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An energy dependent phase shift analysis of proton-proton scattering between 700 and 1 300 MeV

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Résumé. — Une analyse en déphasages avec dépendance en énergie est présentée pour la réaction élastique $pp \Rightarrow pp$ entre 700 et 1 300 MeV. La plupart des déphasages se connectent parfaitement avec ceux trouvés dans l'analyse de Saclay-Genève en dessous de 800 MeV. Grâce aux nouveaux résultats de Saturne II, la base de données est maintenant suffisante pour faire une telle analyse. Par contre nos résultats sont en désaccord avec ceux trouvés par Arndt *et al.*, et ce pour la plupart des déphasages. Le comportement des diagrammes d'Argand suggère un comportement résonnant pour l'onde partielle 1G_4 mais pas pour l'onde 3F_3 .

Abstract. — An energy dependent $pp \Rightarrow pp$ phase-shift analysis for elastic proton-proton scattering is reported in the energy range between 700 to 1 300 MeV. Almost all phase shifts smoothly connect with the previous results of the Saclay-Geneva phase shift analysis below 800 MeV. Due to recent Saturne II results the data base is now sufficiently abundant to perform such an analysis. Present results strongly disagree with the predictions of Arndt *et al.*, for most phase shifts. Argand diagrams suggest a non-resonant behaviour of the 3F_3 partial wave and a resonance-like behaviour for the 1G_4 partial wave.

1. Introduction.

We present an energy dependent phase shift analysis (PSA) of elastic pp scattering in the energy range from 700 to 1 300 MeV. It makes use of a reasonable number of different spin observables measured with a good precision in a large angular domain. The PSA was made possible by the recent results obtained at Saturne II which have considerably enlarged the existing data base. This PSA is an extension and confirmation of the Saclay-Geneva PSA [1] in the energy range from 10 to 800 MeV. With the exception of the narrow energy interval between 970 and 1 040 MeV, the data base above 800 MeV up to now has been too sparse to perform even fixed energy analysis.

The Arndt *et al.* [3, 4] energy dependent PSA from 0 to 1.1 GeV differs considerably from the present PSA results for almost all phase shifts. Note that it makes use of an insufficient data base above 800 MeV, except around 1 GeV.

There exist also several other fixed energy PSA

above 800 MeV. All of them have quite insufficient data base and their results are very dispersed.

The present PSA uses the same amplitude formalism, one pion exchange (OPE) contributions, electromagnetic contributions and a phase shift energy dependence as described in reference [1]. These subjects are briefly reviewed in section 2, while section 3 is devoted to the data base. Results, their discussion and comparison with other representative analyses [3-8] and with the Paris potential predictions [9, 10] are presented in section 4. Conclusions are summarized in section 5.

Throughout this article we will use a four-index notation and the formalism presented in reference [11]. For data published before April 1981 we refer to reference [2] (reference to data already included in Ref. [2] are quoted as in Ref. [2] e.g. 78/B-200).

2. Formalism.

The observables are expressed in terms of five complex scattering amplitudes, the invariant scattering amplitudes a , b , c , d and e , as discussed in

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reference [11]. The angular dependence of the amplitudes is expressed by Legendre polynomial series and partial wave amplitudes α_{LJ} (Ref. [12] and formulae (2), (3) in Ref. [1]). Partial wave amplitudes, parametrized by nuclear bar phase shifts δ_{LJ} are real, while at higher energies the δ_{LJ} can be complex with $\text{Im } \delta \geq 0$. As in reference [1] we use the « five parameter representation » for the partial wave amplitudes assuming all ε_j to be real. This is justified by the fact that the fit to the data is reasonably good and that the number of imaginary phase shifts increases little compared to the lower energy interval. Indeed, only $\text{Im } ^1S_0$, $\text{Im } ^3P_1$ and $\text{Im } ^3H_5$ were added at high energies to the imaginary parts of the 3P_2 , 1D_2 , 3F_2 , 3F_3 and 1G_4 phase shifts used below 800 MeV in reference [1]. Other phase shift analyses in this energy range also use five parameters.

The expansion of the amplitudes was truncated at the total angular momentum J_{max} and orbital angular momentum L_{max} . The higher angular momentum states were replaced, as usual, by the Born approximation of the OPE contribution (Refs. [13, 14] and formulae (4) in Ref. [1]).

The Coulomb amplitudes and Coulomb-nuclear interferences were introduced as in references [12, 14]. For angular momenta L higher than L_{max} , we have added the magnetic moment correction [15] to the amplitude e .

The energy dependence of the phase shifts were fitted by a polynomial expansion of the form

$$\sum_{n=0}^3 \frac{a_{LJn}}{n!} (T - T_0)^n, \quad (1)$$

where T_0 is the central point of the interval (1 000 MeV for the present PSA) and a_{LJn} are variable parameters. In all cases it turned out that $n = 3$ was sufficient (for seven real phase shifts), and in some cases we adopted $n = 2$, $n = 1$ or $n = 0$. Proper threshold behaviour was assured by multiplying equation (1) by the OPE contributions, obtained from the appropriately calculated OPE elements of the K -matrix. The higher waves, as mentioned above, were taken to be pure OPE ones. It is well known that low- L OPE phase shifts do not correspond to the nuclear-bar phase shifts. We have therefore used the scattering length and effective range energy dependence for 1S_0 phase shift.

The energy dependence of the imaginary parts of phase shifts are parametrized as

$$\text{Im } \delta_{LJ}(T) = \begin{cases} 0 & \text{for } T \leq T_{LJ} \\ \sum_{n=0}^2 \frac{a_{LJn}}{n!} (T - T_{LJ})^{(L+n+\frac{1}{2})} & \text{for } T_{LJ} < T \end{cases} \quad (2)$$

where the threshold energy T_{LJ} is proper to each phase shift. It was fixed by the value found in the previous PSA if the imaginary part of a given phase shift was non-zero. For $\text{Im } ^1S_0$ and $\text{Im } ^3P_1$ phase shifts the corresponding T_{LJ} were treated as free parameters. The coefficients a_{LJ0} , a_{LJ1} and a_{LJ2} were varied. The parametrization, equation (2), of the threshold behaviour of the imaginary parts of the phase shifts is in agreement with general analyticity and unitarity requirements. Inelastic unitarity was imposed by constraining the $\text{Im } \delta_{LJ}$ to be non-negative.

3. Data base.

3.1 ANALYSED DATA. — To denote the experimental quantities, we use a four subscript notation [11] X_{srbt} , where the subscripts, s, r, b, t refer to the scattered, recoil, beam and target particle spin orientation, respectively.

In the present analysis we have used all the relevant data available in the compilation of reference [2] as well as recently published data. These latter are listed in table I (Refs. [16-48]). Some data from different experiments at Saturne II, included in this PSA, have only recently been submitted for publication.

A total of 4 356 independent pp data points were analysed. A more detailed repartition of the data is shown in table II. For each type of experiment the total number of data points as well as their occurrences is indicated.

3.2 INELASTIC TOTAL CROSS-SECTIONS. — The inelastic total cross-sections were introduced in the PSA as described in reference [1]. We have fitted the energy dependence of the total cross-section of the reaction j by an expression of the form [49]:

$$\sigma_j(T) = (T - T_{0j}) \exp \left[\sum_{i=0}^n c_{ij} P_i(T) \right], \quad (3)$$

where P_i are polynomials of the energy T and the parameters T_{0j} , c_{0j} , ..., c_{nj} are fitted for each reaction j . The total inelastic cross-section was then calculated as a sum of all the reaction cross-sections, fitted together with the independent measurements of the σ_{tot} (inelastic). The σ_{tot} (inelastic) were introduced into the PSA in 5 MeV steps with the calculated errors. Our fitted pp reaction cross-section is in good agreement with the one obtained by VerWest and Arndt [50] up to 750 MeV. From there on, the 2π -channel contributions, not taken into account in reference [50], become significant, representing ~ 3 mb at 1 300 MeV. Our treatment of total inelastic cross sections is discussed in a separate article (see Ref. [49b]).

Table I. — Recent pp measurements from 1981 to 1986.

Observable	Kin. Energy Range (MeV)	Angular Range CM deg.	Total Points	Accelerator	Ref.
σ_{tot}	767-1 260	0	5	KEK	[16]
$\sigma_{\text{tot}}(\text{el})$	767-1 260	0	5	KEK	[16]
$\Delta\sigma_T$	702-1 220	0	12	Saturne II	[17]
	791	0	1	LAMPF	[18]
	741-791	0	2	LAMPF	[19]
	730-1 271	0	4	ZGS-ANL	[19]
$\Delta\sigma_L$	739-790	0	3	LAMPF	[20]
	752-1 293	0	5	Saturne II	[21]
$d\sigma/d\Omega$	746	7.1-17.3	25	Gatchina	[22]
	795	8.0-17.5	23	Gatchina	[22]
	843	6.8-17.2	25	Gatchina	[22]
	892	7.2-17.3	24	Gatchina	[22]
	942	6.9-16.6	23	Gatchina	[22]
	992	6.5-16.7	24	Gatchina	[22]
	992	3.7-8.0	39	Gatchina	[23]
	699-1 203	90	86	Saturne II	[24]
	797	2.9-10.3	45	LAMPF	[25]
	797	1.2-11.4	41	LAMPF	[25]
	800	5.9-91	70	LAMPF	[26]
	857	22.3-88.6	19	KEK	[27]
	1 078	22.3-88.6	19	KEK	[27]
	1 168	22.3-88.6	19	KEK	[27]
	1 168	22.3-88.6	19	KEK	[27]
1 262	22.3-88.6	19	KEK	[27]	
A_{ortho}	699	34.6-123.0	9	LAMPF	[28]
	750	34.6-122.8	8	LAMPF	[28]
	699-800	~ 40	3	LAMPF	[29]
	797	1.2-14.0	35	LAMPF	[25]
	800	10.4-47.0	9	LAMPF	[26]
	800	40.1-47.0	4	LAMPF	[26]
	800	14.3-75.4	14	LAMPF	[26]
	699-1 015	35-88	1 050	Saturne II	[30]
	702	42.2-77.7	19	Saturne II	[31]
	720	42.3-77.7	19	Saturne II	[31]
	720	42.2-81.7	20	Saturne II	[31]
	790	40.2-77.6	20	Saturne II	[31]
	790	40.2-77.6	20	Saturne II	[31]
	834	43.0-86.6	23	Saturne II	[32]
	874	42.8-86.8	23	Saturne II	[32]
	934	42.8-86.9	23	Saturne II	[32]
	995	42.9-86.9	23	Saturne II	[32]
	1 095	43.4-87.0	23	Saturne II	[32]
	719	46.6-86.1	21	Saturne II	[33]
	719	53.6-88.7	19	Saturne II	[33]
	894	57.2-88.9	17	Saturne II	[33]
	1 145	59.3-91.7	17	Saturne II	[33]
	874	28.6-47.9	11	Saturne II	[34]
870	30.0-47.9	5	Saturne II	[34]	
1 095	28.3-53.5	14	Saturne II	[34]	
1 091	26.8-52.5	8	Saturne II	[34]	
1 295	28.6-55.6	15	Saturne II	[34]	

Table I (continued).

Observable	Kin. Energy Range (MeV)	Angular Range CM deg.	Total Points	Accelerator	Ref.
A_{ono}	1 292	29.9-52.7	7	Saturne II	[34]
	1 295	52.5-90.7	20	Saturne II	[34]
	874	50-85	8	Saturne II	[35]
	890	27.6-88.0	13	Gatchina	[36]
	950	35-70	6	Gatchina	[37]
	761-950	50	8	Gatchina	[38]
	804	50, 85	3	Gatchina	[38]
$D = D_{\text{ono}}$	741-790	90	2	LAMPF	[39]
	800	4.8-23.8	4	LAMPF	[40]
	800	27.3-133.6	11	LAMPF	[41]
	800	23.8-69.2	4	LAMPF	[28]
	800	70-110	3	Gatchina	[42]
	850	50-130	9	Gatchina	[42]
	900	50-130	9	Gatchina	[42]
$R = D_{s'oso}$	699	41.9-123.3	7	LAMPF	[28]
	750	42.3-134.0	7	LAMPF	[28]
	800	4.8-23.8	4	LAMPF	[40]
	800	21.2-133.6	12	LAMPF	[41]
$A = D_{s'oko}$	699	41.1-123.3	8	LAMPF	[28]
	750	42.3-134.0	7	LAMPF	[28]
	800	4.8-23.8	3	LAMPF	[40]
	800	21.2-133.6	12	LAMPF	[41]
	874	95-130	8	Saturne II	[35]
$R' = D_{k'oso}$	800	4.8-23.8	4	LAMPF	[40]
	800	18.8-133.1	11	LAMPF	[28]
$A' = D_{k'oko}$	699	34.6-123.3	8	LAMPF	[28]
	750	34.6-134.0	8	LAMPF	[28]
	800	7.2-23.8	3	LAMPF	[40]
	800	18.8-133.1	11	LAMPF	[28]
A_{onn}	760-950	50, 90	10	Gatchina	[38]
	890	27.6-88.0	13	Gatchina	[36]
	950	35.0-98.0	12	Gatchina	[37]
	700-800	90	3	LAMPF	[43]
	800	5.7-37.0	23	LAMPF	[44]
	702	42.2-77.7	19	Saturne II	[31]
	720	42.3-77.7	19	Saturne II	[31]
	790	40.2-77.6	20	Saturne II	[31]
	834	43.0-86.6	23	Saturne II	[32]
	874	28.6-47.9	11	Saturne II	[45]
	874	42.8-86.8	23	Saturne II	[32]
	934	42.8-86.9	23	Saturne II	[32]
	995	42.9-86.9	23	Saturne II	[32]
	1 095	28.3-52.0	13	Saturne II	[45]
	1 095	43.4-87.0	23	Saturne II	[32]
	1 295	28.6-55.6	15	Saturne II	[45]
1 295	52.5-90.7	20	Saturne II	[45]	

Table I (continued).

Observable	Kin. Energy Range (MeV)	Angular Range CM deg.	Total Points	Accelerator	Ref.
A_{ookk}	706	26.5-50.5	9	ANL-ZGS	[46]
	740	80-98	10	LAMPF	[20]
	790	80-98	10	LAMPF	[20]
	800	4.5-36.3	25	LAMPF	[44]
	806	29.5-50.5	8	ANL-ZGS	[46]
	823	29.5-50.5	8	ANL-ZGS	[46]
	1 012	29.5-53.5	9	ANL-ZGS	[46]
	1 271	29.5-50.5	8	ANL-ZGS	[46]
	719	49.6-87.6	11	Saturne II	[47]
	834	47.5-88.5	22	Saturne II	[47]
	874	34.5-51.9	10	Saturne II	[48]
	874	47.5-88.6	22	Saturne II	[47]
	934	46.7-89.5	23	Saturne II	[47]
	995	46.8-89.6	23	Saturne II	[47]
	1 095	30.5-55.9	14	Saturne II	[48]
1 095	44.7-89.7	23	Saturne II	[47]	
1 295	28.6-93.0	35	Saturne II	[48]	
A_{oosk}	706	29.5-47.5	7	ANL-ZGS	[46]
	806	29.5-50.5	8	ANL-ZGS	[46]
	823	29.5-50.5	8	ANL-ZGS	[46]
	874	50.4-89.4	21	Saturne II	[47]
	1 012	29.5-50.5	8	ANL-ZGS	[46]
	1 271	29.5-50.5	8	ANL-ZGS	[46]
N_{onkk}	874	50-85	8	Saturne II	[35]

Table II. — Summary of pp data points between 700 and 1 300 MeV used in PSA.

Experimental Quantity	697-1 300 MeV	
	Number of Points	Percent
σ_{tot}	108	3.4
$\sigma_{tot}(el)$	9	
$\sigma_{tot}(inel)$	32	
$\Delta\sigma_T$	21	0.7
$\Delta\sigma_L$	11	
$d\sigma/d\Omega$	1 292	29.7
A_{oono}	1 906	43.8
$D = D_{nono}$	58	4.4
$R = D_{s'oso}$	40	
$A = D_{s'oko}$	48	
$R' = D_{k'oso}$	15	
$A' = D_{k'oko}$	30	

Table II (continued).

Experimental Quantity	697-1 300 MeV	
	Number of Points	Percent
A_{oonn}	393	18.0
A_{ookk}	325	
A_{oosk}	60	
N_{onkk}	8	
Total	4 356	100.0
Contributed Points	4 260	97.8
Omitted Points	96	2.2

3.3 FITTING PROCEDURE. — All experimental data were fitted according to the standard χ^2 -method, including the error matrix calculation (see e.g. Ref.

Table III (continued).

T_{kin} (MeV)	Real Parts of pp Phase Shifts [deg]							
	3F_4	1G_4	ε_4	3H_4	3H_5	3H_6	1I_6	ε
700	6.46	4.25	- 1.95	- 0.35	- 1.39	1.68	1.16	- 1.19
750	6.51	4.03	- 2.06	- 0.43	- 1.40	1.69	1.13	- 1.24
800	6.53	3.79	- 2.10	- 0.52	- 1.41	1.68	1.08	- 1.29
850	6.51	3.54	- 2.08	- 0.61	- 1.41	1.65	1.03	- 1.33
900	6.45	3.28	- 2.00	- 0.70	- 1.40	1.61	0.99	- 1.37
950	6.37	3.03	- 1.84	- 0.81	- 1.30	1.55	0.93	- 1.41
1 000	6.26	2.77	- 1.62	- 0.91	- 1.38	1.48	0.87	- 1.45
1 050	6.12	2.50	- 1.32	- 1.02	- 1.36	1.41	0.81	- 1.49
1 100	5.96	2.24	- 0.94	- 1.13	- 1.33	1.32	0.75	- 1.52
1 150	5.77	1.98	- 0.49	- 1.25	- 1.30	1.21	0.68	- 1.55
1 200	5.57	1.71	0.05	- 1.37	- 1.27	1.10	0.62	- 1.58
1 250	5.34	1.45	0.67	- 1.49	- 1.23	0.98	0.55	- 1.61
1 300	5.10	1.19	1.37	- 1.62	- 1.19	0.85	0.48	- 1.64
Free Coef.	2	2	3	2	2	2	2	1

Table III (continued).

T_{kin} (MeV)	Real Parts of pp Phase Shifts [deg]						
	3J_6	3J_7	3J_8	1K_8	ε_8	3L_8	
700	0.20	- 1.14	0.56	0.53	- 0.30	0.09	
750	0.13	- 1.31	0.61	0.39	- 0.32	0.11	
800	0.04	- 1.49	0.65	0.23	- 0.34	0.14	
850	- 0.05	- 1.67	0.69	0.06	- 0.35	0.16	
900	- 0.15	- 1.86	0.73	- 0.12	- 0.37	0.19	
950	- 0.27	- 2.05	0.77	- 0.31	- 0.38	0.22	
1 000	- 0.39	- 2.25	0.81	- 0.50	- 0.39	0.25	
1 050	- 0.52	- 2.46	0.85	- 0.70	- 0.41	0.28	
1 100	- 0.66	- 2.67	0.89	- 0.91	- 0.42	0.31	
1 150	- 0.80	- 2.88	0.92	- 1.12	- 0.43	0.35	
1 200	- 0.96	- 3.10	0.96	- 1.34	- 0.44	0.39	
1 250	- 1.12	- 3.32	0.99	- 1.56	- 0.45	0.42	
1 300	- 1.29	- 3.55	1.02	- 1.78	- 0.46	0.46	
Free Coef.	2	2	1	2	1	2	

« necessary experiments ») allowing to obtain a unique (or more precisely two-valued above the pion production threshold) PSA solution. Klepikov has determined the necessary number of experimental points as well as the number of observables which must be measured. His method can minimize the data taking time at a given energy but necessitates an interactive procedure between measurements and the PSA [64]. Here we restrict ourself to the case when all measured observables are determined with

a good precision in a large angular range. In fact, a precise measured angular dependence of one observable at one energy must be fitted by a polynomial with degree L_{max} and, consequently, allows one to determine about L_{max} free parameters. The total cross sections (σ_{tot} , $\Delta\sigma_T$ and $\Delta\sigma_L$) determine one extra free parameter. For L_{max} equal 8 five independent experiments can determine about 40 free parameters at one energy (we use 22 real parts and 8 imaginary parts of phase shifts). Of course, our

sufficient set of observables does not assure a unique solution of the PSA.

The number of free parameters for an energy dependent PSA increases by a factor two or three with respect to an energy fixed PSA. On the other hand the number of parameters that can be determined increases by the same factor (we use 73 free parameters for the present PSA). Moreover, phase shift energy dependence provides additional powerful constraints. For example the continuity of phase shifts within an energy interval automatically decreases the possible multiplicity of solutions.

Concerning the present PSA we know that the solution is unambiguous at least at two energies namely 800 and 1 000 MeV. In the vicinity of 800 MeV the number of independent observables corresponds to conditions close to a complete set of experiments in a large angular range. At ~ 1 GeV there exist 10 to 11 independent observables in the angular range $40^\circ \leq \theta_{CM} \leq 90^\circ$ but some of these have relatively large experimental errors. From 800 to 970 MeV there exist eight independent observables and above 1.04 GeV their number is reduced to five but covering almost the entire angular range. Imaginary parts of the amplitudes at $\theta_{CM} = 0^\circ$ are well determined by existing measurements of σ_{tot} , $\Delta\sigma_T$ and $\Delta\sigma_L$. The Gatchina $d\sigma/d\Omega$ data at small angles [22, 23] determine the real parts of amplitudes at $\theta = 0^\circ$ below 1 GeV. Energy dependences of the phase shifts are strongly constrained by the $d\sigma/d\Omega$ points at $\theta_{CM} \sim 90^\circ$ measured in a jet-target experiment at Saturne II [24] up to 1.2 GeV. The results of A_{oono} measured in the angular range $35^\circ \leq \theta_{CM} \leq 88^\circ$ in small energy steps [30] give

another excellent constraints for phase shift energy dependence below 1 GeV. The extension of the PSA up to 1.3 GeV was possible only by adding the recent analysing power (A_{oono}) and spin correlation (A_{oonn} and A_{ookk}) data [31-34, 45, 47, 48] measured at Saturne II with a good precision in a large angular region ($20^\circ \leq \theta_{CM} \leq 100^\circ$).

Some of the phase shift energy dependences, mainly above 1 GeV, may change when all results from Saturne II are available.

4. Results and discussion.

Phase shift values found in the present PSA are given in tables III and IV. Both real and imaginary phase shifts are given in degrees. The χ^2 value of the fit per degree of freedom is 1.41.

The starting values for the analysis were taken from our fixed energy solution at 1 GeV [1]. The thresholds of the imaginary parts of S, P, D, F, G and H waves were studied. The thresholds for $\text{Im } ^1S_0$, $\text{Im } ^3P_1$ and $\text{Im } ^3H_5$ are found to be 750, 920 and 900 MeV, respectively, the threshold for other imaginary parts were taken from reference [1] and fixed here. Note, that the contribution of the $\text{Im } ^3P_0$, found to be negligibly small in our analysis, is hard to separate from that of the other P-wave imaginary parts.

The energy dependences of the S, P, D, F, G, H, I, J, K and L phase shifts and mixing parameters are shown in figures 1-14 together with our previous results below 800 MeV (full lines) and the starting points at 1 GeV (black dots). One can see that

Table IV. — *Imaginary parts of pp phase shifts in degrees.*

T_{kin} (MeV)	Imaginary Parts of pp Phase shifts [deg]							
	$\text{Im } ^1S_0$	$\text{Im } ^3P_1$	$\text{Im } ^3P_2$	$\text{Im } ^1D_2$	$\text{Im } ^3F_2$	$\text{Im } ^3F_3$	$\text{Im } ^1G_4$	$\text{Im } ^3H_5$
700			10.03	12.09	1.44	10.33	0.29	
750			12.17	14.44	2.41	13.85	0.50	
800	0.04		14.26	16.74	3.70	17.47	0.81	
850	0.20		16.26	18.91	5.32	20.92	1.22	0.00
900	0.55		18.16	20.89	7.27	23.96	1.73	0.00
950	1.14	0.01	19.96	22.58	9.50	26.41	2.33	0.00
1 000	1.99	0.12	21.63	23.93	11.96	28.20	3.01	0.00
1 050	3.13	0.39	23.19	24.87	14.56	29.42	3.74	0.00
1 100	4.61	0.87	24.63	25.34	17.19	30.37	4.78	0.02
1 150	6.43	1.61	25.96	25.30	19.68	31.62	5.16	0.06
1 200	8.63	2.64	27.20	24.70	21.86	34.09	5.73	0.17
1 250	11.24	3.98	28.35	23.51	23.50	39.12	6.09	0.40
1 300	14.26	5.66	29.43	21.72	24.34	48.48	6.15	0.88
Free Coef.	1	1	3	3	2	3	3	1

almost all phase shifts connect smoothly with our solution at lower energies. The apparent disconnection of the $\text{Im } ^1G_4$ is probably due to the fact that in the lower energy interval (500-800 MeV) we have not allowed 1S_0 to be imaginary. The « corridor of errors », shown only for the fixed energy analysis at 1 GeV, was calculated as the square root of the corresponding diagonal element of the error matrix. It is much narrower than the error corresponding to the « confidence level 1σ » due to the large contribution of the non-diagonal elements in the error matrix, as was discussed in reference [1].

Our results in figures 1-13 are compared with the energy dependent PSA of Arndt *et al.* [3] (dot-dashed line) and with the results of representative fixed-energy phase shift analyses above 800 MeV (triangles [5], open circles [6], diamonds [7]). Below 800 MeV a comparison with the locally energy-dependent PSA of Dubois *et al.* [8] is also shown (black squares). The imaginary parts of phase shifts are also compared with the Paris potential prediction [10] (dashed line). All phase shifts have been converted into our representation ; this transformation mainly changes the imaginary parts. Only the phase shifts left free are shown in the figures. The comparison of the results below 800 MeV was discussed in reference [1]. The energy dependences above 800 MeV show a general agreement with

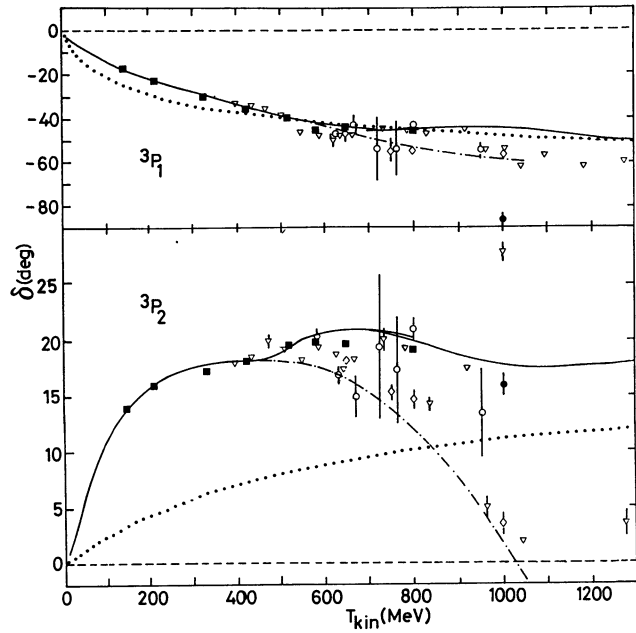


Fig. 2. — Energy dependence of 3P_1 and 3P_2 phase shifts. Symbols are defined in figure 1.

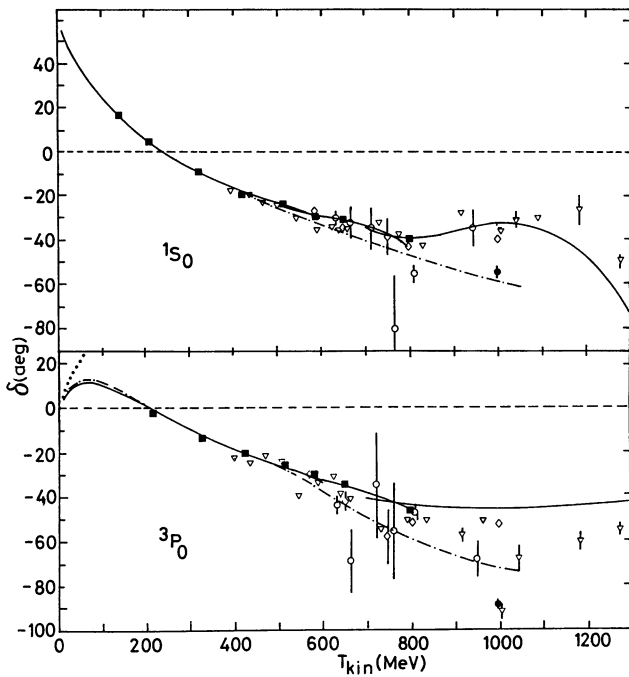


Fig. 1. — Energy dependences of 1S_0 and 3P_0 phase shifts. — Present analysis and PSA of reference [1] ; • Previous fixed energy solution at 1 GeV ; — — — Analysis of reference [3] ; ▽ Analysis of reference [5] ; ○ Analysis of reference [6] ; ◇ Analysis of reference [7] ; ■ Analysis of reference [8] ; OPE and electromagnetic correction.

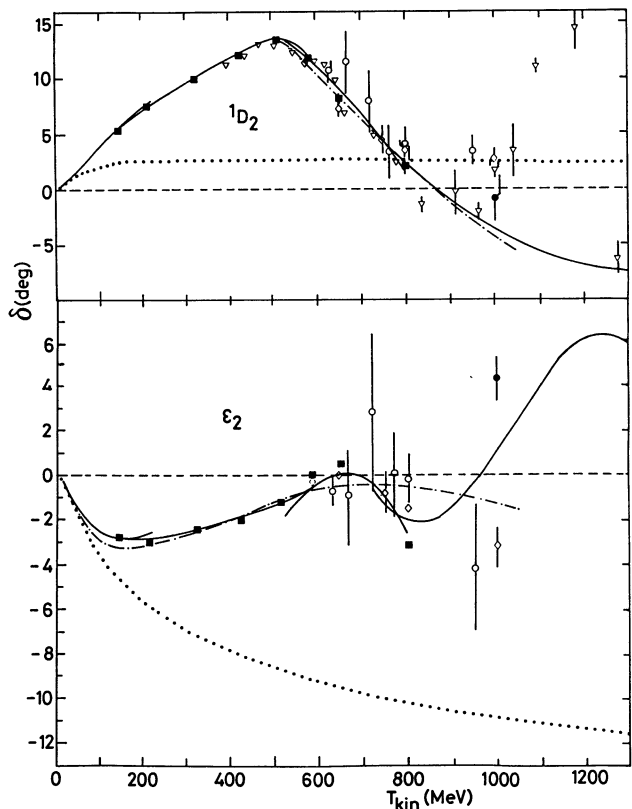


Fig. 3. — Energy dependence of the phase shift 1D_2 , and of the mixing parameter ϵ_2 . Symbols are defined in figure 1.

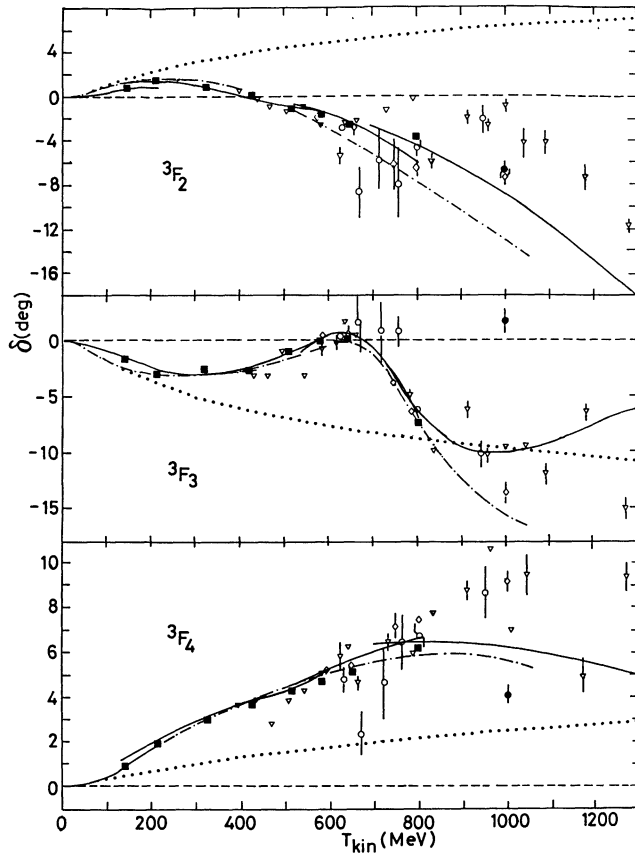


Fig. 4. — Energy dependence of 3F_2 , 3F_3 and 3F_4 . Symbols are defined in figure 1.

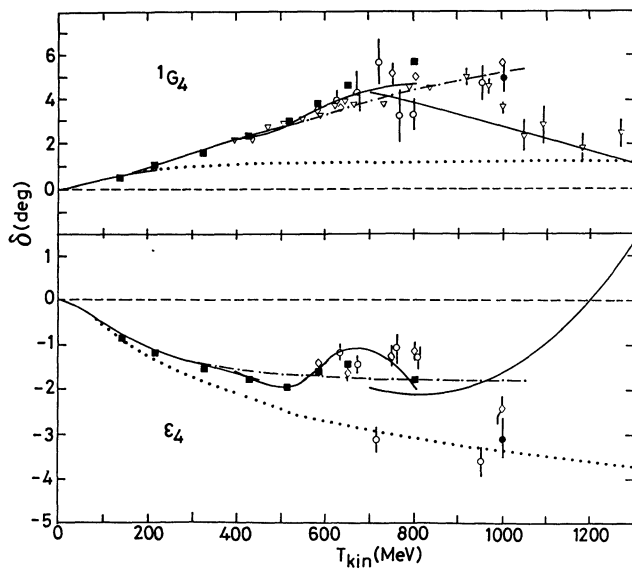


Fig. 5. — Energy dependence of the phase shift 1G_4 and of the mixing parameter ϵ_4 . Symbols are defined in figure 1.

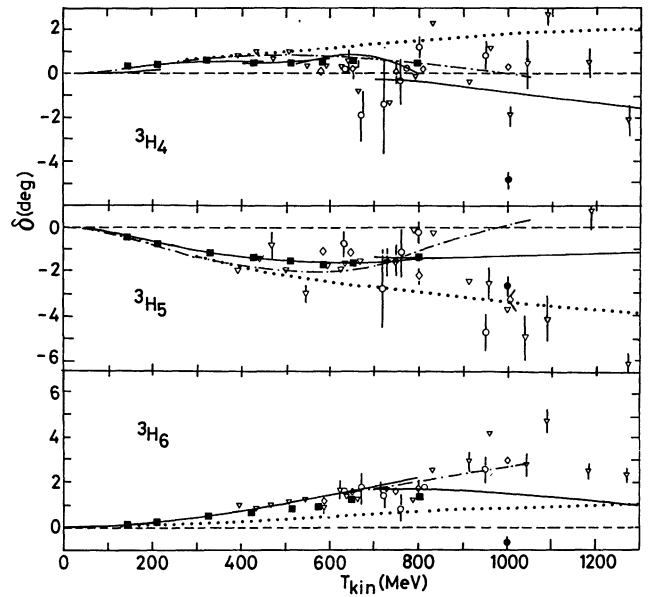


Fig. 6. — Energy dependence of 3H_4 , 3H_5 and 3H_6 . Symbols are defined in figure 1.

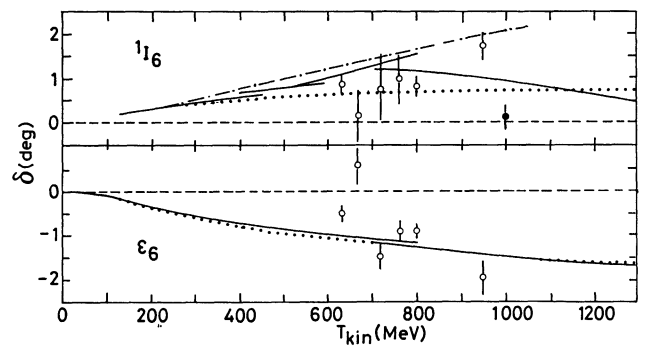


Fig. 7. — Energy dependences of the phase shift 1I_6 and of the mixing parameter ϵ_6 . Symbols are defined in figure 1.

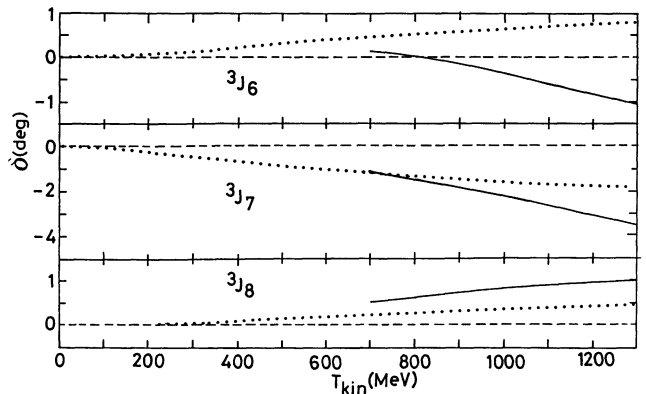


Fig. 8. — Energy dependence of the phase shifts 3J_6 , 3J_7 and 3J_8 . Symbols are defined in figure 1.

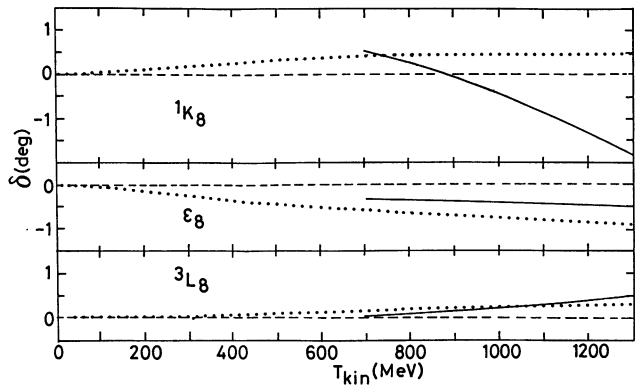


Fig. 9. — Energy dependence of 1K_8 , ϵ_8 and 3L_8 . Symbols are defined in figure 1.

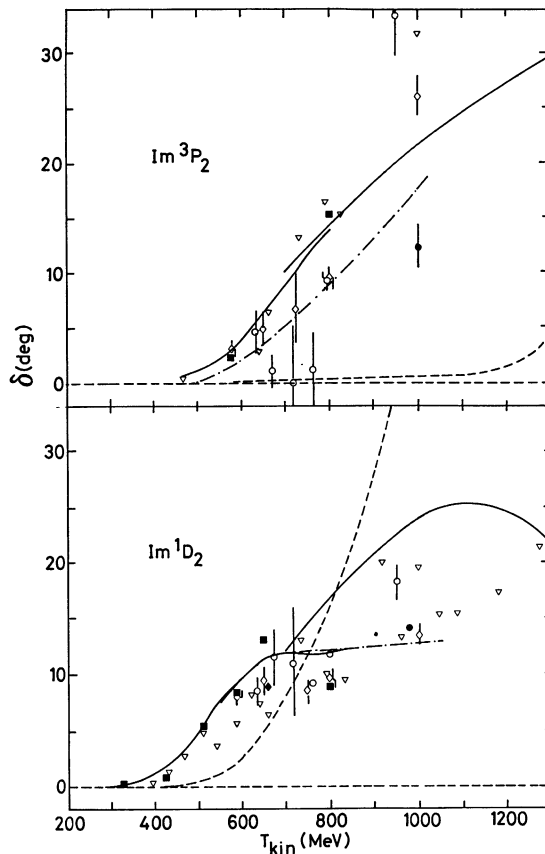


Fig. 11. — Energy dependence of the imaginary parts of 3P_2 and 1D_2 phase shifts. Symbols are defined in figures 1 and 10.

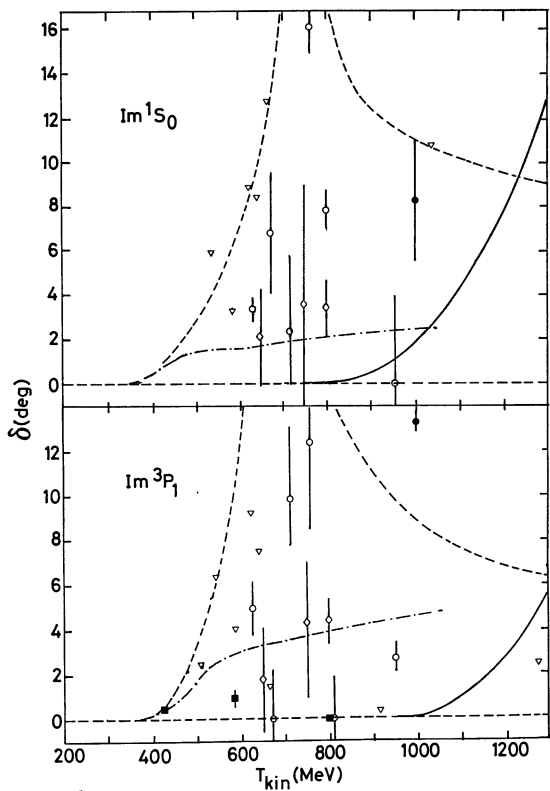


Fig. 10. — Energy dependence of the imaginary parts of the phase shifts 1S_0 and 3P_1 . The dashed line is prediction from the Paris potential [10]. Other symbols have been defined in figure 1.

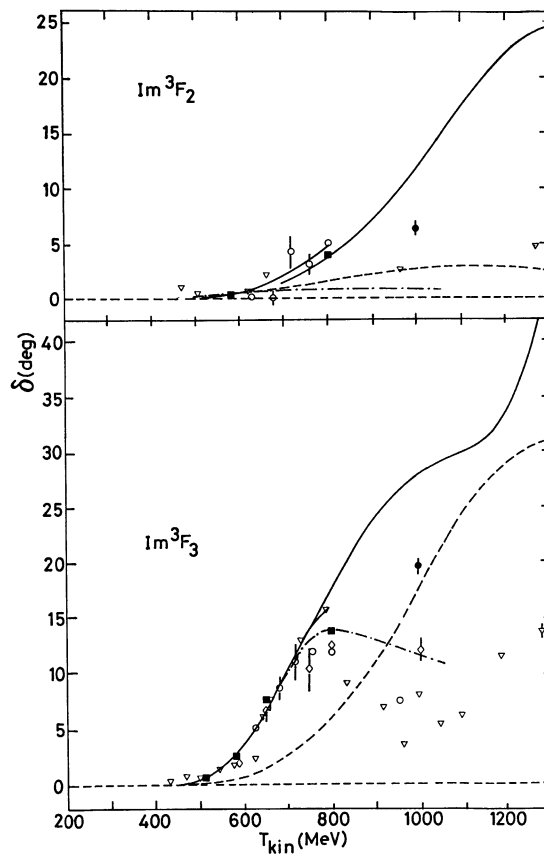


Fig. 12. — Energy dependence of the imaginary parts of 3F_2 , and 3F_3 phase shifts. The symbols are as in figures 1 and 10.

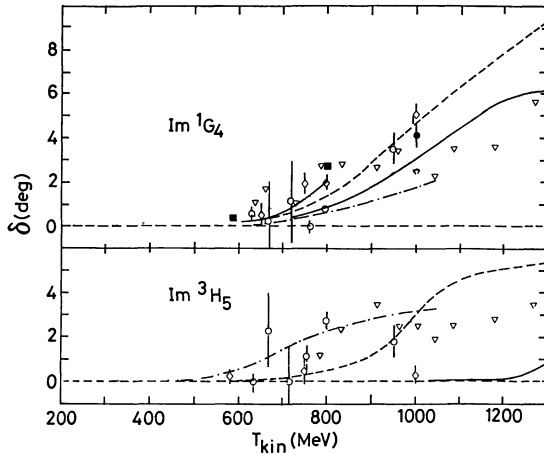


Fig. 13. — Energy dependence of the imaginary parts of 1G_4 and 3H_5 phase shifts.

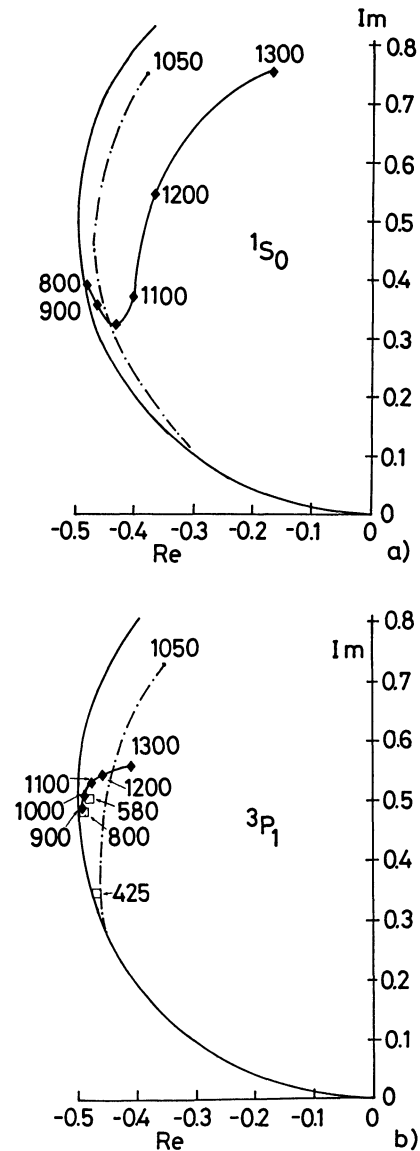
results of Arndt *et al.* [3] only for the real parts of 3P_1 , 1D_2 , 3F_2 , 3F_4 , 3H_4 , 3H_5 and 3H_6 phase shifts and for $\text{Im } ^3P_2$ and $\text{Im } ^1G_4$. All other phase shifts and mixing parameters are in strong disagreement. The results of their very recent version [4] differ only little from reference [3]. A most important disagreement can be seen in figure 2 for the $\text{Re } ^3P_2$ phase shift from 400 MeV on. The present analysis confirms the behaviour observed in reference [1].

This analysis, as well as that of reference [1] below 800 MeV is in good agreement with the results of reference [8]. It is hard to compare our results with those of other analyses at fixed energy above 800 MeV, since fixed-energy PSA results are very dispersed. For the $\text{Re } ^1S_0$ phase shift there is very good agreement between the present PSA and the results of references [5, 7]. For the $\text{Re } ^3P_0$ and $\text{Re } ^3P_1$ the fixed energy PSA results are between the present solution and that of reference [3]. We found the $\text{Re } ^3P_1$ to be very close to the OPE contribution. The energy dependence of $\text{Re } ^3F_3$ agree with the results from references [5, 6]. The imaginary parts differ considerably between different analyses for the 1P_0 , 3P_1 , 1D_2 , 3F_2 and 3F_3 phase shifts. The solution, given by the present analysis will probably change above 1 GeV, but below this energy only small changes should be expected. Finally, we have compared our results for imaginary parts of phase shifts with the Paris potential group latest predictions [10], calculated from a dispersion relation approach [9]. The global energy trend for the 1D_2 , 3F_3 and 1G_4 phase shifts is consistent with ours (see Figs. 11 to 13). A disagreement for S and P phase shifts is not surprising, since the authors of references [9, 10] claim reasonable predictions starting from $J = 3$ on. Considering all their imaginary phase shifts, their total inelastic cross section reproduces the energy dependence of the data.

Our earlier single energy solution at 1 GeV [1] differs significantly from the present energy-dependent solution. In the previous analysis, in order to avoid energy dependent development of phase shifts, data were considered only in a narrow energy range (0.97-1.04 GeV). This provide us with an uncomplet set of observables between 45° CM and 90° CM with relative large experimental errors.

No new data have appeared in this energy range, but a significant amount (~ 210 data points equivalent to $1/3$ of all data used at « 1 GeV ») becomes available mainly from KEK, Saturne II and Gatchina in the interval ± 100 MeV around 1 GeV (see Tab. I). Moreover, these new data cover the small-angle region ($\geq 23^\circ$ CM). Under these circonstances it is not surprizing that the solutions differ, since the energy dependence represent a powerful constraint as discussed in section 3.4.

The Argand diagrams shown in figures 14a-h, also connect smoothly with our previous results. The Argand diagrams for 3P_2 , 1D_2 , 1G_4 and 3H_5 show an



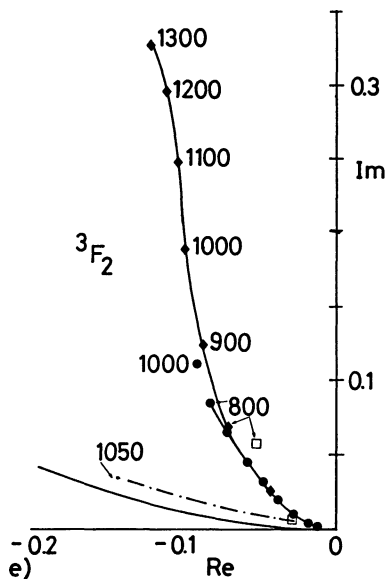
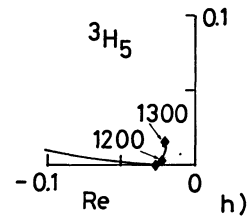
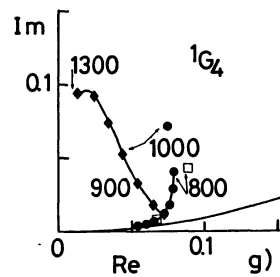
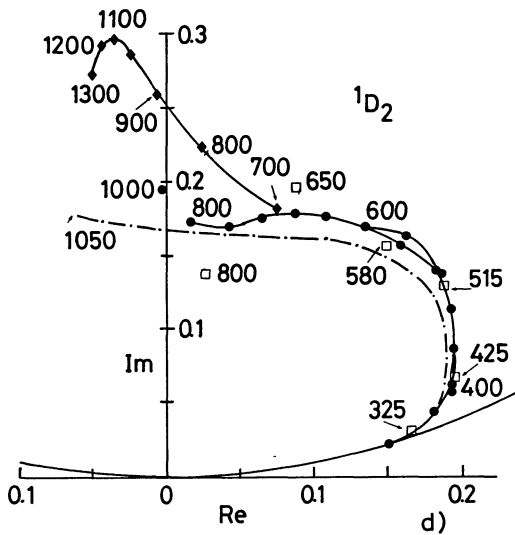
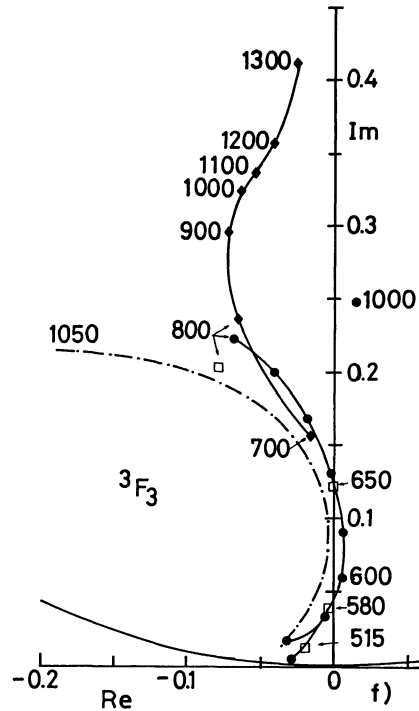
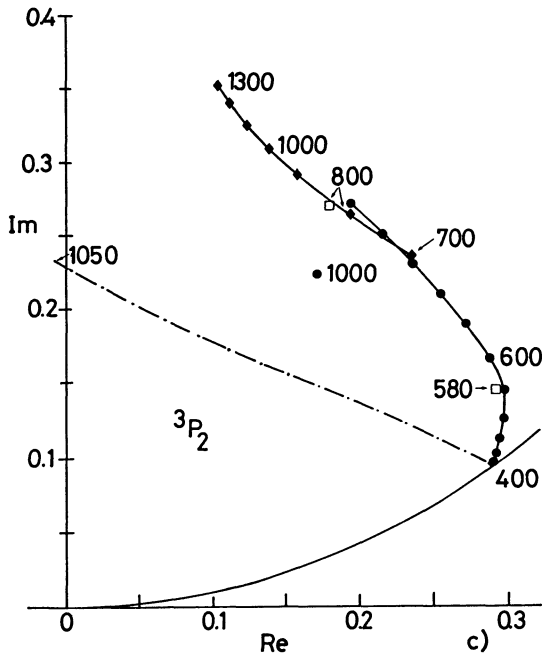


Fig. 14a-h. — Argand diagrams for —●—●— Previous analysis every 40 MeV [1]; ● Previous fixed-energy solution at 1 GeV [1]; —◆—◆— Present analysis every 100 MeV; —·—·— Analysis of reference [3]; □ Analysis of reference [8] at 325, 425, 515, 580, 650 and 800 MeV. For each energy only the phase shifts with model-independent imaginary parts are represented.

anticlockwise behaviour in the entire energy range. The change of the initial anticlockwise behaviour of the 3F_3 phase shift into a slow clockwise movement above 800 MeV bring into question the existence of a resonance at the total energy of 2.22 GeV, as claimed by many authors. The shape of the Argand diagram for 3P_2 (Fig. 14c) may be consistent with a possible resonance structure in one of the P waves, suggested by Lomon [65]. The structure in $\text{Re } ^3P_2$ above 400 MeV (Fig. 2) observed in reference [1] and confirmed here supports this suggestion. The singlet-spin state partial waves 1D_2 and 1G_4 show interesting behaviour between 1.1 and 1.3 GeV. To establish a possible resonance-like structure in the 1G_4 wave, which may occur at ~ 1.3 GeV, an extension of the PSA to higher energies is necessary.

Our results are compared with those of references [3] (dot-dashed line) and [8] (open squares).

The energy dependences of the real to imaginary part ratio of the spin independent forward amplitude $[\text{Re}(a+b)/\text{Im}(a+b)]$ and of the total cross section differences $\sigma_{1\text{tot}} = -\Delta\sigma_T/2$ and $-\Delta\sigma_L$ are shown in figures 15 to 17 (together with all existing data). We have not used dispersion relation predictions [66] as input into our analysis. Nevertheless, these are in excellent agreement with our solution as can be seen in figure 15. The $\sigma_{1\text{tot}}$ (Fig. 16) does not

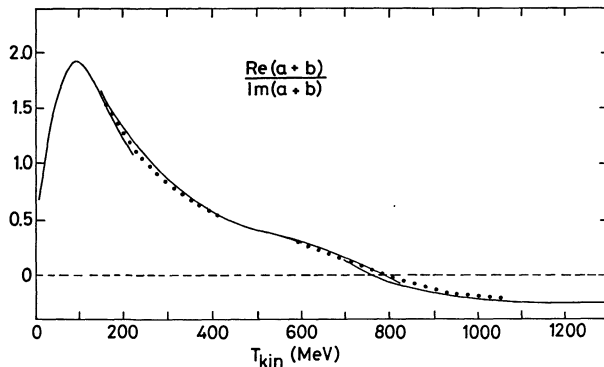


Fig. 15. — Energy dependence of $\text{Re}(a+b)/\text{Im}(a+b)$ at $\theta = 0$. Dispersion relation prediction [64] is shown as dotted line.

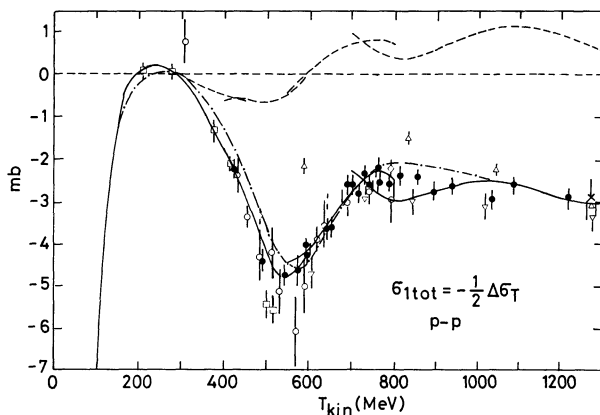


Fig. 16. — Energy dependence of $\sigma_{1\text{tot}}(\text{pp}) = -\Delta\sigma_T(\text{pp})/2$. — this PSA and reference [1]; -·-·- reference [3]; - - - $\sigma_{1\text{tot}}(\text{el})$, this analysis and reference [1]; ● Saturne II, reference [17]; □ TRIUMF, reference [67]; ◇ LAMPF, reference [18]; ○ Rice-LAMPF, reference [19]; ▽ Rice-ZGS, reference [19]; △ ZGS-ANL, reference 78/B-248/ in [2]; × ZGS-ANL, reference 75/D-80/ in [2].

connect smoothly with the energy dependence established in reference [1] even if the experimental points are fairly well fitted. There is a lack of constraints below 700 MeV where the absolute value of $\sigma_{1\text{tot}}$ increases rapidly. Anyway the difference in

the overlapping region is ≤ 0.5 mb which is comparable to the experimental errors. The $-\Delta\sigma_L$ total cross section difference (Fig. 17) connects smoothly with our previous results in the overlapping region. A lack of data around 1.2 GeV allows the decrease in $-\Delta\sigma_L$ values. Our predictions for $\sigma_{1\text{tot}}$ and $\Delta\sigma_L$ are compared with the predictions of reference [3]. In figures. 16 and 17 are also plotted predictions for the elastic parts of $\sigma_{1\text{tot}}$ and $-\Delta\sigma_L$ as dashed line. These results confirm that the elastic part of $\sigma_{1\text{tot}}$ is small and that for $\Delta\sigma_L(\text{el})$ is large.

Our analysis fits well the observed crossing of zero analysing power A_{00n0} at $\theta_{\text{CM}} = 90^\circ$ from negative to positive sign above 1.15 GeV. This behaviour, definitely established at Saturne II [32, 45], can be seen in figure 18 where the angular dependence $A_{00n0} = f(\theta_{\text{CM}})$ is plotted for three energies.

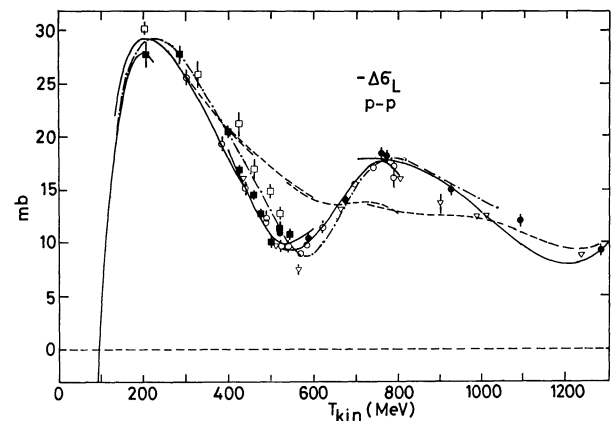


Fig. 17. — Energy dependence of $-\Delta\sigma_L(\text{pp})$. — this PSA and reference [1]; -·-·- reference [3]; - - - $-\Delta\sigma_L(\text{el})$, this PSA and reference [1]; ■ SIN, reference [68]; ● Saturne II, reference [21]; □ TRIUMF, reference [67]; ○ LAMPF, reference [20]; ▽ ZGS-ANL, reference 78/A-164/ in [2].

In figure 19 are shown the energy dependences of the spin correlation parameters A_{00nn} , A_{00kk} and A_{00ss} at $\theta_{\text{CM}} = 90^\circ$ as calculated from this PSA and from reference [1]. Also are plotted all data points above 700 MeV measured close to 90° CM. A local maximum at ~ 1.15 GeV in the A_{00nn} (as well as in A_{00ss}) energy dependence may be due to a lack of experimental data and needs confirmation. On the other hand the decrease observed at 1.3 GeV for all three parameters, is confirmed by existing data above this energy [45, 48]. This supports a possible singlet spin state resonance in 1G_4 partial wave around 1.3 GeV.

5. Conclusions.

The present PSA fits well all existing experimental data points including the new Saclay results. The

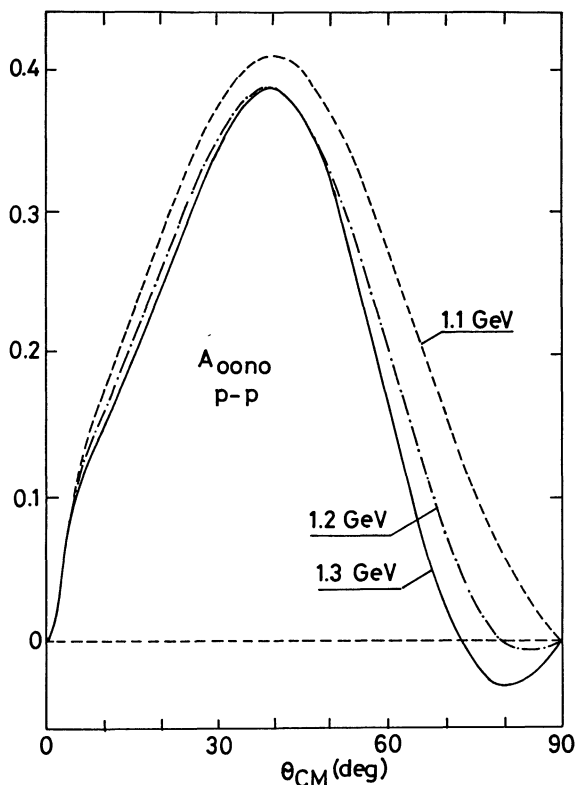


Fig. 18. — Angular dependences of $A_{ooono} = f(\theta_{CM})$ at 1 100, 1 200 and 1 300 MeV.

slope of the analysing power near 90° CM is also well described. The energy dependence of the phase shifts smoothly connects with Saclay-Geneva PSA previous results below 800 MeV. An important disagreement is observed with the results of Arndt *et al.* for almost all phase shifts. The energy dependence of the phase shifts above 1.1 GeV may change when the 2 and 3 spin-index data measured at Saturne II are available.

The present PSA shows the general behaviour of phase shifts and provides good predictions for the design of pp elastic and inelastic scattering experiments as well as for values of nucleon-nucleon amplitudes used as input in nucleon-nucleus scattering models. It also provides useful information about a possible resonance-like behaviour in several partial waves.

Acknowledgments.

We acknowledge J. Arvieux, R. Hess and G.

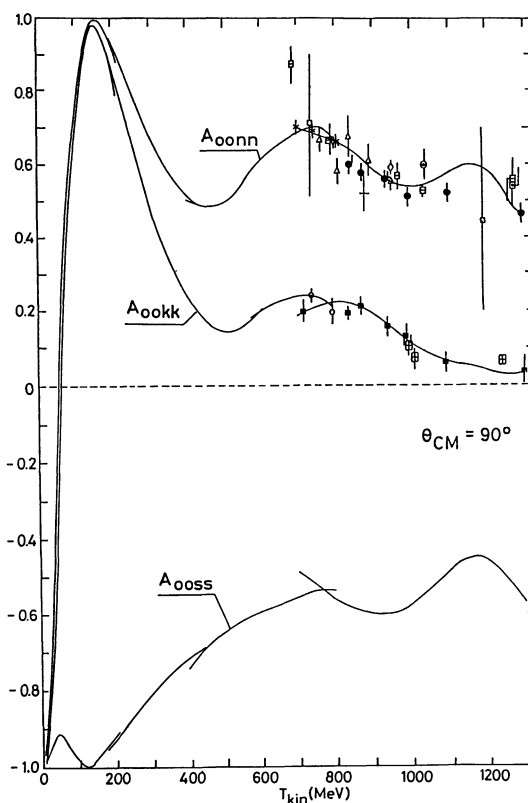


Fig. 19. — Energy dependences of the spin correlation parameters A_{oonn} , A_{ookk} and A_{ooos} at $\theta_{CM} = 90^\circ$. — this PSA and reference [1]; Symbols for A_{oonn} : \square ZGS-ANL, reference 80/B-304/ in [2]; \odot ZGS-ANL, reference 78/L-52/ in [2]; \square ZGS-ANL, reference 77/M-97/ in [2]; \square Saturne I, reference 67/C-26/ in [2]; ∇ LAMPF, reference 80/M-111/ in [2]; * LAMPF, reference [43]; + Gatchina, reference [36]; \diamond Gatchina, reference [37]; \triangle Gatchina, reference [38]; \bullet Saturne II, references [32, 45]; Symbols for A_{ookk} : \circ LAMPF, reference [20]; \oplus ZGS-ANL, reference [69] and reference 78/A-165/ in [2] (numerical values); \blacksquare Saturne II, references [47, 48].

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References

- [1] BYSTRICKY, J. LECHANOINE-LELUC, C. and LEHAR, F., *J. Physique* **48** (1987) 199.
- [2] BYSTRICKY, J. and LEHAR, F., *Physics Data* Nr 11-1, Part I and II (1978) Nr 11-2 and 11-3 (1981), edited by H. Behrens and G. Ebel (Fachinfor-

mationszentrum Karlsruhe) and Bystricky J., Carlson P., Lechanoine C., Lehar F., Monnig F., Schubert K. R., *Landolt-Bornstein Tables, New Series Vol. 9a* (1980), edited by H. Schopper (Springer-Verlag).

- [3] ARNDT, R. A., ROPER, L. D., BRYAN, R. A., CLARK, R. B., VERWEST, B. J., SIGNAL, P., *Phys. Rev. D* **28** (1983) 97 and Private communication.
- [4] ARNDT, R. A., BYSLOP, J. S. and ROPER, L. D., *Phys. Rev. D* **35** (1987) 128.
- [5] HIGUCHI, Y., HOSHIZAKI, N., NAKAO, H. and SUZUKI, K., contributed paper to « 1983 INS Symposium », Tokyo, 1983.
- [6] VOVCHENKO, V. G., KAZARINOV, M. Yu., KAZARINOV, Yu. M., PROKOFIEV, A. N. and STRACHOTA, J., *Yad. Fiz.* **37** (1983) 158, transl. *Sov. J. Nucl. Phys.* **37** (1) (1983) 90.
- [7] GREBENYUK, O. G., KOMAROV, E. N. and SHKLYAREVSKII, G. M., *Yad. Fiz.* **39** (1983) 74, transl. *Sov. J. Nucl. Phys.* **39** (1984) 44.
- [8] DUBOIS, R., AXEN, D., KEELER, R., COMYN, M., LUDGATE, G. A., RICHARDSON, J. R., STEWART, N. M., CLOUGH, A. S., BUGG, D. V. and EDGINGTON, J. A., *Nucl. Phys. A* **377** (1982) 554.
- [9] CÔTÉ, J., LACOMBE, M., LOISEAU, B. and COTTINGHAM, W. N., *Nucl. Phys. A* **379** (1982) 349 ; and LACOMBE, M., LOISEAU, B., RICHARD, J. M., VINH MAU, R., CÔTÉ, J., PIRÈS, P. and DE TOURREIL, R., *Phys. Rev. C* **21** (1980) 861.
- [10] LOISEAU, B., *J. Physique Colloq.* **46** (1984) C2-339 ; and LOISEAU, B., Private Communication, October 1986.
- [11] BYSTRICKY, J., LEHAR, F. and WINTERNITZ, P., *J. Physique* **39** (1978) 1.
- [12] STAPP, H. P., YPSILANTIS, T. J., METROPOLIS, M., *Phys. Rev.* **105** (1957) 302.
- [13] CZIFFRA, P., MACGREGOR, M. H., MORAVCSIK, M. J. and STAPP, H. P., *Phys. Rev.* **114** 880 (1959) ; and MACGREGOR, M. H., MORAVCSIK, M. J. and STAPP, H. P., *Ann. Rev. Nucl. Sci.* **10** (1960) 291.
- [14] HOSHIZAKI, N., *Suppl. Progr. Theor. Phys.* **42** (1968) 107.
- [15] BREIT, G. and RUPPEL, H., *Phys. Rev.* **127** (1962) 2123.
- [16] SHIMIZU, F., KUBOTA, Y., KOISO, H., SAI, K., SAKAMOTO, S. and YAMAMOTO, S. S., *Nucl. Phys. A* **386** (1982) 571.
- [17] PERROT, F., AZAIEZ, H., BALL, J., BYSTRICKY, J., CHAUMETTE, P., CHESNY, Ph., DERÉGEL, J., FABRE, J., FONTAINE, J. M., GOSSET, J., LEHAR, F., LEO, W. R., DE LESQUEN, A., NEWSOM, C. R., ONEL, Y., PENZO, A., VAN ROSSUM, L., SIEMIARCZUK, T., VRZAL, J., WHITTEN, C. A. Jr. and YONNET, J., *Nucl. Phys. B* **278** (1986) 881.
- [18] DITZLER, W. R., HILL, D., IMAI, K., SHIMIZU, H., SPINKA, H., STANEK, R., TOSHIOKA, K., UNDERWOOD, D., WAGNER, R., YOKOSAWA, A., BURLESON, G. R., COTTINGAME, W. B., GREENE, S. J., JARMER, J. J., JEPPESEN, R. H., *Phys. Rev. D* **27** (1983) 680.
- [19] MADIGAN, W. P., BELL, D. A., BUCHANAN, J. A., CALKIN, M. M., CLEMENT, J. M., COPEL, M., CORCORAN, M. D., JOHNS, K. A., LESIKAR, J. D., MIETTINEN, J. E., MUTCHLER, G. S., NAUDET, C. J., PEPIN, G. P., PHILLIPS, G. C., ROBERTS, J. B., TURPIN, S. E., HUNGERFORD, E. V., MAYES, B. W., HANCOCK, A. D., PINSKI, L. S., SEKCHARAN, K. K., HOLLAS, C. L., RILEY, P. J., ALLRED, J. C., BONNER, B. E., CAMERON, P., LINN, T. S., VON WITSCH, W., FURIC, M., VALKOVIC, V., *Phys. Rev. D* **31** (1985) 966.
- [20] AUER, I. P., DITZLER, W. R., HILL, D., IMAI, K., SPINKA, H., STANEK, R., TOSHIOKA, K., UNDERWOOD, D., WAGNER, R., YOKOSAWA, K., HOFFMAN, E. W., JARMER, J. J., BURLESON, G. R., COTTINGAME, W. B., GREENE, S. J. and STUART, S., *Phys. Rev. D* **29** (1984) 2435.
- [21] BYSTRICKY, J., CHAUMETTE, P., DERÉGEL, J., FABRE, J., LEHAR, F., DE LESQUEN, A., VAN ROSSUM, L., FONTAINE, J. M., GOSSET, J., PERROT, F., BALL, J., HASEGAWA, T., NEWSOM, C. R., YONNET, J., LEO, W. R., ONEL, Y., PENZO, A., AZAIEZ, H., MICHALOWICZ, A., *Phys. Lett.* **142B** (1984) 130.
- [22] DOBROVOLSKY, A. V., KHANZADEEV, A. V., KOROLEV, G. A., MAEV, E. M., MEDVEDEV, V. I., SOKOLOV, G. L., TERYTYEV, N. K., TERRIEN, Y., VELICHKO, G. N., VOROBYOV, A. A. and ZALITE, Yu. K., *Nucl. Phys. B* **214** (1983) 1.
- [23] VELICHKO, G. N., VOROBYOV, A. A., ZALITE, Yu. K., KOROLEV, G. A., MAEV, E. M., TERYTYEV, N. K., KHANZADEEV, A. V. and SHCHEGEL'SKII, A. V., *Yad. Fiz.* **35** (1982) 1457 transl. *Sov. J. Nucl. Phys.* **35** (6) (1982) 852.
- [24] GARÇON, M., LEGRAND, D., LOMBARD, R. M., MAYER, B., ROUGER, M., TERRIEN, Y. and NAKACH, A., *Nucl. Phys. A* **445** (1985) 669.
- [25] PAULETTA, G., ADAMS, G., HAJI-SAIED, S. M., IGO, G. J., MCCLELLAND, J. B., WANG, A. T. M., WHITTEN, C. A. Jr., WRIEKAT, A., GAZZALY, M. M., TANAKA, N., *Phys. Rev. C* **27** (1983) 282.
- [26] BARLETT, M. L., HOFFMANN, G. W., MCGILL, J. A., HOISTAD, B., RAY, L., FERGERSON, R. W., MILNER, E. C., MARSHALL, J. A., AMANN, J. F., BONNER, B. E., MCCLELLAND, J. B., BLANPIED, J. S., ARNDT, R. A., *Phys. Rev. C* **27** (1983) 682.
- [27] SHIMIZU, F., KOISO, H., KUBOTA, I., SAI, F., SAKAMOTO, S. and YAMAMOTO, S. S., *Nucl. Phys. A* **389** (1982) 445.
- [28] HOLLAS, C. L., CREMANS, D. J., MCNAUGHTON, K. H., RILEY, P. J., RODEBAUGH, R. F., SHENWU Xu, BONNER, B. E., MCNAUGHTON, M. W., OHNUMA, H., VAN DYCK, O. B., SUN TSU-HSUN, TURPIN, S., AAS, B. and WESTON, G. S., *Phys. Rev. C* **30** (1984) 1251.
- [29] MCNAUGHTON, M. W. and CHAMBERLIN, E. P., *Phys. Rev. C* **24** (1981) 1778.
- [30] GARÇON, M., Thesis, Faculté des Sciences, Orsay, 1985 and Private Communication of Garçon M., Legrand D. and Saudinos J., *Phys. Lett. B*, to be published.

- [31] BYSTRICKY, J., CHAUMETTE, P., DERÉGEL, J., FABRE, J., LEHAR, F., DE LESQUEN, A., VAN ROSSUM, L., FONTAINE, J. M., GOSSET, J., PERROT, F., WHITTEN, C. A. Jr., BALL, J., CHESNY, P., NEWSOM, C. R., YONNET, J., NIINIKOSKI, T., RIEUBLAND, M., MICHALOWICZ, A., DALLA TORRE-COLAUTTI, S., *Nucl. Phys. B* **262** (1985) 727.
- [32] BYSTRICKY, J., CHAUMETTE, P., DERÉGEL, J., FABRE, J., LEHAR, F., DE LESQUEN, A., PETIT, F., VAN ROSSUM, L., FONTAINE, J. M., PERROT, F., BALL, J., MICHALOWICZ, A., ONEL, Y., PENZO, A., *Nucl. Phys. B* **262** (1985) 715.
- [33] BALL, J., GHAZIKHANIAN, V., GORDON, J., LEHAR, F., DE LESQUEN, A., PERROT, F. and VAN ROSSUM, L., *Nucl. Phys. B* **286** (1987) 635.
- [34] PERROT, F., FONTAINE, J. M., BYSTRICKY, J., CHAUMETTE, P., DERÉGEL, J., FABRE, J., LEHAR, F., BALL, J., MICHALOWICZ, A., ONEL, Y., AAS, B., ADAMS, D., GHAZIKHANIAN, V., IGO, G., SPERISEN, F., WHITTEN, C. A., PENZO, A., 7th International Symposium on High Energy Spin Physics, Protvino (USSR), 22-27 September 1986.
- [35] BYSTRICKY, J., CHAUMETTE, P., DERÉGEL, J., FABRE, J., LEHAR, F., DE LESQUEN, A., VAN ROSSUM, L., FONTAINE, J. M., PERROT, F., BALL, J., LAC, C. D., MICHALOWICZ, A., ONEL, Y., PENZO, A., 7th International Symposium on High Energy Spin Physics, Protvino (USSR), 22-27 September 1986.
- [36] PROKOFIEV, A. N., Rapport LIYaF, Gatchina 1984.
- [37] BORISOV, N. S., VOVCHENKO, V. G., EFIMOVYCH, V. A., ZHDANOV, A. A., KAZARINOV, M. Yu., KAZARINOV, Yu. M., KISELEV, Yu. F., KOVALEV, A. I., LIBURG, M. Yu., MATAFONOV, V. I., POLYAKOV, V. V., POPOV, V. E., PROKOFIEV, A. N., TRAUTMAN, V. Yu., USOV, Yu. A., FEDOROV, O. Ya., and SCHVEDCHIKOV, A. V., *Pisma Zh. Eksp. Teor. Fiz.* **34** (1981) 137, transl. *JETP Lett.* **34** (1981) 130.
- [38] BORISOV, N. S., VOVCHENKO, V. G., EFIMOVYCH, V. A., ZHDANOV, A. A., KAZARINOV, M. Yu., KAZARINOV, Yu. M., KISELEV, Yu. F., KOVALEV, A. I., LIBURG, M. Yu., NEGANOV, B. S., POLYAKOV, V. V., POPOV, V. E., PROKOFIEV, A. N., TRAUTMAN, V. Yu., USOV, Yu. A., FEDOROV, O. Ya., and SCHVEDCHIKOV, A. V., *Zh. Eksp. Teor. Fiz.* **81** (1981) 1583, transl. *Sov. Phys. JETP* **54** (5) (1981) 841.
- [39] HOLLAS, C. L., CREMANS, D. J., RANSOME, R. D., RILEY, P. J., BONNER, B. E., MCNAUGHTON, M. W. and WOOD, S., *Phys. Lett.* **143B** (1984) 343.
- [40] BARLETT, M. L., HOFFMANN, G. W., MCGILL, J. A., FERGERSON, R. W., MILNER, E. C., MARSHALL, J. A., AMANN, J. P., BONNER, B. E. and MCCLELLAND, J. B., *Phys. Rev. C* **30** (1984) 279.
- [41] MCNAUGHTON, M. W., BONNER, B. E., CORNELIUS, W. D., HOFFMAN, E. W., VAN DYCK, O. B., YORK, R. L., RANSOME, R. D., HOLLAS, C. L., RILEY, P. J., TOSHIOKA, K., *Phys. Rev. C* **25** (1982) 1967.
- [42] BORISOV, N. S., VOVCHENKO, V. G., EFIMOVYCH, V. A., ZHDANOV, A. A., KAZARINOV, Yu. M., KISELEV, Yu. F., KOVALEV, A. I., KOZLENKO, N. G., MIKUSHKIN, M. Yu., POLYAKOV, V., POPOV, V. E., TRAUTMAN, V. V., FEDOROV, O. Ya., CHERNIKOV, A. N., SCHVEDCHIKOV, A. V., *JETP Lett. (USSR)* **43** (1986) 559, transl. *JETP Lett.* **43** (1986).
- [43] BHATIA, T. S., GLASS, G., HIEBERT, J. C., KENEFICK, R. A., NORTHCLIFFE, L. C., TIPPENS, W. B., BOISSEVAIN, J. G. J., JARMER, J. J., SIMMONS, J. E., TRIPARD, G. E., FITZGERALD, D., HOLT, J. and MOKHTARI, A., *Phys. Rev. Lett.* **49** (1982) 1135.
- [44] PAULETTA, G., Private Communication Feb. 1985 and Gazzaly M. M., Private Communication, Nov. 1985.
- [45] BYSTRICKY, J., CHAUMETTE, P., DERÉGEL, J., FABRE, J., LEHAR, F., DE LESQUEN, A., VAN ROSSUM, L., FONTAINE, J. M., PERROT, F., BALL, J., MICHALOWICZ, A., ONEL, Y., AAS, B., ADAMS, D., GHAZIKHANIAN, V., IGO, G., SPERISEN, F., WHITTEN, C. A., PENZO, A., 7th International Symposium on High Energy Spin Physics, Protvino (USSR), 22-27 September 1986.
- [46] AUER, I. P., COLTON, E., DITZLER, W. R., HILL, D., MILLER, R., SPINKA, H., THEODOSIOU, G., TAVERNIER, J.-J., TAMURA, N., TOSHIOKA, K., UNDERWOOD, D., WAGNER, R., YOKOSAWA, A., KROLL, P. and JAUCH, W., *Phys. Rev. Lett.* **51** (1983) 1411.
- [47] BYSTRICKY, J., CHAUMETTE, P., DERÉGEL, J., FABRE, J., LEHAR, F., DE LESQUEN, A., VAN ROSSUM, L., FONTAINE, J. M., PERROT, F., BALL, J., HASEGAWA, T., NEWSOM, C. R., PENZO, A., AZAIEZ, H., MICHALOWICZ, A., *Nucl. Phys. B* **258** (1985) 483.
- [48] BYSTRICKY, J., CHAUMETTE, P., DERÉGEL, J., FABRE, J., LEHAR, F., DE LESQUEN, A., VAN ROSSUM, L., FONTAINE, J. M., PERROT, F., BALL, J., LAC, C. D., MICHALOWICZ, A., GHAZIKHANIAN, V., 7th International Symposium on High Energy Spin Physics, Protvino (USSR), 22-27 September 1986.
- [49] LEHAR, F., SIEMIARCZUK, T., WINTERNITZ, P. and BYSTRICKY, J., Preprint, DPhPE 79-28, Saclay 1979.
- [49b] BYSTRICKY, J., LA FRANCE, P., LEHAR, F., PERROT, F., SIEMIARCZUK, T., WINTERNITZ, P., Preprint DPhPE 87-03, Saclay 1987, to be published in *J. Physique* n° 10 (Paris).
- [50] VERWEST, B. J. and ARNDT, R. A., *Phys. Rev. C* **25** (1982) 1979.
- [51] ARNDT, R. A. and MACGREGOR, M. H., *Meth. Comp. Phys.* **6** (1966) 263.
- [52] PUZIKOV, L. D., RYNDIN, R. M. and SMORODINSKII, Ya. A., *Zh. Eksp. Teor. Fiz.* **32** (1957) 592, transl. *Sov. Phys. JETP* **5** (1957) 489.
- [53] KLEPIKOV, N. P., *Zh. Eksp. Teor. Fiz.* **47** (1964) 757, transl. *Sov. Phys. JETP* **20** (1965) 505.

- [54] BYSTRICKY, J., LEHAR, F. and WINTERNITZ, P., *Experientia Suppl.* **25** (1976) 462.
- [55] BYSTRICKY, J., LEHAR, F., PATERA, J. and WINTERNITZ, P., *Lecture Notes Phys.* **87** (1978) 509.
- [56] LA FRANCE, P. and WINTERNITZ, P., *Phys. Rev. D* **27** (1983) 112.
- [57] GOLDSTEIN, G. R., MORAVCSIK, M. J., *NIM* **227** (1984) 108.
- [58] APRILE, E., EISENEGGER, C., HAUSAMMANN, R., HEER, E., HESS, R., LECHANOINE-LELUC, C., LEO, W. R., MORENZONI, S., ONEL, Y. and RAPIN, D., *Phys. Rev. Lett.* **46** (1981) 1047.
- [59] LECHANOINE-LELUC, C., Invited Talk to 7th International Symposium on High Energy Spin Physics, Protvino (USSR), 22-27 September 1986.
- [60] BYSTRICKY, J., LEHAR, F. and WINTERNITZ, P., Preprint CRM-763, Université de Montréal, Montréal, 1978 and *J. Physique Lett.* **40** (1979) L 117.
- [61] KLEPIKOV, N. P., *Zh. Eksp. Teor. Fiz.* **41** (1961) 1187, transl. *Sov. Phys. JETP* **14** (1962) 846.
- [62] KLEPIKOV, N. P., *Zh. Eksp. Teor. Fiz.* **44** (1963) 376, transl. *Sov. Phys. JETP* **17** (1963) 269.
- [63] KLEPIKOV, N. P., SMORODINSKII, Ya. A., *Zh. Eksp. Teor. Fiz.* **43** (1962) 2173, transl. *Sov. Phys. JETP* **16** (1963) 1458.
- [64] LEHAR, F. and WINTERNITZ, P., *Fortschritte Phys.* **15** (1967) 495.
- [65] LOMON, E. L., Aussois Meeting, March 1980 and Private Communication (1984).
- [66] KROLL, P., Phenomenological Analyses of Nucleon-Nucleon Scattering, Physics Data Nr 22-1 (1981), edited by H. Behrens and G. Ebel (Fachinformationszentrum Karlsruhe).
- [67] STANLEY, J. P., STEWART, N. M., BUGG, D. V., EDGINGTON, J. A., STEVENSON, N. R., CLOUGH, A. S., AXEN, D. A., SHYPIT, R., COMYN, M., HEALEY, D. and LUDGATE, G. A., *Nucl. Phys. A* **403** (1983) 525.
- [68] APRILE-GIBONI, E., BYSTRICKY, J., DERÉGEL, J., DROMPT, Ph., EISENEGGER, C., FONTAINE, J. M., HEER, E., HESS, R., JACCARD, S., LEHAR, F., LEO, W. R., MANGO, S., MORENZONI, S., ONEL, Y., PERROT, F., RAPIN, D., VRZAL, J. and YONNET, J., *Nucl. Phys. A* **431** (1984) 637.
- [69] AUER, I. P., BERETVAS, A., COLTON, E., HALPERN, H., HILL, D., NIELD, K., SANDLER, B., SPINKA, H., THEODOSIOU, G., UNDERWOOD, D., WATANABE, Y. and YOKOSAWA, A., *Phys. Rev. Lett.* **41** (1978) 1436.
-