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Strain and stress at the fracture of percolative media

L. Benguigui ⁽¹⁾, P. Ron ⁽¹⁾ and D. J. Bergman ⁽²⁾⁽¹⁾ Department of Physics and Solid State Institute, Technion-Israel Institute of Technology, Haifa 32000, Israel⁽²⁾ School of Physics and Astronomy, The Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel-Aviv 69978, Israel

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Résumé. — La déformation d'une feuille de métal perforée et d'un réseau élastique dilué ainsi que la contrainte ont été déterminées au voisinage du seuil de percolation, d'une part en augmentant régulièrement la déformation jusqu'à la rupture et d'autre part en augmentant régulièrement la contrainte appliquée. On a déterminé l'exposant critique T_f de la contrainte σ_f à la rupture ($\sigma_f \sim (p_c - p)^{T_f}$), $T_f = 2,5 \pm 0,4$ et l'exposant critique T_u de la déformation ε_f à la rupture ($\varepsilon_f \sim (p_c - p)^{-T_u}$), $T_u = 1,4 \pm 0,2$. Une interprétation théorique des résultats est proposée.

Abstract. — We measured the strain and the stress of a perforated metal foil and of a diluted elastic network near the percolation threshold. We used two different techniques: in the first, the strain is increased continuously and monotonically until the system ruptures, whereas in the second, it is the stress which is increased monotonically. We determined the critical exponent of the fracture stress $\sigma_f \sim (p_c - p)^{T_f}$, with $T_f = 2.5 \pm 0.4$ and the critical exponent of the fracture strain $\varepsilon_f \sim (p_c - p)^{-T_u}$ with $T_u = 1.4 \pm 0.2$. A theoretical interpretation of the results is proposed within the framework of the nodes-links-blobs picture.

In this paper, we present the results of an experimental determination of the evolution of the stress and the strain of a percolative medium in two dimensions, as it is stretched through a sequence of plastic and fracture transitions until complete rupture occurs. Recently, interest has grown in the problem of fracture in a medium in which increasing amounts of non-linear and irreversible damage is created by mechanical stress up to a point where total rupture occurs [1-4]. While it is now well established that the linear elasticity of real perforated solids and that of diluted networks with bond-bending forces belong to the same universality class, it is not clear whether the fracture phenomenon also exhibits a similar universality: it is possible that the properties of the medium itself will be of importance. It is well known that for stresses above the linear regime, different behaviours are observed in brittle and in plastic materials.

First, we have to define the stress and the strain at the fracture point unambiguously. Two methods can be devised to rupture a two-dimensional sample; i.e. a sheet of the chosen material. In the first

situation, the strain is increased and one measures the corresponding stress. Since near the percolation threshold the fracture takes place through breaking of a finite number of bonds, one measures the stress at the first bond breaking, at the second, and so on. One can define the fracture stress in different ways, such as either the stress at which the first bond breaks, or the largest value of stress. In our experiment this causes no problem, because we always found that the stress at the first break is also the largest stress. Thus we adopted this definition of the fracture stress. The fracture strain is defined as the relative elongation of the sample when it ruptures into two separate pieces. At that time, the stress drops to zero.

In the second situation, we impose the stress, which is increased until the breaking of the different bonds begins. Here too, one can imagine different possibilities. It is possible that once one bond is broken others immediately follow until complete rupture occurs. Another possibility is that after one bond is broken, one has to increase the force further in order to break another bond, and this continues

until the force is sufficient to fracture the sample. As shown below, we always observed only the first possibility. Here too, the fracture strain can be defined as the relative elongation when the rupture occurs. We note that the definition of the fracture strain is always free of ambiguity, which may not be the case for the fracture stress if the first break does not occur at the largest value of stress.

The material used in this experiment is a copper foil of thickness 0.2 mm. We performed two types of experiments, following the two procedures described above, and we found complete agreement between them. In the first type [4] we used sheets of size 20×21 cm in which we randomly punched holes of 1.1 cm diameter on a square lattice of 1 cm unit cell. The sample was elongated in time by means of a small motor and we increased very slowly (0.05 mm/s) the sample length. At the same time, we recorded the stress by means of a pressure sensor (Fig. 1). The time t_c of the fracture is thus proportional to the strain. In figures 2 and 3 we plotted the fracture stress F_c (the stress at the first bond breaking) and the fracture time t_c versus $p_c - p$, where p is the hole fraction and p_c is the

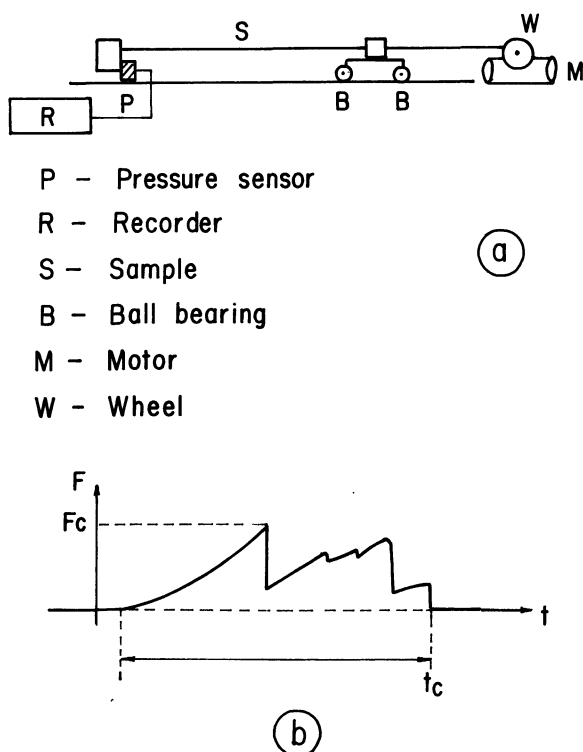


Fig. 1. — a) Experimental set-up of the fracture experiment when the sample is elongated. b) Record of the stress versus the time during the fracture process. Each discontinuity corresponds to the breaking of a bond. F_c is the stress at the first breaking and is the largest of the stress at the different bond breakings. The initial curvature of the $F(t)$ curve is due to the fact that, at the beginning the sample is not stretched because of its loose shape. This curvature is not observed after each bond breaking.

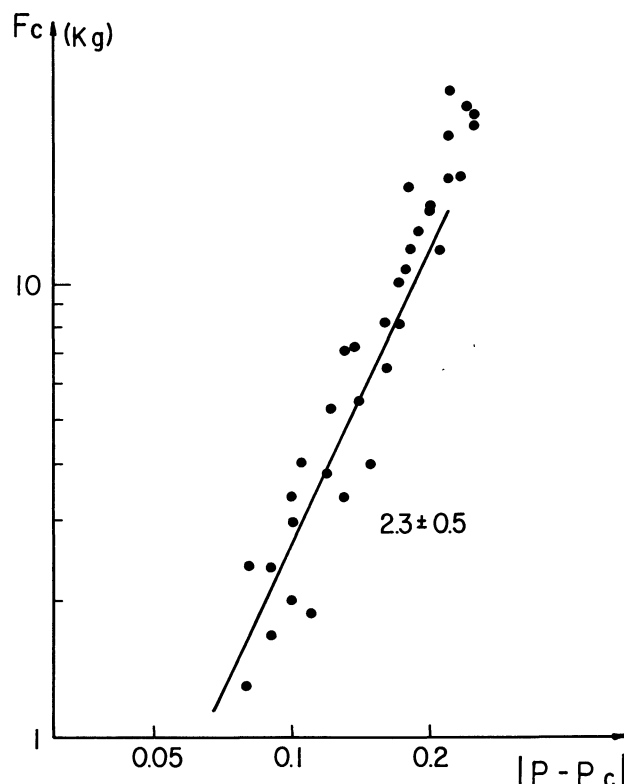


Fig. 2. — The fracture stress F_c versus $(p_c - p)$ in a log-log plot.

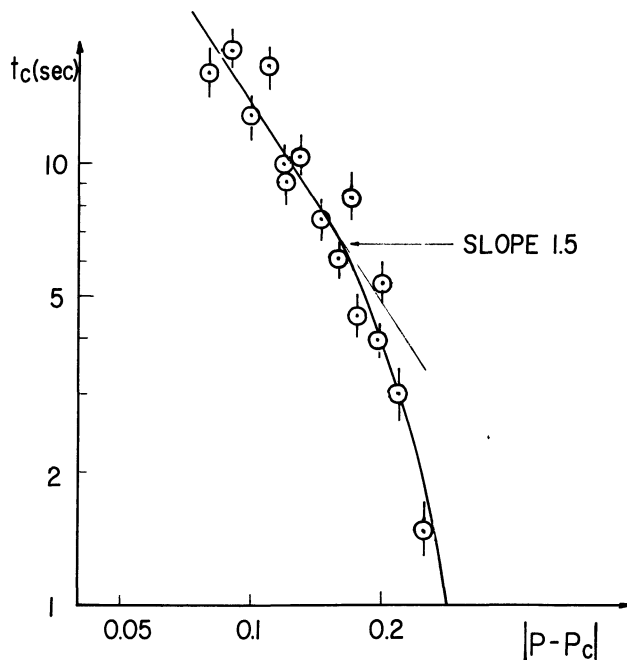


Fig. 3. — The fracture time t_c versus $(p_c - p)$ in a log-log plot. t_c is proportional to the strain.

threshold [5] ($p_c \approx 0.6$). We see that F_c goes too zero like $F_c \sim (p_c - p)^{T_f}$, $T_f = 2.3 \pm 0.5$ and t_c diverges like $t_c \sim (p_c - p)^{-T_u}$, $T_u = 1.5 \pm 0.5$. Each measurement was made on a different sample and in

the case of the same value of p , this corresponds to a different random distribution of holes.

In the second experiment, we applied a known force (as in Ref. [6]) and measured the elongation of the sample. The difference from the set-up of reference [6] is that the elongation is determined using a different technique: in the present case a ferrite rod is mounted on the side of the sample to which the force is applied. This rod can move inside the two coils of a small transformer, and an AC voltage (1 kHz) is applied to one of the coils. The magnitude of the voltage induced in the second coil, depends on the position of the rod inside the coils. This voltage is rectified and recorded through a DC amplifier (Fig. 4).

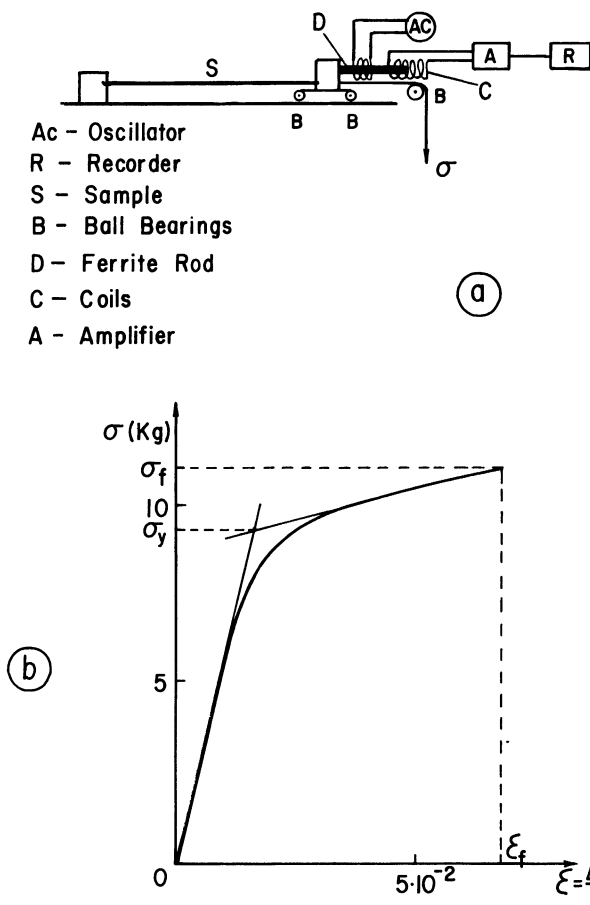


Fig. 4. — a) Experimental set-up when the stress is increased. b) The stress (σ) — strain (ϵ) curve until the fracture (σ_f , ϵ_f). σ_y is the plastic yield stress.

As in the preceding experiment, the mobile side of the sample is mounted on four ball bearings in order to keep it always parallel to the clamped side. The sample is made of copper, but it is different from the sample of the first experiment: we first made a uniform square lattice of 900 (30×30) holes in the sheet. Defects were then introduced by cutting the interhole bonds at random. The measurements were

made in the following way: we apply a known force and measure the corresponding strain. The force is then increased by smaller and smaller increments, until the sample ruptures into two pieces. The stress-strain curve can be drawn up to the point of fracture (Fig. 4). Of course, the fracture takes place through successive breaking of several bonds. This feature cannot quantitatively be observed in the present experiment because the fracture of the sample occurs rapidly.

In figures 5 and 6, we show the two basic quantities: the fracture stress σ_f and the fracture strain

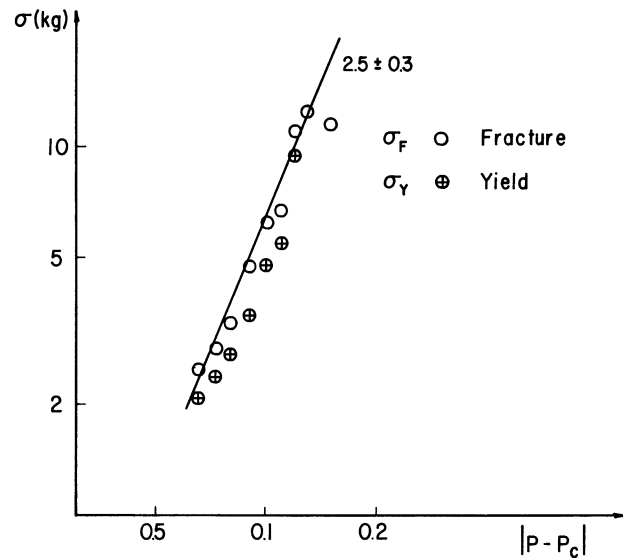


Fig. 5. — The fracture stress (σ_f) and the yield stress (σ_y) versus $(p_c - p)$ in a log-log plot.

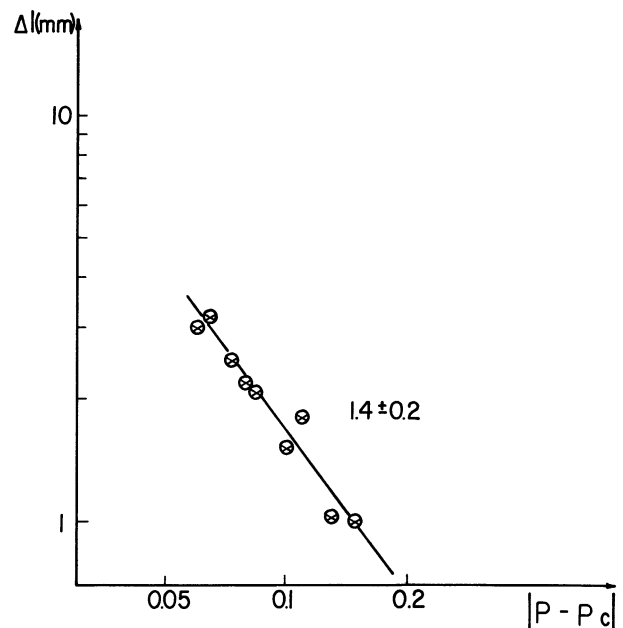


Fig. 6. — The sample elongation versus $(p_c - p)$ in a log-log plot.

ε_f versus $(p_c - p)$, where p is the fraction of disconnected bonds and $p_c = 0.50$. As above, we determined the critical indices $T_f = 2.5 \pm 0.4$ and $T_u = 1.4 \pm 0.2$, and they are in very good agreement with the values determined in the first experiment. In figure 5, we show also the yield stress (defined in Fig. 4), versus $(p_c - p)$ and one sees that σ_y behaves in the same manner as σ_f .

Ray and Chakrabarti [1] have recently proposed a lower bound for T_f

$$T_f \geq (T + (d - d_B) \nu) / 2 \quad (1)$$

where T is the elasticity exponent, d the dimensionality, d_B the fractal dimensionality of the percolative backbone and ν the correlation length exponent. The derivation was made under the explicit assumption that the system is microscopically brittle, i.e. linearity between the stress and the strain of a bond up to its fracture. Inserting the known values of $T = 3.96$ (see Ref. [8]) $d = 2$, $d_B = 1.62$ (Ref. [7]) and $\nu = 4/3$ one finds $T_f \geq 2.12$, which is in agreement with our results. However, caution must be taken, if one recalls that our samples are at least not macroscopically brittle (Fig. 4).

Under similar assumptions, Bergman [2] has obtained both an upper and a lower bound on T_f , namely :

$$T - \nu z < T_f < T - 1 \quad (2)$$

where z is the exponent that characterizes the shortest path ξ_c between neighbouring nodes of the backbone (also known as the chemical path) $\xi_c \sim \xi^z$. Taking $z \simeq 1.10$ (see Ref. [9]) we find

$$2.49 < T_f < 2.96 \quad (3)$$

which is in good agreement with our results.

We now argue that this inequality is applicable not only to brittle materials as envisioned originally, but to a much wider class. The basic idea used to derive equation (2) was that in a diluted elastic network with both bond-stretching and bond-bending force constants, it is only the latter that are important near the percolation threshold p_c . This idea, due to Kantor and Webman [10], explains not only the linear elastic properties near p_c , but the fracture properties as well. In reference [2], a bond angle was assumed to rupture as soon as its angular distortion $\delta\phi$ exceeded some critical value $\delta\phi_c$, with the elastic properties remaining linear up to that point (this is the assumption of microscopic brittleness). Instead of that, we shall assume here that when $\delta\phi$ exceeds $\delta\phi_c$ the bond angle does not rupture, but that its bending force constants drops discontinuously and irreversibly from its original value m_0 to a much smaller value m_1 . In this way the inequality of equation (2) now refers to the plastic yield stress, exponent T_y i.e.

$$\sigma_y \sim (p_c - p)^{T_y} \quad (4)$$

where σ_y is the value of macroscopic stress at which the material first undergoes irreversible plastic distortions. As the macroscopic stress applied to the system increases, a critical value is reached σ_y at which for the first time a bond angle undergoes this irreversible transition — this value is called the plastic yield stress. In order to estimate σ_y for a diluted elastic network near p_c , we invoke the links-nodes-blobs (LNB) picture for the percolating backbone. In that picture, the macroscopic intensive properties are determined by the properties of a typical link which connects two neighbouring nodes of the backbone, at a typical separation given by the correlation length $\xi \sim (p_c - p)^{-\nu}$. When the ends of such a link are stretched (or compressed), most of the strain is carried by the singly connected bond angles (SCBA), and they will therefore be the first to exhibit nonlinear behaviour or undergo the irreversible plastic change described above. If only the SCBA's were distorted, then the angular strain of a SCBA would be related to the macroscopic strain ε and to the end-to-end displacement of a link δR by $L_l \delta\phi_{SCBA} \sim \delta R / \xi \sim \varepsilon$, where $L_l \sim (p_c - p)^{-1}$ is the total number of SCBA's in a link. Since, in fact, the other bond angles and bond lengths may also be somewhat distorted, we get an inequality

$$L_l \delta\phi_{SCBA} \leq \delta R / \xi \sim \varepsilon. \quad (5)$$

An inequality in the opposite sense is obtained if we replace L_l in this equation by the length of a shortest path from end to end of a link ξ^z , namely

$$\xi^z \delta\phi_{SCBA} \geq \frac{\delta R}{\xi} \sim \varepsilon. \quad (6)$$

Plastic yield will occur when $\delta\phi_{SCBA}$ equals $\delta\phi_c$, so that

$$\delta\phi_c (p_c - p)^{T-1} \leq \sigma_y \sim \varepsilon (p_c - p)^T \leq \delta\phi_c (p_c - p)^{T-\nu z}$$

$$T - \nu z < T_y < T - 1. \quad (7)$$

We have thus regained the inequality of equation (2), but with T_f replaced by T_y .

After one SCBA of a link has undergone the transition $m_0 \rightarrow m_1$, it will carry a smaller stress, and consequently the stresses carried by other links will increase. Thus they too will reach the same kind of plastic threshold where one SCBA of each link undergoes the transition $m_0 \rightarrow m_1$. If the macroscopic strain is increased slowly with time, a series of such discontinuous transitions should occur. At each transition the macroscopic stress jumps to a lower value and then increases continuously until the next transition. If the microscopic bending constants

m_0 and critical angular strains $\delta\phi_c$ have the same values for all bond angles, then the critical value of the macroscopic stress is expected to decrease monotonically for consecutive transitions. Thus, if an experiment is performed in which the stress is under control and is increased monotonically, then as soon as σ_y is reached, not only will plastic yield occur, but total fracture as well, i.e., $\sigma_y = \sigma_f$. If the values of m_0 and $\delta\phi_c$ have a statistical distribution, we can expect deviations from the strict monotonicity rule. That is how we interpret the behaviour exhibited in figure 1, where the first transition indeed occurs at the maximum stress, but the subsequent critical stress values are not entirely monotonic with time (and hence with strain). It is also conceivable that for a sufficiently broad distribution of the bond angle parameters m_0 , $\delta\phi_c$ the first plastic transition does not occur at the maximum value of stress. In that case $\sigma_y < \sigma_f$ and moreover the critical behaviour of the fracture process might then belong to a different universality class. Since in reference [3] it is reported that « the load at which fracture of the first bond occurs does not in general coincide with the maximum load in the load — displacement curve », we would like to suggest that their system indeed had such a broad distribution of microscopic parameters. Consequently, the fact that they found a value of $T_f = 1.7 \pm 0.1$, which differs considerably from our value and is even outside the bounds of equation (3), may be due to the fact that their system belongs to a different universality class.

After an entire cross section of links of the LNB superlattice has undergone the plastic transition, the system can be stretched further until the m_1 bond angles become flat. If the system is stretched even

beyond that point it can be expected to rupture into two separate and unconnected pieces. When this occurs, if only the m_1 bond angles are distorted then the absolute elongation of the system L_f should be of the order of ξ . Since in fact other angles and also bond lengths may be distorted, is only a lower bound on L_f . If we define the critical strain exponent T_u , as above by

$$\varepsilon_f = \frac{L_f}{L} \sim (p_c - p)^{-T_u}$$

where L is the total length in the direction of stretching, then what we have found can be summarized as $L_f \geq \xi$, $T_u \geq \nu$. Since $\nu = 4/3$ for $d = 2$, this is in good agreement with our measurements (see Figs. 3 and 6).

We note that despite the fact that for total rupture to occur some links must achieve a relative strain on the order of 1 or more, it nevertheless follows from our picture that the macroscopic relative strain can be much less than 1 if the length L of the system is much greater than ξ . This too is at least in qualitative agreement with our measurements (see Fig. 6).

In a recent article [11], it has been suggested that electrical breakdown thresholds, which are somewhat similar to the mechanical fracture threshold discussed here, vanish in the thermodynamic limit of an infinitely large system. We were unable to observe such an effect since we did not vary systematically the size of our samples.

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