameters. In this case diffusional mechanisms are of primary importance for constructing the unique solution and for the calculation of the physical characteristics of ferrofluid flows.

2. The general system of equations and jump conditions.

In quasistationary magnetic fields the following equations, describing the flows of a nonconductive micropolar magnetic fluid in the regions of continuity of constitutive parameters with their derivatives up to the second order included, hold [5]:

Conservation of mass

$$\rho \frac{d\rho}{dt} + \rho v_{k,k} = 0. \quad (2.1)$$

Conservation of momentum

$$\rho \frac{dv_i}{dt} = p_{ik,k}.$$

Conservation of moment of momentum

$$\rho \frac{dk_i}{dt} = Q_{ik,k} + \varepsilon_{ijk} p_{kj}.$$

Balance of magnetization

$$\rho \frac{d}{dt} \frac{M_i}{\rho} = m_{ik,k} + r_i.$$

Conservation of energy

$$\rho \frac{d}{dt} (U + v^2/2) = -G_{k,k}.$$

Equations of quasistationary electrodynamics

$$\operatorname{div} \mathbf{B} = 0, \quad \operatorname{rot} \mathbf{H} = 0, \quad \operatorname{rot} \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}.$$

In this system of equations the comma before the subscripts denotes the covariant derivative, ε_{ijk} is the antisymmetric unit tensor, \mathbf{E} , the electric field strength, $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$, the magnetic induction, and \mathbf{v} the velocity; the summation rule over repeated indices is meant. The conditions on discontinuity surfaces are the following:

$$\begin{aligned} \langle \rho v_n \rangle &= 0, & \langle p_{ik} n_k - \rho v_i v_n \rangle &= 0 & (2.2) \\ \langle Q_{ik} n_k - \rho v_n k_i \rangle &= 0, & \langle m_{ik} n_k - M_i v_n \rangle &= 0 \\ \langle G_n + \rho v_n (U + v^2/2) \rangle &= 0, \\ \langle B_n \rangle &= 0, & \langle H_\tau \rangle &= 0 \end{aligned}$$

where \mathbf{n} and $\boldsymbol{\tau}$ are unit vectors along the normal and tangent of the discontinuity surface, and v_n is the normal component of vector \mathbf{v} relative discontinuity surface. The sign $\langle \dots \rangle$ denotes the jump of the quantity through the discontinuity surface.

The systems of equations (2.1) and jump conditions (2.2) are closed by the following expressions for the components of stress tensor $\{p_{ik}\}$, couple stress tensor $\{Q_{ik}\}$, magnetic momentum stress tensor $\{m_{ik}\}$, vector of flux energy density $\{G_k\}$ and relaxation term \mathbf{r} for magnetization:

$$\begin{aligned} p_{ik} &= -p \delta_{ik} + \frac{H_i B_k}{4\pi} + 2\eta v_{ik} + \\ &+ \left(\xi - \frac{2}{3}\eta \right) \operatorname{div} \mathbf{v} \delta_{ik} + \frac{\rho I}{2\tau_s} \varepsilon_{ikn} (\Omega_{en} - \Omega_n) \\ Q_{ik} &= \frac{\delta_1}{I} k_{i,k} + \frac{\delta_2 - \delta_3}{I} \operatorname{div} \mathbf{k} \delta_{ik} + \frac{\delta_3}{I} k_{k,i} & (2.3) \\ m_{ik} &= m_1 h_{i,k} + (m_2 - m_3) \delta_{ik} \operatorname{div} \mathbf{h} + m_3 h_{k,i} \\ G_k &= - \left(p_{ik} + \frac{BH}{4\pi} \delta_{ik} - \frac{H_i B_k}{4\pi} \right) v_i + \\ &+ \frac{c}{4\pi} [\mathbf{E} \times \mathbf{H}]_k - \Omega_{ei} Q_{ik} - h_i m_{ik} + q_k \\ r_i &= -\varepsilon_{ikn} M_k k_n / I - h_i / \tau \\ q_k &= -\Lambda T_{,k}, \quad \Omega_k = \frac{1}{2} \varepsilon_{kij} v_{j,i}, \\ \Omega_e &= \mathbf{k} / I, \quad \mathbf{h} = \mathbf{M} / \chi - \mathbf{H}. \end{aligned}$$

In these expressions $\{v_{ik}\}$ stands for the rate deformation tensor, and \mathbf{q} for the vector of heat flux density.

We have introduced phenomenological parameters [1] which have the following meaning: I , the average inertial momentum of ferroparticles in unit mass, χ , the equilibrium magnetic susceptibility, and Ω_e , the average angular velocity of ferroparticles. Only direct effects are taken into account and gyromagnetic phenomena in ferrofluid are neglected.

Further more I and χ are considered as constants. Then the pressure p is expressed as follows:

$$p = p_0 + \frac{H^2}{8\pi} + \frac{M^2}{2\chi} = p_1 + \frac{H^2}{8\pi} \quad (2.4)$$

where p_0 is the pressure in the absence of magnetic field. The dissipative coefficients τ and τ_s are the characteristic relaxation times of magnetization and internal moments of momentum respectively, η and ξ are shear and volume viscosities, δ_i and m_i ($i = 1, 2, 3$), the momentum viscosities, and Λ is the thermoconductivity.

According to the second law of thermodynamics, dissipative coefficients satisfy the following inequalities:

$$\Lambda \geq 0, \quad \xi \geq 0, \quad \eta \geq 0, \quad \tau_s \geq 0, \quad \tau \geq 0 \quad (2.5)$$

$$m_1 - m_3 \geq 0, \quad m_1 + m_3 \geq 0,$$

$$\delta_1 - \delta_3 \geq 0, \quad \delta_1 + \delta_3 \geq 0$$

$$3\delta_2 - 2\delta_3 + \delta_1 \geq 0, \quad 3m_2 - 2m_3 + m_1 \geq 0.$$

Assuming in equations (2.1), (2.3) $m_i = 0$, the

equations quoted in paper [4] are obtained. With the additional assumption $\delta_i = 0$, the equations of reference [1] are obtained. (It should be noted that the pressure p in Ref. [1] differs from that in Eq. (2.4). But such a distinction can be only relevant in the case of flows of compressible fluid.)

The role of momentum tensions in ferrohydrodynamics will be studied as an example of the simplest flows of incompressible ferrofluid with strong discontinuities in an external homogeneous magnetic field.

3. Appearance of tangential discontinuities in partially dissipative ferrofluid flows.

The characteristic time of the processes described by equations (2.1) with closing relations (2.3) does not exceed $\tau_1 = \chi\tau$ in order of magnitude. Otherwise the magnetisation in the fluid flow can be considered at equilibrium and the angular velocity Ω_e of ferromagnetic particles equals the angular velocity Ω of flow (because of $\tau_s < \tau_1$ [1]). In this case equations (2.1) are reduced to the form of equations with an equilibrium magnetization [2, 3]. Let us consider the case of constant phenomenological and dissipative coefficients which allows the consideration of the energy equation independently of other equations.

Taking τ_1 as the characteristic time and introducing characteristic scales of length L , velocity $v_0 = L/\tau_1$, magnetic field strength — the value of the external homogeneous field H_0 , magnetization — the value M_0 of ferrofluid saturation magnetization, pressure p_0 , internal moment of momentum $k = I/\tau_1$, we can write the basic system of equations in the following dimensionless form :

$$\operatorname{div} \mathbf{v} = 0 \quad (3.1)$$

$$\dot{\mathbf{v}} = (1 + \lambda) Re^{-1} \Delta \mathbf{v} + 2 \lambda Re^{-1} \operatorname{rot} \mathbf{k} + \kappa Al^2 (M \mathbf{V}) \mathbf{H} - Eu \nabla p_1$$

$$\dot{\mathbf{k}} = Re^{-1} (\lambda_1 \Delta \mathbf{k} + \lambda_2 \nabla \operatorname{div} \mathbf{k}) + F_m^{-1} \mathbf{M} \times \mathbf{H} - \tau_1 \tau_s^{-1} (\mathbf{k} - \Omega)$$

$$\dot{\mathbf{M}} = Re^{-1} (\lambda_3 \Delta \mathbf{M} + \lambda_4 \nabla \operatorname{div} \mathbf{M}) + \mathbf{k} \times \mathbf{M} - \mathbf{M} + 4 \pi \chi \kappa^{-1} \mathbf{H}$$

$$\operatorname{div} (\mathbf{H} + \kappa \mathbf{M}) = 0, \quad \operatorname{rot} \mathbf{H} = 0.$$

Here : $Re = \rho v_0 L \eta^{-1}$ (Reynold's number), $Eu = p_0 (\rho v_0^2)^{-1}$ (Euler's number), $Al = H_0 (4 \pi \rho v_0^2)^{-1/2}$ (Alven's number), $F_m = M_0 H_0 \tau^2 I^{-1}$ (Frude's momentum number), $\lambda = \rho I (4 \eta \tau_s)^{-1}$ (ratio of angular rotational viscosity to shear viscosity), $\lambda_i = \delta_i (I \eta)^{-1}$ ($i = 1, 2$) (ratio of momentum viscosities to shear viscosity); $\lambda_{i+2} = D_i \eta^{-1}$ ($i = 1, 2$) (ratio of coefficients of magnetization diffusion to shear viscosity); and $\kappa = 4 \pi M_0 H_0^{-1}$.

The coefficients of magnetization diffusion are as follows :

$$D_1 = m_1 \chi^{-1}, \quad D_2 = (m_2 + 4 \pi \chi (m_1 + m_2)) \chi^{-1}.$$

As it follows from equations (3.1), the magnetic body couple and the couple of viscous friction of rotating ferroparticles in the liquid carrier introduce the main contribution to the variation of internal angular momentum when $|\nabla k|$ are small. But, in the case of gradient catastrophe ($|\nabla k| \rightarrow \infty$) the gradients of couple stress (momentum tensions) are of the same order as the indicated couples. Analogically, it is necessary to take into account the magnetic momentum tensions in the equation for the magnetization when $|\nabla M| \rightarrow \infty$. The latter describes, in particular, the contribution of the magnetodipole interaction between the ferroparticles to the correlations of directions of ferroparticles' magnetic moments in ferrofluid [5].

Let us consider steady flows with straight streamlines of partially dissipative ferrofluid neglecting the momentum and magnetic momentum tensions in equations (3.1) ($\lambda_i = 0$, $i = 1, 2, 3, 4$). Assuming that in the Cartesian frame of reference (x, y, z) the stream velocity is directed along the x -axis, the external homogeneous magnetic field lies in the (x, y) plane and all variables except pressure depend on the coordinate (y) alone, we derive from equation (3.1) the formulae :

$$\begin{aligned} v &= v_x, \quad k = k_z, \quad H_x = H_{x0} \quad (3.2) \\ H_y &= \frac{H_{y0}(1 + k^2) - 4 \pi \chi H_{x0} k}{1 + 4 \pi \chi + k^2}, \\ M_x &= \frac{4 \pi \chi (1 + 4 \pi \chi) H_{x0} - k H_{y0}}{\kappa (1 + 4 \pi \chi + k^2)}, \\ M_y &= \frac{4 \pi \chi (k H_{x0} + H_{y0})}{\kappa (1 + 4 \pi \chi + k^2)}. \end{aligned}$$

Here k is solution of an algebraic equation of degree five and v of a differential equation of the first order :

$$\beta k - \frac{\alpha k}{(1 + 4 \pi \chi + k^2)^2} [8 \pi \chi H_{x0} H_{y0} k - H_{x0}^2 (k^2 + (1 + 4 \pi \chi)^2) - H_{y0}^2 (1 + k^2)] = \omega \quad (3.3)$$

$$v' + 2(1 - \beta) k = -2 \omega, \quad \omega = \gamma y + \omega_0$$

$$\alpha = \frac{4 \pi \chi \tau_s}{\kappa \tau_1 F_m}, \quad \beta = \frac{1}{1 + \lambda}, \quad \gamma = -\frac{Eu Re \Delta p}{1 + \lambda \Delta x}.$$

The prime denotes a derivative with respect to (y). The pressure gradient is constant along the x -axis and is equal to $\Delta p (\Delta x)^{-1}$. In equation (3.2), (3.3) H_{x0}, H_{y0}, ω_0 are the constants which are found from the boundary conditions.

Let us consider the situation of strong external fields ($4 \pi \chi \sim \kappa$, $\kappa \ll 1$). Then $H_y = H_{y0}$, and

$H^2 = 1$, so that the magnetic field in the flow equals the external one. For k we get the cubic equation :

$$k \left(\beta + \frac{\alpha}{1+k^2} \right) = \omega . \quad (3.4)$$

Note that $k(-\omega) = -k(\omega)$, hence it is sufficient to consider values $\omega \geq 0$. We have the following asymptotic behaviour of solution $k(\omega)$ of equation (3.4) :

$$\begin{aligned} k(\omega) &\sim \frac{\omega}{\alpha + \beta} & \text{at } \omega \rightarrow 0, \\ k(\omega) &\sim \frac{\omega}{\beta} & \text{at } \omega \rightarrow \infty. \end{aligned} \quad (3.5)$$

At $\alpha < 8\beta$ equation (3.4) has only one real root for any real ω , the dependence $k(\omega)$ being monotonically increasing. At $\alpha > 8\beta$ in region of values $k > 0$ equation (3.4) has two specific points

$$k_{*1,2} = \frac{\alpha}{2\beta} - 1 \pm \frac{\sqrt{\alpha}}{2\beta} \sqrt{\alpha - 8\beta}$$

which are the branch points of the solutions of this equation. For $0 < k < k_{*1}$ and $k_{*2} < k < \infty$, $k(\omega)$ is monotonically increasing, for $k_{*1} < k < k_{*2}$ it is monotonically decreasing. The plot $k(\omega)$ for $\alpha = 10$ and $\beta = 0.5$ is given in figure 1. In the interval of values $\omega \in [\omega_{*2}, \omega_{*1}]$, where

$$\omega_{*1,2} = k_{*1,2} \frac{3\beta \sqrt{\alpha} \pm \beta \sqrt{\alpha - 8\beta}}{\sqrt{\alpha} \pm \sqrt{\alpha - 8\beta}}$$

equation (3.4) has three real roots for any ω . As seen from figure 1, when ω changes from 0 to ∞ , $k(\omega)$ in this interval should change discontinuously from values $k = k_1(\omega)$ on the lower branch OA to the values $k = k_2(\omega)$ on the upper branch BC. The appearing discontinuity is weak for the velocity of the fluid and strong for the internal moment of

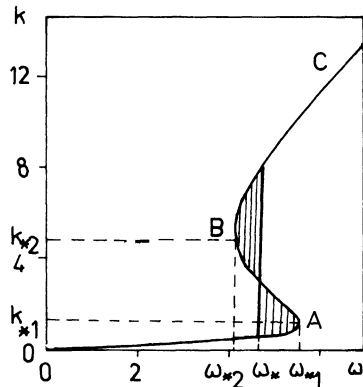


Fig. 1. — The dependence $k = k(\omega)$ in ferrofluid flow with direct streamlines. The discontinuity position is determined by the condition of equality of dashed figures areas ($\alpha > 8\beta$).

momentum and angular velocity of the ferroparticles.

Conditions (2.2) in the approximation under consideration are reduced to the form

$$\langle p \rangle = 0, \quad \langle v' \rangle = 2(\beta - 1)\langle k \rangle, \quad \langle H \rangle = 0 \quad (3.6)$$

and make it possible to calculate such a discontinuity, provided its position in the stream is known.

4. Determination of the discontinuity position.

For this purpose it is necessary to take into account diffusional addenda in equation (3.1). A similar method connected with the study of jump structure, has been applied in magnetohydrodynamics, particularly in the study of ionization shocks [8].

In the present paper we only consider the diffusion of internal moment of momentum neglecting magnetic momentum tensions ($\lambda_3 = \lambda_4 = 0$) in equations (3.1). Then equations (3.3) for the velocity is unchanged, and the equation for the internal moment of momentum has the following form

$$\varepsilon \frac{d^2k}{d\omega^2} = f(k, \omega), \quad \varepsilon = \gamma^2 \frac{\tau_s \lambda_1}{\tau_1 Re} \quad (4.1)$$

$$f(k, \omega) = \frac{\alpha k}{1+k^2} + \beta k - \omega .$$

In the limiting case $\varepsilon = 0$, the differential equation (4.1) coincides with the algebraic equation (3.4). But, in contrast to equation (3.4), equation (4.1) describes the change of k directly in a thin layer $[\omega_* - \delta, \omega_* + \delta]$ of large gradients of k ; this layer in the limiting case $\varepsilon = 0$ degenerates into the discontinuity surface $\omega = \omega_*$, which is sought.

We set the following boundary conditions for equation (4.1) :

$$k(0) = 0, \quad k(\omega) \sim \frac{\omega}{\beta} \quad \text{at } \omega \rightarrow \infty . \quad (4.2)$$

Such conditions follow from the demand that, in the limiting case $\varepsilon = 0$, the solution $k = k(\omega, \varepsilon)$ of equation (4.1) must coincide with the following discontinuous solution of equation (3.4) :

$$\begin{aligned} k &= k_1(\omega) & \text{at } 0 \leq \omega < \omega_* \\ k &= k_2(\omega) & \text{at } \omega_* < \omega < \infty, \quad \omega_* \in [\omega_{*2}, \omega_{*1}]. \end{aligned} \quad (4.3)$$

Let us take $\ell > \omega_{*1}$. Then in the interval $[\ell, \infty]$, the solution of equation (4.1) can be obtained by direct asymptotic expansion in powers of ε in the form

$$k(\omega, \varepsilon) = k_2(\omega) + \varepsilon^i \varphi_i(\omega) . \quad (4.4)$$

It follows from equation (4.1) that $\varphi_1 = \frac{1}{2} \frac{d}{d\omega} \left(\frac{dk}{d\omega} \right)^2$ and $\varphi_i (i > 1)$ are functions of φ_1 ,

..., φ_{i-1} , such that $\varphi_i \rightarrow 0$ when $\varphi_{i-1} \rightarrow 0$. Since $\lim_{\omega \rightarrow \infty} \varphi_1 = 0$, then $\lim_{\omega \rightarrow \infty} \varphi_i = 0$. Thus, the solution (4.4) satisfies the boundary condition (4.2) at infinity due to (3.5). In the interval $[0, \ell]$ equation (4.1) should be integrated with the following boundary conditions

$$k(0) = 0, \quad k(\ell) = k_2(\ell) + \varepsilon^i \varphi_i(\ell). \quad (4.5)$$

The problem of integration of equation (4.1) in a limited interval with the values of k known on the bounds of the interval (in particular, in the form (4.5)) is widely studied [9]. Using the results of [9] we conclude that the problem of (4.1) and (4.2) has a solution $k = k(\omega, \varepsilon)$, satisfying conditions (4.3) in the limiting case $\varepsilon = 0$.

Integrating equation (4.1) in the discontinuity layer interval $[\omega_* - \delta, \omega_* + \delta]$, we get

$$\begin{aligned} \varepsilon \left[\frac{dk}{d\omega} (\omega_* + \delta) - \frac{dk}{d\omega} (\omega_* - \delta) \right] = \\ = \int_{\omega_* - \delta}^{\omega_* + \delta} f(k, \omega) d\omega. \end{aligned} \quad (4.6)$$

Equation (4.1) allows us to obtain the expression of the derivative in the discontinuity structure in the following form:

$$\begin{aligned} \frac{dk}{d\omega} &= \left(\frac{2}{\varepsilon} \varphi + \left[\frac{dk}{d\omega} (\omega_* - \delta) \right]^2 \right)^{1/2}, \\ \varphi &= \int_{k(\omega_* - \delta)}^{k(\omega)} f(k, \omega) dk. \end{aligned} \quad (4.7)$$

Hence, it follows that $k(\omega)$ is a monotonically increasing function. Using (4.7), we can integrate over k instead of integrating over ω in equation (4.6). As a result, equation (4.6) is written in the form

$$\begin{aligned} \frac{\varepsilon^{3/2}}{\sqrt{2}} \left[\frac{dk}{d\omega} (\omega_* + \delta) - \frac{dk}{d\omega} (\omega_* - \delta) \right] = \\ = \int_{k(\omega_* - \delta)}^{k(\omega_* + \delta)} \frac{f dk}{\left[\varphi + \frac{\varepsilon}{2} \left(\frac{dk}{d\omega} (\omega_* - \delta) \right)^2 \right]^{1/2}}. \end{aligned} \quad (4.8)$$

Setting here $\varepsilon \rightarrow 0$ and taking into account the restriction of $\frac{dk}{d\omega} (\omega_* \pm \delta)$ (in accordance with (4.3)), we obtain the integral condition which is satisfied by the solution of problem (4.1) and (4.2) in the limiting case of momentum viscosities equal to zero:

$$\int_{k(\omega_* - 0)}^{k(\omega_* + 0)} f(k, \omega_*) dk = 0. \quad (4.9)$$

The resulting obtained condition (4.9) on the discontinuity has a simple geometrical interpretation:

the discontinuity line $\omega = \omega_*$ cuts off curvilinear figures of equal area on the plot $f(k, \omega) = 0$ in the plane (ω, k) . In our case of partial form of function $f(k, \omega)$, the discontinuity position is uniquely defined (Fig. 1).

For $0 \leq \omega \leq \omega_*$ it is necessary to choose the root $k = k_1(\omega)$ of equation (3.4), corresponding to the branch OA in figure 1, and for $\omega > \omega_*$, the root $k = k_2(\omega)$ corresponding to the branch BC.

5. Poiseuille flow of micropolar ferrofluid.

Let us consider a ferrofluid flow between two parallel infinite plates under a constant pressure gradient along the x -axis (Fig. 2). The plates are

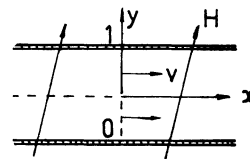


Fig. 2. — The geometry of Poiseuille flow of micropolar ferrofluid in external homogeneous magnetic field H .

made of nonmagnetic material, the distance between them equals $2a$. The external homogeneous magnetic field lies in the (x, y) plane. Let us take $L = a$, so the flow region corresponds to $-1 \leq y \leq 1$. Then the problem is described by equation (3.3) for the velocity, equation (3.4) for the internal moment of momentum and the following boundary conditions for the velocity

$$v(1) = v(-1) = 0. \quad (5.1)$$

For the Poiseuille flow $\omega = \gamma y$. When $\alpha < 8\beta$, then the discontinuities in the flow region are absent. When $\alpha > 8\beta$, then we can find $\omega_* = \omega_*(\alpha, \beta)$ from the condition (4.9). The following two cases are possible:

1) $\gamma < \omega_*$, so that in the flow region the inequalities are satisfied: $|\omega| < \gamma, |\omega| < \omega_*$. Then for the internal moment of momentum we get $k = k_1(\omega)$. The equation for the velocity under the condition (5.1) has been integrated numerically. The solution is continuous, as in the case of $\alpha < 8\beta$. The dependence $k = k(y)$ is close to a linear one, the dependence $v = v(y)$ is close to a parabolic one (Fig. 3).

2) $\gamma > \omega_*$. In this case the discontinuity surfaces are planes $|y| = |y_*| = \omega_*/\gamma$, which are located in the flow region. Note that the higher the pressure gradient, the nearer the discontinuity surfaces to the central region $y = 0$ of flow. For $|y| < \omega_*/\gamma$ the internal moment of momentum is equal to $|k| = k_1(\omega)$, for $\omega_*/\gamma < |y| < 1$ it is equal to $|k| = k_2(\omega)$.

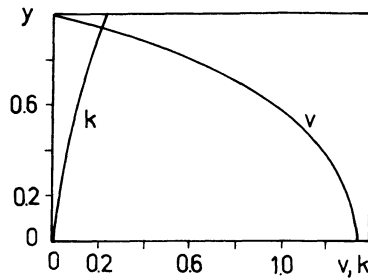


Fig. 3. — The profiles of velocity v and internal moment of momentum k in Poiseuille flow when $\alpha > 8\beta$, $\gamma < \omega_*$ ($\alpha = 6$, $\beta = 0.5$, $\gamma = 0.5$).

The integration of equation (3.3) for the velocity in this case is carried out in the following way. In the interval $y_* \leq y \leq 1$, equation (3.3) is integrated with boundary conditions (5.1). As a result, $v(y_*) = v_*$ and $v'(y_* + 0)$ are determined. Using conditions (3.5) it can be determined then

$$v'(y_* - 0) = v'(y_* + 0) + 2(1 - \beta) \langle k \rangle \Big|_{y=y_*}$$

which allows one to continue the numerical integration in the interval $0 \leq y \leq y_*$. The plots of velocity and internal moment of momentum are represented in figures 4, 5. As it follows from figure 5, the dependences $k = k(y)$ in the continuity regions are close to linear dependences.

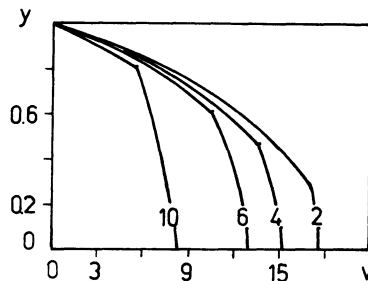


Fig. 4. — The continuous profiles of velocity v when $\alpha > 8\beta$, $\gamma > \omega_*$. The numbers on the curves denote the corresponding values of parameter α ; $\beta = 0.2$, $\gamma = 4$.

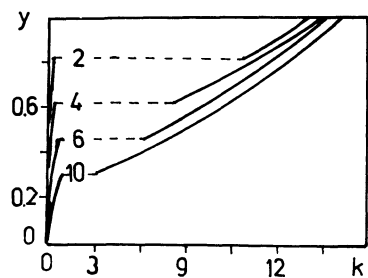


Fig. 5. — The discontinuous profiles of internal moment of momentum k when $\alpha > 8\beta$, $\gamma > \omega_*$. The numbers on the curves denote the corresponding values of parameter α ; $\beta = 0.2$, $\gamma = 4$.

We get from equations (3.2) in the approximation under consideration

$$M_x = \frac{4 \pi \chi}{\kappa} \frac{H_{x0} - kH_{y0}}{1 + k^2},$$

$$M_y = \frac{4 \pi \chi}{\kappa} \frac{H_{y0} + kH_{x0}}{1 + k^2}$$

so that on the discontinuity surface the value and direction of vector of magnetization change :

$$\langle M \rangle = \frac{4 \pi \chi}{\kappa} \left\langle \frac{1}{(1 + k^2)^{1/2}} \right\rangle$$

$$\left\langle \frac{M_x}{M_y} \right\rangle = - \frac{\langle k \rangle}{(H_{y0} + H_{x0} k_1(\omega_*))(H_{y0} + H_{x0} k_2(\omega_*))}$$

The pressure on the discontinuity surface does not change. The volume rate of flow Q

$$Q = \int_{-1}^1 v \, dy$$

is a one-to-one function of γ . From (3.3)-(3.5) it follows :

$$Q = \frac{4}{3} \gamma \frac{\alpha + 1}{\alpha + \beta}, \quad \gamma < \omega_*$$

$$Q = \frac{4}{3} \frac{\gamma}{\beta} - \frac{4}{3} \frac{\omega_*^3}{\gamma^2} \frac{\alpha(1 - \beta)}{\beta(\alpha + \beta)}, \quad \gamma > \omega_*$$

So, the dependence Q on γ is different for $\gamma < \omega_*$ and $\gamma > \omega_*$. This allows one to verify the obtained results experimentally.

Concluding remarks.

In the present paper the momentum tensions in ferrohydrodynamic equations are taken into account only for obtaining condition (4.9) on the discontinuity surface. The boundary conditions for the internal moment of momentum and magnetization are not formulated, the corresponding values of M and k on the streamlined surfaces are determined by the process of the solution under the known boundary conditions for the velocity. When the couple stress is not taken into account in equations (3.1), then, starting from the equation (3.4) obtained for k , it is possible to reach false conclusions about : 1) the hysteresis character of the solutions under consideration (transition of k from the lower branch OA to the upper branch BC in points $\omega_{*1,2}$ (see Fig. 1)) or 2) the stochastic character of the ferrofluid flow (transition of k on any point within the interval $[\omega_{*2}, \omega_{*1}]$).

In this paper the case $\gamma = \omega_*$ in Poiseuille flow is not considered. In this case the discontinuity surfaces coincide with streamlined plates. So, the solution to Poiseuille's problem can only be obtained through

the concretisation of boundary conditions for \mathbf{k} (or streamlined surface) and through the investigation for the angular velocity Ω_e of ferroparticles on of the structure of discontinuity surfaces.

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