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Classification

Physics Abstracts

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Interference effects and magnetoresistance oscillations in normal-metal networks.

2. Periodicity of the probability distribution

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Résumé. — Nous étudions les propriétés de transmission de systèmes désordonnés quasi unidimensionnels, en présence d'un champ magnétique. Pour une boucle seule, le coefficient de transmission oscille, en fonction du flux magnétique traversant la boucle, avec la période h/e pour une réalisation du désordre, et la période $h/2e$ pour la distribution de probabilité. Dans le cas de boucles connectées en série (collier), nous montrons que la distribution de probabilité présente la période h/e à faible désordre et $h/2e$ en désordre fort. Le cas d'un problème à deux canaux de transmission couplés (échelle) est également abordé dans la limite de faible désordre, faisant apparaître la période h/e . Nos résultats montrent que les oscillations du type Altshuler-Aronov-Spivak avec la période $h/2e$ sont obtenues à condition que la longueur caractéristique associée au désordre (longueur de localisation) soit inférieure au périmètre des boucles. Nous soulignons l'influence des détails microscopiques du désordre et nous comparons ces résultats avec ceux obtenus dans la référence [1].

Abstract. — We study the transmission properties of quasi one dimensional disordered systems in the presence of a magnetic field. For a single loop, the transmission coefficient as a function of the magnetic flux across the loop oscillates with the period h/e for a given realization of disorder and the period $h/2e$ for the whole probability distribution. For the series of identical connected loops (necklace geometry), we show that the probability distribution has the period h/e at weak disorder, and $h/2e$ at strong disorder. The case of a two-channel system (ladder geometry) is also investigated in the weak disorder limit, and the period h/e is shown to appear. Our results show that the Altshuler-Aronov-Spivak oscillations with the period $h/2e$ are recovered if the characteristic length scale associated with disorder (localization length) is less than the perimeter of the loops. The influence of the microscopic features of disorder is outlined and compared with the results of the first paper in this series (Ref. [1]).

1. Introduction.

It is now well established, both theoretically [1, 2] and experimentally [3-7], that interference effects of the Aharonov-Bohm type can exist in disordered conductors. But the presence of disorder may change dramatically the features of this phenomenon as compared to the case of a pure metal. The basic effect is the oscillation of all physical observables (energy spectrum, thermodynamical properties, electric conductance) of a ring or a regular assembly of loops, with a period h/e in the applied magnetic flux per elementary loop [8]. In the case of a disordered system however, Altshuler, Aronov and Spivak have shown, using weak-localization theory [1], that the conductance of such a multiconnected system, averaged over the realizations of disorder, should oscil-

late with the period $h/2e$. This effect is directly related to the fact that weak-localization corrections to classical transport theory comes from the interference between the two possible ways of travelling around a closed loop [9]. Experiments performed on a hollow cylinder [3], or on regular arrays [4, 5] have shown this period $h/2e$ with great accuracy. But in the case of a single loop, an oscillation at h/e has been predicted [2] and seen [6], as well as a superposition of both h/e and $h/2e$ components [7].

Recently, Stone and Imry [2] gave a coherent description of these different experimental results. They use the concept of lack of self-averaging in small disordered systems, such as submicronic loops. A given sample can oscillate with the period h/e , while the period for the probability distribution is $h/2e$. A self-averaging effect is obtained by

increasing the number of loops [10], or the temperature [2, 11] and this leads to the recovery of the $h/2e$ oscillation. This explanation is also consistent with the fact that h/e oscillations survive at high magnetic field [6], while $h/2e$ oscillations do not [1].

However, some recent investigations [11-15] have shown that the oscillation period of the whole probability distribution can be either h/e or $h/2e$. This is not in contradiction with the weak-localization results where only the interference between time reversed closed paths survives. In a closed ring geometry the usual Aharonov-Bohm interference (period h/e) is of main importance [6, 11]. For instance, reference [11] discussed a situation where the averaging procedure does not lead to the period $h/2e$. In that case the average is a thermal one and not over disorder configurations : for small transmission, the thermal ensemble is not broad enough to wash out the h/e period. In this respect, our averages are taken over disorder configurations and should not be confused with thermal averages. Other situations where both h/e and $h/2e$ periods appear have been studied in reference [14]. Such a result has been derived for the averaged transmission properties as the density of impurities is modified. It is important to notice that the impurity potential in that case is assumed to be larger than the band width and therefore comparison with weak-localization is inappropriate. Similar results have been obtained in reference [15] through explicit calculations done on a very small system.

In this paper, we study the periodicity of the probability distribution of the conductance in different quasi one-dimensional geometries. The case of a single loop is investigated in section 2. Section 3 considers the case of a necklace of loops, both in weak and strong disorder limits. Because this geometry still involves a single effective channel, a detailed study can be done. Then we examine the case of two coupled channels (ladder geometry) in section 4, in the weak disorder limit. Two principal conclusions can be drawn from our study : the period $h/2e$ for the probability distribution is recovered when the localization length is smaller than the perimeter of a single loop (strong disorder condition) ; and microscopic features of disorder may have a great influence on this period. This result can be viewed as a one-dimensional counterpart of the transition from h/e to $h/2e$ discovered by Nguen, Shklovskii and Spivak in a two-dimensional disordered model [14].

2. Single loop.

This geometry has been investigated by many authors [11, 12, 16, 17]. Here, we wish to give a simple model for describing scattering events which can occur in the upper or lower branch of the ring, or at contact points between the ring and the outside

world. Let us consider the single ring geometry as shown in figure 1. The model is a usual disordered tight-binding Hamiltonian :

$$H = \sum_i \varepsilon_i a_i^\dagger a_i + \sum_{i,j} V_{ij} a_j^\dagger a_i$$

where ε_i is zero if site i belongs to the external arms. Along the ring, the ε_i are independent random variables, such that $\langle \varepsilon_i \rangle = 0$. The hopping matrix elements are : $V_{ij} = V$ if i and j are nearest neighbours and $V_{ij} = 0$ otherwise. In the following, we shall take V as the energy-unit. The dispersion relation for waves propagating along the ordered arms is simply :

$$E = 2 \cos k \quad (2.1)$$

where k denotes the wave vector and E the incident wave energy.

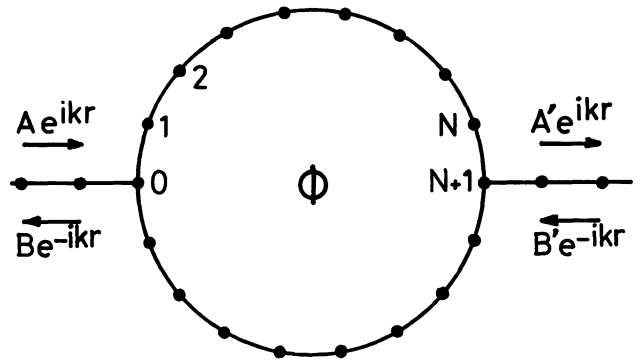


Fig. 1. — A one-dimensional disordered ring. ϕ denotes the magnetic flux through the ring.

The transmission matrix for the whole system is expressed in terms of the transmission coefficients t , t' and the reflection coefficient r as :

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = \begin{bmatrix} 1/t^* & -r^*/t^* \\ -r/t' & 1/t' \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}. \quad (2.2)$$

In the absence of magnetic fields, time reversal invariance holds and then $t = t'$. Let us introduce the transmission matrix m_1 and m_2 for the upper and the lower branch of the ring in the absence of magnetic field. For finite magnetic fields, and with a particular choice of the gauge, we have to replace m_1 by $e^{-i\gamma/2} m_1$ and m_2 by $e^{i\gamma/2} m_2$. Here $\gamma = 2\pi\phi/\phi_0$, where $\phi_0 = h/e$.

In this model, scattering at nodes is described by the following 3×3 S matrix (see Fig. 2 for the notations) :

$$\begin{bmatrix} B \\ A_1 \\ A_2 \end{bmatrix} = S \begin{bmatrix} A \\ B_1 \\ B_2 \end{bmatrix} \quad (2.3)$$

where

$$S = \begin{bmatrix} a & b & b \\ b & a & b \\ b & b & a \end{bmatrix} \quad (2.4)$$

and

$$a = -e^{ik}/(2e^{ik} - e^{-ik}); \quad |a|^2 = 1/(1 + 8\sin^2 k) \quad (2.5)$$

$$b = 2i \sin k/(2e^{ik} - e^{-ik}); \quad |b|^2 = 4\sin^2 k/(1 + 8\sin^2 k) \quad (2.6)$$

Using this notation, we obtain the transmission matrix m for the whole system :

$$m = \frac{1}{2iG \sin k} \begin{bmatrix} -[|G|^2 + (U - i \sin k)(V + i \sin k)] & -[|G|^2 + (U - i \sin k)(V - i \sin k)] \\ [|G|^2 + (U + i \sin k)(V + i \sin k)] & [|G|^2 + (U + i \sin k)(V - i \sin k)] \end{bmatrix} \quad (2.7)$$

U, V and G are related to t_1, r_1, t_2, r_2 and γ . For this purpose, it is interesting to write :

$$t_1 = T_1^{1/2} e^{i\varphi_1}; \quad r_1 = \left(\frac{1 - T_1}{T_1} \right)^{1/2} e^{i\psi_1} \quad (2.8)$$

(similar definitions hold for t_2 and r_2) and this leads to :

$$U = \cos k - \sin k \left[\frac{\cos(k(N+1) + \varphi_1) + (1 - T_1)^{1/2} \cos(k(N+1) - \psi_1)}{\sin(k(N+1) + \varphi_1) + (1 - T_1)^{1/2} \sin(k(N+1) - \psi_1)} + (1 \leftrightarrow 2) \right] \quad (2.9)$$

$$V = \left[\frac{\cos(k(N+1) + \varphi_1) - (1 - T_1)^{1/2} \cos(k(N+1) - \psi_1)}{\sin(k(N+1) + \varphi_1) + (1 - T_1)^{1/2} \sin(k(N+1) - \psi_1)} + (1 \leftrightarrow 2) \right] \sin k - \cos k \quad (2.10)$$

$$G = \sin k \left[T_1^{1/2} \frac{e^{i\gamma/2}}{\Delta_1} + T_2^{1/2} \frac{e^{-i\gamma/2}}{\Delta_2} \right] \quad (2.11)$$

where

$$\Delta_\alpha = \sin(k(N+1) + \varphi_\alpha) + (1 - T_\alpha)^{1/2} \sin(k(N+1) - \psi_\alpha), \quad \alpha = 1, 2. \quad (2.12)$$

As a consequence of equation (2.7) the transmission coefficient T of the system is given by :

$$T = \frac{4|G|^2 \sin^2 k}{|G|^4 + 2(UV + \sin^2 k)|G|^2 + (U^2 + \sin^2 k)(V^2 + \sin^2 k)} \quad (2.13)$$

The conductance g is given by Landauer's formula [18] :

$$g = \frac{e^2}{\pi \hbar} \frac{T}{1 - T}. \quad (2.14) \quad T = 4 \left(\frac{T_1}{\Delta_1^2} + \frac{T_2}{\Delta_2^2} + 2 \frac{(T_1 T_2)^{1/2}}{\Delta_1 \Delta_2} \cos \gamma \right) / D \quad (2.15a)$$

It is interesting to show explicitly the flux dependence of T :

$$D = \left(\frac{T_1}{\Delta_1^2} + \frac{T_2}{\Delta_2^2} + 2 \frac{(T_1 T_2)^{1/2}}{\Delta_1 \Delta_2} \cos \gamma \right)^2 + 2 \left(\frac{T_1}{\Delta_1^2} + \frac{T_2}{\Delta_2^2} + 2 \frac{(T_1 T_2)^{1/2}}{\Delta_1 \Delta_2} \cos \gamma \right) \times \\ \times \left[\left(\cotan k - \frac{\alpha_1}{\Delta_1} - \frac{\alpha_2}{\Delta_2} \right) \left(\frac{\beta_1}{\Delta_1} + \frac{\beta_2}{\Delta_2} - \cotan k \right) + 1 \right] + \left[\left(\frac{\alpha_1}{\Delta_1} + \frac{\alpha_2}{\Delta_2} - \cotan k \right)^2 + 1 \right] \\ \times \left[\left(\frac{\beta_1}{\Delta_1} + \frac{\beta_2}{\Delta_2} - \cotan k \right)^2 + 1 \right] \quad (2.15b)$$

and

$$\alpha_1 = \cos(k(N+1) + \varphi_1) + (1 - T_1)^{1/2} \cos(k(N+1) - \psi_1) \quad (2.15c)$$

$$\beta_1 = \cos(k(N+1) + \varphi_1) - (1 - T_1)^{1/2} \cos(k(N+1) - \psi_1) \quad (2.15d)$$

(α_2 and β_2 are defined similarly).

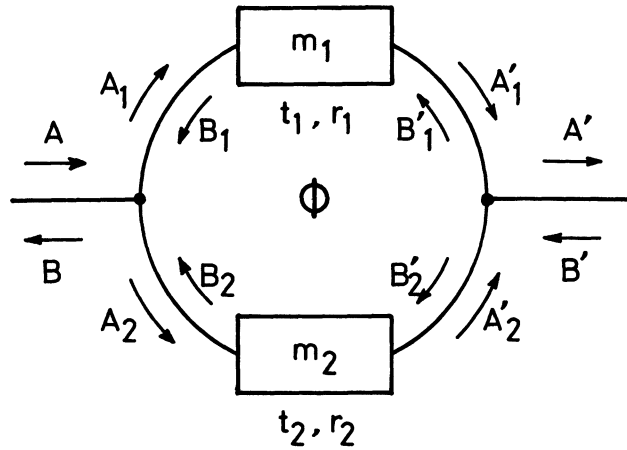


Fig. 2. — Propagation of waves in the disordered ring of figure 1.

This expression for T has the usual form [16]

$$T = \frac{a + b \cos \gamma}{c + d \cos \gamma + e \cos^2 \gamma}. \quad (2.16)$$

The term $e \cos^2 \gamma$ in the denominator comes from the interference between the two possible orientations of paths around the ring. Indeed the coefficient e is proportional to $T_1 T_2$. Its net effect is to induce a negative magnetoresistance at low fields and an oscillating component with the period $\phi_0/2$.

2.1 CASE OF A PURE SYSTEM. — In this case ($t_1 = t_2 = 1$, $r_1 = r_2 = 0$, $\varphi_1 = \varphi_2 = 0$), equation (2.15) becomes $T = N/D$ where :

$$N = 8(1 + \cos \gamma) \sin^2(k(N+1))$$

and

$$D = 4(1 + \cos \gamma)^2 + 4 \sin^2 k(N+1) \times [1 - (\cotan k - 2 \cotan^2 k(N+1))(1 + \cos \gamma) + \sin^4 k(N+1)[(\cotg^2 k(N+1)) + 1]^2].$$

Two possibilities arise, according to the value of

$$K = \sin^2 k(N+1) \times [(\cotan k - 2 \cotan^2 k(N+1)) + 1].$$

a) $K > 4$

$T(\gamma)$ exhibits the behaviour shown in figure 3a.

b) $K < 4$

Here there is a resonance for some values of the magnetic field, as can be seen in figure 3b.

The qualitative difference between these two situations can be understood from a direct analysis of the S matrix structure (Eqs. (2.4), (6)) which describes the scattering at nodes [16]. When $k \rightarrow 0$, one is always in case a, but as k approaches $\pi/2$, case b is recovered and a negative magnetoresistance in weak field appears. This behaviour is related to the fact that, at $k \rightarrow 0$, a wave coming

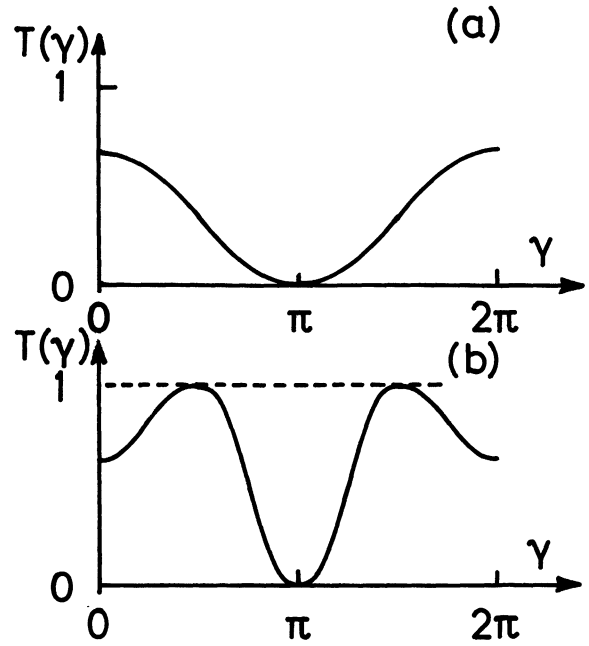


Fig. 3. — Transmission coefficient of a single ring without disorder for cases a and b (see text).

towards a node is reflected with a probability close to one. As a consequence, the wave cannot turn many times around the ring. In the opposite limit, $k \rightarrow \frac{\pi}{2}$, the probability of a perfect reflection is rather small ($1/9$) and then, the interference mechanism between closed paths takes place. However, it is important to notice that we always have $T = 0$ for $\gamma = \pi$ (i.e. $\phi = h/2e$).

2.2 CASE OF A DISORDERED LOOP. — Let us assume that the length of the ring is larger than the localization length ξ at energy E . As a result T_1 and T_2 are typically very small. But the important fact here is the distribution of phases $\varphi_1, \psi_1, \varphi_2, \psi_2$. The probability distribution of the phase of the reflection coefficient in the strongly localized regime has been studied carefully in reference [19] where it has been shown that this distribution is not uniform, and has maxima for the values $\pi + 2\pi m$ (m integer). With our notation :

$$r = (1 - T)^{1/2} e^{i(\varphi + \psi)} \\ r' = -(1 - T)^{1/2} e^{i(\varphi - \psi)}.$$

Taking the length L of the system into account, the result of reference [19] implies that the probability distribution of φ and ψ has strong correlations and exhibits maxima when :

$$\varphi + \psi = \pi + 2\pi m \\ \varphi - \psi = 2\pi m' - 2kL.$$

The probability distribution $W(\varphi, \psi)$ is a 2π periodic function of both variables $\varphi + \psi$ and $\varphi - \psi + 2kL$. This is true in the stationary regime

and this requires the hypothesis of a ring larger than the localization length ξ . From this, one deduces that the probability distribution W satisfies :

$$W(\varphi + \pi, \psi + \pi) = W(\varphi, \psi). \quad (2.17)$$

It is important to notice that this property remains true although the phase of the reflection coefficient is not uniformly distributed.

The general expression for T is invariant under the transformation :

$$\begin{aligned} \varphi_1 &\rightarrow \varphi_1 + \pi; \varphi_2 \rightarrow \varphi_2; \psi_1 \rightarrow \psi_1 + \pi; \\ \psi_2 &\rightarrow \psi_2; T_1 \rightarrow T_1; T_2 \rightarrow T_2; \gamma \rightarrow \gamma + \pi. \end{aligned}$$

As a consequence, the probability distribution of T is periodic, with the period $h/2e$. It is important to notice here that this result holds in general, without any assumption as done usually in the literature : for instance, a uniform distribution for $\varphi_1, \varphi_2, \psi_1,$

ψ_2 in reference [11], or $\psi = -\frac{\pi}{2}$ in reference [12].

It is not easy to calculate exactly the probability distribution of T , mainly because of the complexity of equation (2.15). Indeed, perfect resonances ($T = 1$) can always occur when :

$$\begin{cases} (1 - T_1)^{1/2} \cos(k(N+1) - \psi_1)/\Delta_1 + \\ + (1 - T_2)^{1/2} \cos(k(N+1) - \psi_2)/\Delta_2 = 0 \\ \frac{T_1}{\Delta_1^2} + \frac{T_2}{\Delta_2^2} + \frac{2(T_1 T_2)^{1/2}}{\Delta_1 \Delta_2} \cos \gamma = \\ = 1 + \left(\cotan k - \left(\frac{\alpha_1}{\Delta_1} + \frac{\alpha_2}{\Delta_2} \right) \right)^2. \end{cases} \quad (2.18)$$

However, for small T_1 and T_2 , and outside the resonance sectors :

$$T \simeq 4 \left(\frac{T_1}{\Delta_1^2} + \frac{T_2}{\Delta_2^2} + 2 \frac{(T_1 T_2)^{1/2}}{\Delta_1 \Delta_2} \cos \gamma \right) / \left[\left(\frac{\alpha_1}{\Delta_1} + \frac{\alpha_2}{\Delta_2} - \cotan k \right)^2 + 1 \right] \left[\left(\frac{\beta_1}{\Delta_1} + \frac{\beta_2}{\Delta_2} - \cotan k \right)^2 + 1 \right]. \quad (2.19)$$

This expression has the form :

$$T = \lambda^2 + \mu^2 + 2\lambda\mu \cos \gamma. \quad (2.20)$$

Here, λ, μ are two (non-independent) random variables with a probability distribution which is even in both variables.

In this limit,

$$\frac{\langle (T(\gamma=0) - T(\gamma=\pi))^2 \rangle}{\langle T(\gamma=0) + T(\gamma=\pi) \rangle^2} = \frac{4 \langle \lambda^2 \mu^2 \rangle}{\langle \lambda^2 + \mu^2 \rangle^2} = 1$$

if the correlations between λ and μ can be neglected. This shows that for a given sample, the magnetoresistance always oscillates with the period h/e . Fluctuations from sample to sample are very strong. Now, if m such loops are connected in series, and assuming that the phase of the transmission coefficient of a single loop is uniformly distributed, we have :

$$\begin{aligned} \langle \ln T_{(m)} \rangle &= m \langle \ln T_{(m=1)} \rangle \\ \langle (\ln T_{(m)})^2 \rangle &= m \langle (\ln T_{(m=1)})^2 \rangle. \end{aligned}$$

Then $\ln T_{(m)}$ is a self-averaging variable, and oscillates with the period $h/2e$ for a given sample. This result is a consequence of the phase spreading over $[0, 2\pi]$. A necessary condition for its occurrence is that the localization length being smaller than the loop perimeter.

3. Necklace of loops.

Let us consider the necklace geometry shown in figure 4 and describing a tight-binding model. The

disordered region has a finite length n and is connected on each side to a semi-infinite ordered necklace. A magnetic field is also present in the disordered region. Here γ denotes the reduced flux per elementary loop ($\gamma = 2\pi\phi/\phi_0$, $\phi_0 = h/e$). This system is interesting because the dynamical variables χ_m and ψ_m can be eliminated and a one channel problem is recovered.

3.1 ORDERED SYSTEM. — Using the notation of figure 4, the Schrödinger equation can be written as :

$$\begin{aligned} E\varphi_m - e^{-i\gamma/4} \chi_{m-1} - e^{i\gamma/4} \psi_{m-1} - \\ - e^{i\gamma/4} \chi_m - e^{-i\gamma/4} \psi_m &= 0 \\ E\chi_m - e^{-i\gamma/4} \varphi_m - e^{i\gamma/4} \varphi_{m+1} &= 0 \\ E\psi_m - e^{i\gamma/4} \varphi_m - e^{-i\gamma/4} \varphi_{m+1} &= 0. \end{aligned} \quad (3.1)$$

Here, E refers to the eigenvalue (energy), φ_m, χ_m and ψ_m being the wave function amplitudes. Eliminating χ_m and ψ_m leads to a one-dimensional

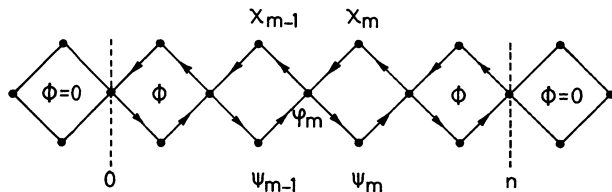


Fig. 4. — Necklace geometry. A magnetic field is present, corresponding to the magnetic flux ϕ across a single loop.

problem :

$$\left(E - \frac{4}{E}\right) \varphi_m - \frac{2}{E} \cos \frac{\gamma}{2} \varphi_{m-1} - \frac{2}{E} \cos \frac{\gamma}{2} \varphi_{m+1} = 0. \quad (3.2)$$

We then obtain the dispersion relation :

$$E^2 = 4 \left[1 + \cos \frac{\gamma}{2} \cos k \right],$$

where

$$-\pi \leq k \leq \pi \text{ denotes a wave vector.} \quad (3.3)$$

The band structure thus obtained is shown in figure 5. Note that only the value $E = 2$ belongs to the spectrum for any magnetic flux γ .

It is easy to calculate the transmission coefficient of an ordered system consisting of n loops.

Indeed, setting $E^2/4 - 1 = \cos k = \cos \frac{\gamma}{2} \cos k_1$, the transmission coefficient T is given by :

$$T = 4 \cos^2 \frac{\gamma}{2} \sin^2 k \sin^2 k_1 / D \quad (3.4)$$

with

$$D = 4 \cos^2 \frac{\gamma}{2} \sin^2 k \sin^2 k_1 + \left(1 + \cos^2 \frac{\gamma}{2} - 2 \cos(k + k_1)\right) \times \left(1 + \cos^2 \frac{\gamma}{2} - 2 \cos(k - k_1)\right) \sin k_1 n.$$

The γ dependence is not a simple one since k_1 is a function of γ . An interesting feature is that $T = 0$ when $\gamma = \pi$, for any value of n . A consequence of this simple property is the following one. When a dilute disorder is introduced (each site may be occupied by an impurity with probability x), the total probability that each loop has at least one impurity is given by $(2x - x^2)^n$. As long as x is less than one this probability goes to zero when n becomes large. Therefore, in the limit of an infinite system, there is a loop with no impurity, and this with a probability of one. Such a loop has a transmission coefficient which vanishes at $\gamma = \pi$ and then, the transmission coefficient of the whole system vanishes. In the case of a dilute disorder, the period of magnetoresistance oscillations is then h/e . This result holds for any value of the impurity potential. From this we deduce that $h/2e$ oscillations may exist only in the case $x = 1$ and in the strong disorder limit (impurity potential much greater than the bandwidth). The strong disorder hypothesis is necessary because otherwise, the period is controlled by the oscillation of the band structure, and is then equal to h/e . From these remarks we shall investigate the two limiting cases :

- weak and dilute disorder
- strong disorder at $x = 1$.

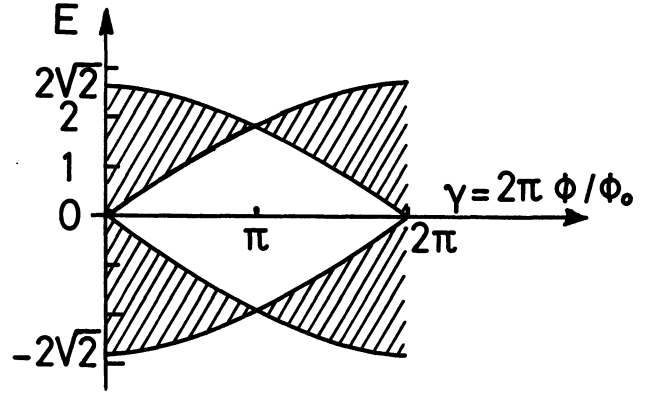


Fig. 5. — Band structure of a tight binding Hamiltonian on the necklace geometry, as a function of the magnetic field (dashed region).

3.2 WEAK AND DILUTE DISORDER. — This case corresponds to $x \ll 1$ and $\langle \varepsilon^2 \rangle \ll 1$ where ε denotes the impurity potential. In this section we shall calculate the localization length. Here the resistance due to the interfaces between the central and the two semi-infinite regions can be neglected. This allows us to use the eigenmodes of the necklace imbedded in a uniform magnetic field. One way to calculate the localization length is to write down a Fokker-Planck equation for the evolution of the probability distribution of the resistance, the length of the necklace being the « time » axis [20]. Let us first fix the energy E of incoming particles at :

$$E = 4 \left[1 + \cos \frac{\gamma}{2} \cos k \right],$$

where k denotes the wave vector of the incident wave.

In the weak disorder limit ($x \ll 1$), the probability for two adjacent loops to have impurities is of order x^2 and such events can be neglected in a calculation to first order in x . The transmission matrix associated to a disordered loop, as shown in figure 6, can be evaluated starting from the following equations (after elimination of the ψ and χ degrees of freedom) :

$$\begin{aligned} & \left(E - \varepsilon_0 - \frac{2}{E} - \frac{1}{E - \eta} - \frac{1}{E - \theta} \right) \varphi_0 - \frac{2}{E} \times \\ & \times \cos \frac{\gamma}{2} \varphi_{-1} - \left(\frac{e^{i\gamma/2}}{E - \eta} + \frac{e^{-i\gamma/2}}{E - \theta} \right) \varphi_1 = 0 \quad (3.5a) \\ & \left(E - \varepsilon_1 - \frac{2}{E} - \frac{1}{E - \eta} - \frac{1}{E - \theta} \right) \varphi_1 - \\ & - \left(\frac{e^{-i\gamma/2}}{E - \eta} + \frac{e^{i\gamma/2}}{E - \theta} \right) \varphi_0 - \varphi_2 = 0. \end{aligned}$$

Here, ε_1 , ε_2 , η and θ denote impurity potentials.

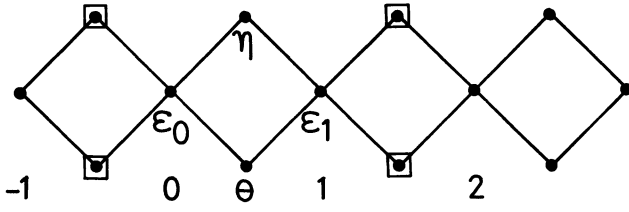


Fig. 6. — Impurity potentials ε , η , θ on a disordered loop. Sites surrounded by squares are assumed to have no impurity.

The transmission matrix m of the single loop is defined as usual by [20] (3.5b) :

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = m \begin{bmatrix} A \\ B \end{bmatrix} \quad (3.5b)$$

where

$$\begin{aligned} \varphi_{-1} &= A e^{-ik} + B e^{ik} \\ \varphi_0 &= A + B \end{aligned}$$

denote the scattering data on the left, and

$$\begin{aligned} \varphi_1 &= A' e^{ik} + B' e^{-ik} \\ \varphi_2 &= A' e^{i2k} + B' e^{-i2k} \end{aligned}$$

on the right.

Note that in the absence of disorder,

$$m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Starting from equation (3.5) and up to second order in the impurity potentials ε_0 , ε_1 , η , θ , one obtains :

$$m = \frac{1}{2i \sin k(1 + \beta)} \times \begin{bmatrix} -|1 + \beta|^2 e^{-ik} + (e^{ik} - \alpha_0)(1 - \alpha_1 e^{-ik}) & -|1 + \beta|^2 e^{-ik} + (e^{-ik} - \alpha_0)(1 - \alpha_1 e^{-ik}) \\ |1 + \beta|^2 e^{ik} - (e^{ik} - \alpha_0)(1 - \alpha_1 e^{ik}) & |1 + \beta|^2 e^{ik} - (e^{-ik} - \alpha_0)(1 - \alpha_1 e^{ik}) \end{bmatrix} \quad (3.6)$$

with the following notations :

$$\begin{aligned} \alpha_0 &= \frac{1}{2 \cos \left(\frac{\gamma}{2} \right)} (E \varepsilon_0 + (\eta + \theta)/E + (\eta^2 + \theta^2)/E^2) \\ \beta &= \frac{1}{2 \cos \left(\frac{\gamma}{2} \right)} \left(\left(\frac{\eta}{E} + \frac{\eta^2}{E^2} \right) e^{i\gamma/2} + \left(\frac{\theta}{E} + \frac{\theta^2}{E^2} \right) e^{-i\gamma/2} \right) \end{aligned}$$

α_1 is defined as α_0 , where ε_0 is replaced by ε_1 .

If M denotes the transmission matrix of the whole necklace, it is interesting to follow the evolution of the Hermitian matrix $J = MM^+$, which is the relevant matrix of interest here [20].

$$J = \begin{bmatrix} (1 + R)/T & 2r'/T \\ 2r'^*/T & (1 + R)/T \end{bmatrix} \equiv \begin{bmatrix} F & G e^{i\varphi} \\ G e^{-i\varphi} & F \end{bmatrix} \quad \text{with} \quad F^2 - G^2 = 1.$$

Here, r' is the reflection amplitude (for the right side) of the necklace, $R = |r'|^2$ and $T = 1 - R$.

The occurrence of a disordered loop in the system, at position n , results in the following modification of F :

$$\begin{aligned} F_{n+2} &= (|m_{11}|^2 + |m_{12}|^2) F_n + \\ &+ (m_{11} m_{12}^* e^{i(\varphi_n + 2kn)} \\ &+ m_{11}^* m_{12} e^{-i(\varphi_n + 2kn)}) (F_n^2 - 1)^{1/2}. \end{aligned}$$

Here the matrix coefficients m_{ij} are defined by equation (3.6).

The problem can be further simplified by averaging over the strongly oscillating terms $e^{i(\varphi_n + 2kn)}$, and one can obtain a Fokker-Planck equation for the evolution of the probability distribution of F [20] : $W(L, F)$, where L denotes the length of the necklace. This equation can be written as follows :

$$\frac{\partial W}{\partial x} = \frac{\partial}{\partial F} \left[(F^2 - 1) \frac{\partial W}{\partial F} \right] \quad (3.7)$$

where

$$x = L/\xi$$

and

$$\xi = \frac{8 \sin^2 k \cos^2 \left(\frac{\gamma}{2} \right)}{x \langle \varepsilon^2 \rangle \left(E^2 + \frac{4}{E^2} \left(\cos \frac{\gamma}{2} + \cos k \right)^2 \right)}. \quad (3.8)$$

The length scale ξ as given by equation (3.8) is the localization length, since we have :

$$\langle \ln F \rangle = L/\xi$$

and

$$\langle (\ln F)^2 \rangle - \langle \ln F \rangle^2 = 2L/\xi.$$

From the dispersion relation (Eq.(3.3)) it is easy to check that ξ is a decreasing function of γ for $\gamma \ll \pi$ as long as E belongs to the energy spectrum of the necklace in the presence of a magnetic field. For example, $E = 2$, belongs to the energy spectrum for any value of γ and :

$$\xi(E=2) = \frac{8 \cos^2 \frac{\gamma}{2}}{x \langle \varepsilon^2 \rangle \left(4 + \cos^2 \frac{\gamma}{2} \right)}. \quad (3.9)$$

In equations ((3.8) and (3.9)) $\langle \varepsilon^2 \rangle$ is the mean square of impurity potentials, which are assumed to be independent random variables.

In the limit considered here, the probability distribution of the resistance of the necklace oscillates with the period h/e . To see this, one can use Landauer's formula [18]: $g = \frac{e^2 T}{\pi \hbar R}$ relating the transmission coefficient T to the conductance g of the system.

Furthermore, this system exhibits a positive magnetoresistance $\left(\frac{d\xi}{d\gamma} < 0 \right)$ at low magnetic fields and this can be understood from the narrowing of the energy spectrum as the field is increased. In another words, the group velocity decreases when the magnetic field increases and then the scattering by impurities becomes larger and larger. In this weak disorder limit, the band structure governs the periodicity of the magnetoresistance. Therefore it is natural to consider the strong disorder limit when the amplitude of the impurity potentials is much greater than the bandwidth. According to the last paragraph, a $h/2e$ oscillation might only appear in the limit $x = 1$ and $\langle \varepsilon^2 \rangle \gg 1$. The following paragraph will be devoted to this limit. Before, let us remark that the case $\langle \varepsilon^2 \rangle \ll 1$ and $x \simeq 1$ is much more difficult that the two limiting cases investigated in this paper. This is because the problem becomes equivalent to a pure 1D disordered Anderson model with an off diagonal disorder, with a strong correlation between hopping matrix elements. Such correlations can be neglected when $x \ll 1$ or $\langle \varepsilon^2 \rangle \gg 1$ as will be shown below. However, in the opposite case ($\langle \varepsilon^2 \rangle \ll 1$ and $x \simeq 1$) we expect that the periodicity of the magnetoresistance will also be governed by the band structure.

3.3 STRONG DISORDER AT $x = 1$. — In this case it is not easy to write a simple Fokker-Planck equation for the probability distribution of the transmission coefficient. Instead, we shall use a transfer matrix method, as introduced in reference [21].

Let us start from the Schrödinger equation

$$\begin{aligned} & \left(E - \varepsilon_n - \frac{1}{E - \eta_n} - \frac{1}{E - \theta_n} - \frac{1}{E - \eta_{n-1}} - \right. \\ & \quad \left. - \frac{1}{E - \theta_{n-1}} \right) \varphi_n \\ & - \left(\frac{e^{-i\gamma/2}}{E - \eta_{n-1}} + \frac{e^{i\gamma/2}}{E - \theta_{n-1}} \right) \varphi_{n-1} \\ & - \left(\frac{e^{i\gamma/2}}{E - \eta_n} + \frac{e^{-i\gamma/2}}{E - \theta_n} \right) \varphi_{n+1} = 0. \quad (3.10) \end{aligned}$$

Dividing this equation by the factor in front of φ_n leads to a tight binding model with correlated off-diagonal disorder, as previously noted. This model can be simplified by setting $\varepsilon_n = 0$ for any n and $\eta_n = \pm W$, $\theta_n = \pm W$ with probability 1/2. Furthermore, let us assume that $W \gg E$. In this limit, the diagonal matrix element of the Hamiltonian (Eq. (3.10)) can be replaced by a constant ($= E$), and equation (3.10) gives

$$\varphi_n - a_{n-1}^* \varphi_{n-1} - a_n \varphi_{n+1} = 0 \quad (3.11)$$

with

$$a_n = \frac{1}{E} \left(\frac{e^{i\gamma/2}}{E - \eta_n} + \frac{e^{-i\gamma/2}}{E - \theta_n} \right).$$

Equation (3.11) can also be written in the following appropriate form :

$$\begin{aligned} \begin{bmatrix} \varphi_{n+1} \\ \varphi_n \end{bmatrix} &= \begin{bmatrix} 1/a_n & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \times \\ &\quad \times \begin{bmatrix} 1 & 0 \\ 0 & a_{n-1}^* \end{bmatrix} \begin{bmatrix} \varphi_n \\ \varphi_{n-1} \end{bmatrix}. \end{aligned}$$

Now assume that the disordered region is restricted to the loops 0, 1, ..., $n-1$. We have :

$$\begin{aligned} \begin{bmatrix} \varphi_{n+2} \\ \varphi_{n+1} \end{bmatrix} &= \begin{bmatrix} 2 \cos k & 0 \\ 0 & 1 \end{bmatrix} JH(n) J \dots JH(-1) J \times \\ &\quad \times \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2 \cos k} \end{bmatrix} \begin{bmatrix} \varphi_{-1} \\ \varphi_{-2} \end{bmatrix} \quad (3.12) \end{aligned}$$

where J denotes the matrix

$$\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

and

$$H(m) = \begin{bmatrix} 1/a_m & 0 \\ 0 & a_m^* \end{bmatrix}; \quad -1 \leq m \leq n.$$

Multiplying φ_{n+2} and φ_{n+1} by $\sum_{m=-1}^n \frac{|a_m|}{a_m^*}$ allows us to transform the matrices $H(m)$ into real matrices and this phase transformation keeps the transmission coefficient $T = |t|^2$ unchanged.

In the absence of disorder,

$$JH(m) = \begin{bmatrix} 2 \cos k & -1/2 \cos k \\ 2 \cos k & 0 \end{bmatrix}.$$

This matrix can be written as :

$$JH(m) = Q \begin{bmatrix} e^{ik} & 0 \\ 0 & e^{-ik} \end{bmatrix} Q^{-1},$$

where

$$Q = \begin{bmatrix} e^{ik} & e^{-ik} \\ 2 \cos k & 2 \cos k \end{bmatrix}.$$

Introducing the following unitary transformation :

$$\begin{bmatrix} \varphi_{m+1} \\ \varphi_m \end{bmatrix} = \begin{bmatrix} e^{ik} & e^{-ik} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} A_m \\ B_m \end{bmatrix}.$$

Equation (3.12) becomes :

$$\begin{bmatrix} A_{n+1} \\ B_{n+1} \end{bmatrix} = \left(\prod_{m=-1}^n M_m \right) \begin{bmatrix} e^{ik} & 0 \\ 0 & e^{-ik} \end{bmatrix} \begin{bmatrix} A_{-2} \\ B_{-2} \end{bmatrix} \quad (3.13)$$

where

$$M_m = Q^{-1} J \begin{bmatrix} 1/|a_m| & 0 \\ 0 & |a_m| \end{bmatrix} Q.$$

Explicitly

$$M_m = \frac{1}{2i \sin 2k} \begin{bmatrix} \frac{e^{i2k}}{|a_m|} - 4 \cos^2 k |a_m| & \frac{1}{|a_m|} - 4 \cos^2 k |a_m| \\ -\frac{1}{|a_m|} + 4 \cos^2 k |a_m| & \frac{-e^{-i2k}}{|a_m|} + 4 \cos^2 k |a_m| \end{bmatrix} \quad (3.14)$$

M_m satisfies unitarity conditions as expected for a transmission matrix from the current conservation.

In the present considered limit : $W \rightarrow \infty$, $|a| \rightarrow 0$ and $\frac{1}{|a|}$ is the leading term.

$$\frac{1}{|a|} = \frac{EW}{2 \left| \cos \frac{\gamma}{2} \right|} \quad \text{or} \quad \frac{EW}{2 \left| \sin \frac{\gamma}{2} \right|}$$

with probability 1/2.

The localization length ξ in such a strong disorder limit has been evaluated in reference [21] :

$$\xi^{-1} = \left\langle \ln \frac{1}{|a|} \right\rangle = \ln \left(\frac{EW}{(2|\sin \gamma|)^{1/2}} \right). \quad (3.15)$$

Equation (3.15) shows that the localization length ξ oscillates with the period $h/2e$ in the magnetic flux. Furthermore ξ increases at low fields and this corresponds to a negative magnetoresistance, which is also the case in the standard weak-localization theory [1].

Let us conclude this section by recalling the main results. The necklace geometry exhibits an h/e oscillation in the weak disorder limit $\langle \varepsilon^2 \rangle \ll 1$ or when $x \neq 1$. However an $h/2e$ periodicity appears in the strong disorder case $\langle \varepsilon^2 \rangle \gg 1$ at $x = 1$. This behaviour is consistent with the numerical results of reference [13] for the ladder geometry. In this work, the period of the localization length oscillations has been found to be h/e at small disorder, and $h/2e$ at stronger disorder. The cross-over is determined by comparing the localization length and the length of a single loop. In order to have an $h/2e$ oscillation, the localization length has to be smaller than the loop perimeter. This is necessary to

randomize the phases of the scattered waves in the upper and lower branches of a given loop. In our case, the necklace geometry exhibits a particular feature : in the infinite W limit, the $h/2e$ oscillation is recovered only at $x = 1$, because the lack of connectedness of the structure. This influence of the precise structure of disorder has been investigated in reference [14] in a two-dimensional model. The mechanism considered here is of similar nature. The probability distribution exhibits a crossover between h/e and $h/2e$ periods when the sign of the probability amplitude to travel along half a loop (in the absence of the field) is positive or negative with equal probabilities. For low concentrations of impurities, the period h/e dominates and this because of the non-symmetric nature of the distribution of the sign. Such a behaviour contrasts with the usual weak and dilute disorder limit ($\langle \varepsilon^2 \rangle \ll 1$ and $x \ll 1$) where only the combination : $x \langle \varepsilon^2 \rangle$ is involved. When $\langle \varepsilon^2 \rangle$ is much larger than the bandwidth, the concentration x becomes a pertinent parameter and then the magnetic field appears as an interesting probe.

4. Ladder geometry.

In the previous section, we have shown that the necklace geometry can be transformed into a pure one-dimensional disordered model. The ladder geometry is interesting because it is the simplest non-trivial multichannel system, where numerical data are available (Ref. [13]). According to the last remarks, one should consider both the weak ($x \langle \varepsilon^2 \rangle \ll 1$) and strong ($x \langle \varepsilon^2 \rangle \gg 1$) disorder limits. The multichannel localization problem has already been studied at zero magnetic field in reference [22].

Here we shall use a similar method and restrict ourselves to the weak disorder limit. The strong disorder limit can be analysed through an extension of the transfer matrix methods to multichannel disordered problems [23] but will not be discussed here.

4.1 ENERGY SPECTRUM OF THE LADDER STRUCTURE. — Let us consider the ladder geometry as shown in figure 7. It will be interesting to consider the interchain hopping matrix element as a relevant parameter t . The hopping matrix element along each chain will be taken equal to unity. As above, we use the notation $\gamma = 2\pi\phi/\phi_0$ where ϕ is the magnetic flux per elementary loop and $\phi_0 = h/e$. According to the notation of figure 7, the eigenmodes have the following forms :

$$\varphi_n = \phi e^{ikn}; \quad \psi_n = \psi e^{ikn}$$

and the eigenvalue equations are given by

$$\begin{aligned} E\varphi_n - e^{-i\gamma/2}\varphi_{n-1} - e^{i\gamma/2}\varphi_{n+1} - t\psi_n &= 0 \\ E\psi_n - e^{i\gamma/2}\psi_{n-1} - e^{-i\gamma/2}\psi_{n+1} - t\varphi_n &= 0. \end{aligned} \quad (4.1)$$

The corresponding dispersion relation is as follows

$$E = 2 \cos k \cos \frac{\gamma}{2} \pm \left(t^2 + 4 \sin^2 \frac{\gamma}{2} \sin^2 k \right)^{1/2}. \quad (4.2)$$

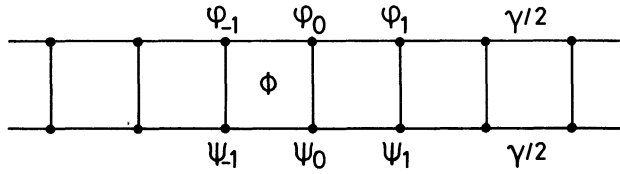


Fig. 7. — Ladder geometry.

At fixed E , the number of real solutions k , $-k$ of equation (4.2) gives the number of transmission channels. Of course this number can change with E . For the ladder geometry, the situation is summarized in figure 8, in the case $t = 1$. In the limit $t \rightarrow 0$, the region with only one propagating channel shrinks and disappears at $t = 0$ (uncoupled chains). In what follows, we shall study the weak coupling limit ($t \rightarrow 0$) and the isotropic chains ($t = 1$) limit respectively.

4.2 ONE-CHANNEL LIMIT AT $t = 1$. LOW FIELD EXPANSION. — This situation cannot occur at $\gamma = \frac{\pi}{2}$. However, around $\gamma = 0$, the energy region defined by

$$2 \cos \frac{\gamma}{2} - t < |E| < 2 \cos \frac{\gamma}{2} + t$$

exhibits just one propagating channel, the other is an evanescent one. In the following the localization

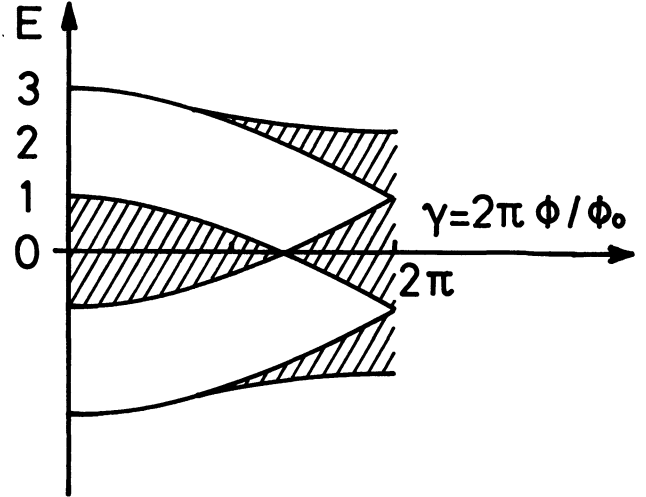


Fig. 8. — Energy spectrum of a tight binding Hamiltonian on the ladder geometry in the case of isotropic coupling $t = 1$. The system exhibits two propagating channels in dashed regions, and only one elsewhere in the spectrum.

length will be calculated in the weak disorder limit, up to the second order in the magnetic field. Note that at $\gamma = 0$, the dispersion relation reduces to $E - 2 \cos k = \pm 1$. Let us assume $1 < |E| < 3$, and take $E > 0$ for instance. We define k_1 and δ by :

$$k_1 = \text{Arc cos} \left(\frac{E-1}{2} \right)$$

and

$$\delta^{-1} = \text{Arg cosh} \left(\frac{E+1}{2} \right).$$

Here δ denotes the attenuation length of the evanescent mode ($k_2 = \pm i/\delta$) and k_1 is the wave vector of the propagating mode.

In the presence of a small magnetic field, the wave vectors and the wave functions are modified, according to :

$$\begin{cases} k_1 \rightarrow k_1 + \Delta k_1, \\ \Delta k_1 = \frac{\gamma^2}{4 \sin k_1} \left[\sin^2 k_1 - \frac{1}{2} \cos k_1 \right] \\ \lambda_1 = 1 \\ \mu_1 = 1 + \gamma \sin k_1 + \frac{\gamma^2}{2} \sin^2 k_1 \end{cases} \quad (4.3)$$

$$\begin{cases} k_2 = -i/\delta \rightarrow k_2 + \Delta k_2, \\ \Delta k_2 = \frac{i\gamma^2}{4 \sinh \delta^{-1}} \left(\sinh^2 \delta^{-1} - \frac{1}{2} \cosh \delta^{-1} \right) \\ \lambda_2 = 1 \\ \mu_2 = -1 - i\gamma \sinh \delta^{-1} + \frac{\gamma^2}{2} \sinh^2 \delta^{-1} \end{cases} \quad (4.4)$$

where λ_1 , λ_2 , μ_1 and μ_2 will be defined below.

Let us consider the weak and dilute disorder limit : $\langle \varepsilon^2 \rangle \ll 1$ and $x < 1$ (same notation as for the

necklace geometry). When a propagating wave is scattered by an impurity, an outgoing wave is emitted but superposed to a localized wave function with the characteristic length δ . However, if δ is much smaller than the average distance between two neighbouring impurities ($x\delta \ll 1$), one can neglect the overlap between two such localized wave functions. This approximation is equivalent to selecting the first term in a perturbation expansion with a perturbation parameter of order $\exp\left(-\frac{1}{x\delta}\right)$. In

perturbation calculations of the transmission coefficient, or of the Green function, this corresponds to keeping the oscillating part of the bare propagator.

It is then necessary to consider the diffusion by an impurity (with a potential ε or η as shown in Fig. 9).

On the left side ($n \leq 0$), we have

$$\begin{bmatrix} \varphi_n \\ \psi_n \end{bmatrix} = A \begin{bmatrix} \lambda_1 \\ \mu_1 \end{bmatrix} e^{i(k_1 + \Delta k_1)n} + B \begin{bmatrix} \mu_1 \\ \lambda_1 \end{bmatrix} e^{-i(k_1 + \Delta k_1)n} + C \begin{bmatrix} \lambda_2 \\ \mu_2 \end{bmatrix} e^{i(k_2 + \Delta k_2)n}$$

and on the right side ($n \geq 0$), we have

$$\begin{bmatrix} \varphi_n \\ \psi_n \end{bmatrix} = A' \begin{bmatrix} \lambda_1 \\ \mu_1 \end{bmatrix} e^{i(k_1 + \Delta k_1)n} + B' \begin{bmatrix} \mu_1 \\ \lambda_1 \end{bmatrix} e^{-i(k_1 + \Delta k_1)n} + C' \begin{bmatrix} \mu_2 \\ \lambda_2 \end{bmatrix} e^{-i(k_2 + \Delta k_2)n}$$

(the signs in front of k_2 have been chosen in order to have a non-divergent wave function). The transmission matrix by the impurity, up to first order in ε and η and to second order in γ is then given by :

$$\begin{aligned} A' &= A + \frac{i(\varepsilon + \eta)}{4 \sin k_1} (A + B) - i \frac{\gamma}{4} (\varepsilon - \eta) A + \\ &+ \frac{i\gamma^2}{32 \sin k_1} (\varepsilon + \eta) [(1 + \cotan^2 k_1 + 2 \cos k_1) A \\ &+ (1 + \cotan^2 k_1 + 2 \cos k_1 - 4 \sin^2 k_1) B] \\ B' &= B - \frac{i(\varepsilon + \eta)}{4 \sin k_1} (A + B) - i \frac{\gamma}{4} (\varepsilon - \eta) \times \\ &\times B - \frac{i\gamma^2}{32 \sin k_1} (\varepsilon + \eta) \\ &\times [(1 + \cotan^2 k_1 + 2 \cos k_1 - 4 \sin^2 k_1) A \\ &+ (1 + \cotan^2 k_1 + 2 \cos k_1) B]. \end{aligned} \quad (4.5)$$

This relation can be cast into the following form :

$$\begin{bmatrix} A' - A \\ B' - B \end{bmatrix} = \begin{bmatrix} 1 + i\alpha & i\beta \\ -i\beta & 1 - i\delta \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

where α , β and δ are real numbers given by equation (4.5).

The breaking of time reversal invariance due to the presence of the magnetic field appears through the difference between α and δ . This difference comes from the term $\gamma(\varepsilon - \eta)$ which is of first order

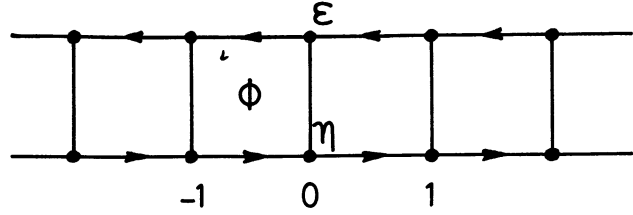


Fig. 9. — Impurity potentials ε or η located at the origin, in the ladder geometry.

in γ as expected, and vanishes when no dissymmetry between the upper and lower chains is present ($\varepsilon = \eta$). For an impurity at position n , the transmission matrix becomes :

$$\begin{bmatrix} 1 + i\alpha & i\beta e^{-i2k_1 n} \\ -i\beta e^{i2k_1 n} & 1 - i\delta \end{bmatrix}$$

Using again the method of reference [20], described in section 3, we can write a Fokker-Planck equation for the evolution of the probability distribution $W(L, F)$ along the ladder. Here L is the length of the ladder and $F = (1 + R)/T$.

This equation can be written as :

$$\begin{aligned} \frac{\partial}{\partial L} W &= -(\langle \alpha^2 \rangle + \langle \beta^2 \rangle) \frac{\partial}{\partial F} (FW) + \\ &+ \langle \beta^2 \rangle \frac{\partial^2}{\partial F^2} [(F^2 - 1)W]. \end{aligned} \quad (4.6)$$

In the limit of large L and $F \gg 1$ (large resistance), one obtains :

$$\begin{aligned} \langle \ln F \rangle &= \langle \alpha^2 \rangle L \\ \langle \ln^2 F \rangle - \langle \ln F \rangle^2 &= 2 \langle \beta^2 \rangle L \end{aligned} \quad (4.7)$$

with

$$\begin{aligned} \langle \alpha^2 \rangle &= \frac{x \langle \varepsilon^2 \rangle}{8 \sin^2 k_1} \left[1 + \frac{\gamma^2}{4} \times \right. \\ &\times (1 + \cotan^2 k_1 + 2 \cos k_1 + 4 \sin^2 k_1) \left. \right] \\ &= \frac{x \langle \varepsilon^2 \rangle}{8 \sin^2 k_1} \left[1 + \frac{\gamma^2}{4} \times \right. \\ &\times (1 + \cotan^2 k_1 + 2 \cos k_1 - 4 \sin^2 k_1) \left. \right]. \end{aligned} \quad (4.8)$$

The localization length ξ is equal to $\langle \alpha^2 \rangle^{-1}$. Therefore, (Eq. (4.8)), ξ decreases at low magnetic field. The magnetoresistance is then positive, and strongly enhanced near the band edge ($k \rightarrow 0$). This result is similar to our result for the necklace geometry, and comes from the narrowing of the energy spectrum in the presence of a magnetic field.

4.3 TWO CHANNELS LIMIT AT $\gamma = 0$ AND $\gamma = \pi$. — In this section, we restrict ourselves to the limits $\gamma = 0$ and $\gamma = \pi$, for both $t \ll 1$ and $t = 1$. The calculation for a general value of the flux γ is much

more cumbersome. Unfortunately $\gamma = 0$ and $\gamma = \pi$ both correspond to a system which satisfies the time reversal symmetry. So these limits are quite particular.

As before, we start by calculating the transmission matrix for an impurity at the origin (see Fig. 9). This is a 4×4 matrix in the case of two propagating channels. In all the cases considered below, it will take the form :

$$m(\varepsilon, \eta) = \mathbb{I} + i \begin{bmatrix} \alpha(\varepsilon, \eta) & \alpha(\varepsilon, \eta) \\ -\alpha(\varepsilon, \eta) & -\alpha(\varepsilon, \eta) \end{bmatrix}$$

where $\alpha(\varepsilon, \eta)$ denotes a 2×2 real, symmetric matrix.

For an impurity located at the position n , we have similarly :

$$m(\varepsilon, \eta, n) = \mathbb{I} + i \begin{bmatrix} e^{-iKn} \alpha e^{iKn} & e^{iKn} \alpha e^{iKn} \\ -e^{iKn} \alpha e^{iKn} & -e^{iKn} \alpha e^{-iKn} \end{bmatrix}$$

where

$$K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix},$$

k_1 and k_2 being the values of the two wave vectors.

Let us consider the explicit form of the matrix

$$\alpha(\varepsilon, \eta) \equiv \begin{bmatrix} a & b \\ b & c \end{bmatrix}.$$

i) $t = 1, \gamma = 0$:

k_1 and k_2 are defined by $E = 1 + 2 \cos k_1 = -1 + 2 \cos k_2$.

k_1 corresponds to a symmetric mode ($\phi = \psi$) and k_2 to an antisymmetric one ($\phi = -\psi$) (ϕ and ψ have been defined with the wave function by $\varphi_n = \phi e^{ikn}$ and $\psi_n = \psi e^{-ikn}$). We have chosen a normalization of the wave function in order to keep a simple form for the current $J \simeq |A_1|^2 - |B_1|^2 + |A_2|^2 - |B_2|^2$, and this leads to

$$a = \frac{\varepsilon + \eta}{4 \sin k_1}, \quad b = \frac{\varepsilon - \eta}{4 (\sin k_1 \sin k_2)^{1/2}}, \quad c = \frac{\varepsilon + \eta}{4 \sin k_1}.$$

ii) $t = 1, \gamma = \pi$:

Taking a gauge in which all the hopping matrix elements are real (Fig. 10), the dispersion relation reads $E^2 = 1 + 4 \cos^2 k$.

If k_1 is a solution, we take $k_2 = k_1 - \pi$ in order to have the same sign for the group velocity of both channels.

k_1 is associated to the wave function $\phi = 1$, $\psi = \lambda$, where $\lambda = (E + \sqrt{E^2 - 1})^{-1}$ and k_2 to the wave function $\phi = \lambda$, $\psi = 1$.

With these definitions, one obtains :

$$a = \frac{\varepsilon + \eta \lambda^2}{2 \sin k_1 (1 - \lambda^2)}, \quad b = \frac{(\varepsilon + \eta) \lambda}{2 \sin k_1 (1 - \lambda^2)},$$

$$c = \frac{\varepsilon \lambda^2 + \eta}{2 \sin k_1 (1 - \lambda^2)}.$$

iii) $t \ll 1, \gamma = 0$:

In comparison with i), the dispersion relations are modified :

$E = t + 2 \cos k_1$ (symmetric mode) and $E = -t + 2 \cos k_2$ (antisymmetric mode).

iv) $t \ll 1, \gamma = \pi$:

The dispersion relation is $E^2 = 4 \cos^2 k + t^2$, with the gauge of figure 10. The result is the same as in ii) with $\lambda = t/2E$. Keeping only the first order in t , one obtains

$$a = \frac{\varepsilon}{2 \sin k_1}, \quad b = \frac{\varepsilon + \eta}{2 \sin k_1} \frac{t}{2E}, \quad c = \frac{\eta}{2 \sin k_1}.$$

Let us denote by \mathcal{T} the transmission matrix of the whole system. \mathcal{T} is a 4×4 matrix. An appropriate parametrization of $M = \mathcal{T} \mathcal{T}^+$ is as follows [22]

$$M = \begin{bmatrix} u \cosh \Gamma u^+ & u \sinh \Gamma v^+ \\ v \sinh \Gamma u^+ & v \cosh \Gamma v^+ \end{bmatrix},$$

where u and v are 2×2 unitary matrices and Γ is a diagonal 2×2 matrix with positive real coefficients : Γ_1 and Γ_2 . Given an incoming wave on the left side, which is a linear superposition of the two channels with the amplitudes A_1 and A_2 , the transmitted wave on the right side is a quadratic form of A_1 and A_2 . The eigenvalues of this quadratic form are the two transmission coefficients T_1 and T_2 . Then :

$$T_i = \frac{2}{1 + \cosh \Gamma_i} \quad (i = 1, 2).$$

From time reversal invariance, we can choose the following form for u and v :

$$u = \begin{bmatrix} \cos \theta & \sin \theta e^{-i\varphi} \\ -\sin \theta e^{i\varphi} & \cos \theta \end{bmatrix}$$

and

$$v = \begin{bmatrix} \cos \theta & \sin \theta e^{i\varphi} \\ -\sin \theta e^{-i\varphi} & \cos \theta \end{bmatrix} \begin{bmatrix} e^{-iD_1} & 0 \\ 0 & e^{-iD_2} \end{bmatrix}.$$

Considering the evolution along the ladder leads to a 6-parameter stochastic process : $\Gamma_1, \Gamma_2, \theta, \varphi, D_1$ and D_2 . The effect of an impurity on them, up to the order of two in the impurity potential (ε, η) can be calculated from the transmission matrices given

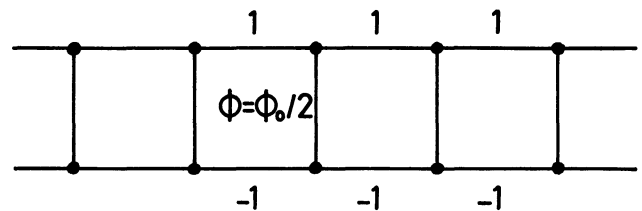


Fig. 10. — Particular gauge for the case $\gamma = \pi$. The hopping matrix elements along the chains are chosen to be real and of opposite signs.

above. The result of this calculation is then used to derive a Fokker Planck equation for the probability distribution $W(\Gamma_1, \Gamma_2, \theta, \varphi, D_1, D_2; L)$. After a

rather lengthy algebra, and making the change of variables : $F_i = \cosh \Gamma_i$ ($i = 1, 2$), the first terms of this Fokker-Planck equation take the form :

$$\begin{aligned} \frac{\partial}{\partial L} W = & (\langle a^2 \rangle \cos^4 \theta + \langle c^2 \rangle \sin^4 \theta + 4 \langle b^2 \rangle \sin^2 \theta \cos^2 \theta) \frac{\partial}{\partial F_1} \left[(F_1^2 - 1) \frac{\partial W}{\partial F_1} \right] \\ & - 2((\langle a^2 \rangle + \langle c^2 \rangle) \sin^2 \theta \cos^2 \theta + \langle b^2 \rangle \cos^2 2\theta) \frac{\partial}{\partial F_1} \left[\frac{F_1^2 - 1}{F_1 - F_2} W \right] \\ & + (\langle a^2 \rangle \sin^4 \theta + \langle c^2 \rangle \cos^4 \theta + 4 \langle b^2 \rangle \sin^2 \theta \cos^2 \theta) \frac{\partial}{\partial F_2} \left[(F_2^2 - 1) \frac{\partial W}{\partial F_2} \right] \\ & - 2((\langle a^2 \rangle + \langle c^2 \rangle) \sin^2 \theta \cos^2 \theta + \langle b^2 \rangle \cos^2 2\theta) \frac{\partial}{\partial F_2} \left[\frac{F_2^2 - 1}{F_2 - F_1} W \right] \\ & + \dots \end{aligned} \quad (4.9)$$

(here we have only written the terms necessary for our calculation).

In the limit $L \rightarrow \infty$, one expects an exponential increase of F_1 and F_2 , with two different characteristic lengths [22, 24]. With the hypothesis that $1 \ll F_1 \ll F_2$, equation (4.9) leads to :

$$\frac{d \langle \ln F_1 \rangle}{dL} = \langle a^2 \rangle \langle \cos^4 \theta \rangle + \langle c^2 \rangle \langle \sin^4 \theta \rangle + 4 \langle b^2 \rangle \langle \sin^2 \theta \cos^2 \theta \rangle \quad (4.10a)$$

and

$$\begin{aligned} \frac{d \langle \ln F_2 \rangle}{dL} = & \langle a^2 \rangle \langle \sin^4 \theta \rangle + \langle c^2 \rangle \langle \cos^4 \theta \rangle + 4 \langle b^2 \rangle \langle \sin^2 \theta \cos^2 \theta \rangle + \\ & + 2(\langle a^2 \rangle + \langle c^2 \rangle) \langle \sin^2 \theta \cos^2 \theta \rangle + 2 \langle b^2 \rangle \langle \cos^2 2\theta \rangle. \end{aligned} \quad (4.10b)$$

The knowledge of the probability distribution of θ is then required. However, as noticed in reference [22], one has a simple evolution law for $(F_1 F_2)^{1/2}$:

$$\begin{aligned} \frac{d}{dL} \left[\frac{\langle \ln F_1 \rangle + \langle \ln F_2 \rangle}{2} \right] = \\ = \frac{\langle a^2 \rangle + \langle c^2 \rangle}{2} + \langle b^2 \rangle. \end{aligned} \quad (4.11)$$

In the limit $L \rightarrow \infty$, F_1 and F_2 are very large and the evolution of θ becomes independent of F_1 and F_2 . It is then possible to write down a Fokker-Planck for the evolution of the probability distribution $W(\theta, L)$. Setting $h = \cos 2\theta$, this equation reads as :

$$\begin{aligned} \frac{\partial}{\partial L} W(h, L) = & \frac{\partial}{\partial h} [(\alpha h(1 - h^2) + \\ & + \beta h(1 + h^2) + \zeta(1 - h^2))W] \\ & + \frac{1}{2} \frac{\partial^2}{\partial h^2} [(\alpha(1 - h^2)^2 + \beta(1 - h^4))W] \end{aligned} \quad (4.12)$$

with

$$\begin{aligned} \alpha &= (\langle a^2 \rangle + \langle b^2 \rangle)/2 \\ \beta &= 2 \langle b^2 \rangle \\ \zeta &= (\langle a^2 \rangle - \langle c^2 \rangle)/2. \end{aligned}$$

When L is large, the probability distribution of h can be approximated by the stationary solution of equation (4.12). This distribution is given by

i) if $\beta > \alpha$

$$\begin{aligned} W(h) = & \frac{\mu}{\alpha(1 - h^2) + \beta(1 + h^2)} \times \\ & \times \exp \left\{ -\frac{2\zeta}{(\beta^2 - \alpha^2)^{1/2}} \text{Arctan} \left(h \left(\frac{\beta - \alpha}{\beta + \alpha} \right)^{1/2} \right) \right\} \end{aligned}$$

where

$$\begin{aligned} \mu = & \zeta / \sinh \left(\frac{2\zeta}{(\beta^2 - \alpha^2)^{1/2}} \times \right. \\ & \left. \times \text{Arctan} \left(\frac{\beta - \alpha}{\beta + \alpha} \right)^{1/2} \right). \end{aligned} \quad (4.13)$$

ii) if $\beta < \alpha$

$$\begin{aligned} W(h) = & \frac{\mu}{\alpha(1 - h^2) + \beta(1 + h^2)} \times \\ & \times \exp \left\{ -\frac{2\zeta}{(\alpha^2 - \beta^2)^{1/2}} \text{Argtanh} \left(h \left(\frac{\alpha - \beta}{\alpha + \beta} \right)^{1/2} \right) \right\} \end{aligned}$$

where

$$\begin{aligned} \mu = & \zeta / \sinh \left(\frac{2\zeta}{(\alpha^2 - \beta^2)^{1/2}} \times \right. \\ & \left. \times \text{Argtanh} \left(\frac{\alpha - \beta}{\alpha + \beta} \right)^{1/2} \right). \end{aligned} \quad (4.14)$$

The net effect of α , β and ζ on the shape of $W(h)$ can be understood from the meaning of θ : θ is the rotation angle which is necessary to diagonalize the quadratic form associated with the transmitted current. Let us first suppose $\zeta = 0$. Then $W(h) = \frac{\mu}{\alpha(1-h^2) + \beta(1+h^2)}$. If $\beta > \alpha$, the distribution has a maximum at $h = 0$, i.e. at $\theta = \pi/4 + (\pi/2)n$, n integer. This corresponds to a strong diffusion from one channel to another. If $\beta < \alpha$, the maximum of $W(h)$ occurs at $h^2 = 1$, i.e. for $\theta = \frac{\pi}{2}n$. Here the diffusion between the two channels is weaker. ζ introduces an asymmetry in the probability distribution. If $\zeta > 0$, $W(h)$ increases for $h < 0$ and decreases for $h > 0$. This implies that the eigenmode of the transmission quadratic form corresponding to T_1 (largest transmission coefficient) is closer to channel 2 than to channel 1. This is a natural result since $\zeta > 0$ means that scattering is weaker in channel 2 than in channel 1.

Using equations (4.13), (4.14) and (4.10a), one obtains the localization length l_1 associated with the largest transmission coefficient:

$$l_1^{-1} = \alpha + \beta - \zeta \coth \left\{ \frac{2\zeta}{(\beta^2 - \alpha^2)^{1/2}} \times \right. \\ \left. \times \operatorname{Arctan} \left(\frac{\beta - \alpha}{\beta + \alpha} \right)^{1/2} \right\} \quad \text{if } \beta > \alpha \quad (4.15a)$$

$$l_1^{-1} = \alpha + \beta - \zeta \coth \left\{ \frac{2\zeta}{(\alpha^2 - \beta^2)^{1/2}} \times \right. \\ \left. \times \operatorname{Argtanh} \left(\frac{\alpha - \beta}{\alpha + \beta} \right)^{1/2} \right\} \quad \text{if } \alpha > \beta. \quad (4.15b)$$

To summarize, we shall give now the values of l_1 and $(l_1^{-1} + l_2^{-1})/2$ obtained from equations (4.11) and (4.15), l_2 being the localization length associated with the smallest transmission coefficient T_2 and consequently $l_2 < l_1$.

i) $\phi = 0, t = 1$:

$$(l_1^{-1} + l_2^{-1})/2 = \frac{x \langle \varepsilon^2 \rangle}{4} [((1+E)(3-E))^{-1/2} + ((1-E)(3+E))^{-1/2}]^2 \\ l_1^{-1} = \frac{x \langle \varepsilon^2 \rangle}{2} \left[\frac{3-E^2-2E\mathcal{F}(E)}{(1-E^2)(9-E^2)} + 2((1-E^2)(9-E^2))^{-1/2} \right].$$

The function $\mathcal{F}(E)$ is defined by

$$\mathcal{F}(E) = \coth \left[\frac{4E}{(27-34E^2+3E^4)^{1/2}} \operatorname{Arctan} \left(\frac{2((1-E^2)(9-E^2))^{1/2} - (3-E^2)}{2((1-E^2)(9-E^2))^{1/2} + (3-E^2)} \right)^{1/2} \right] \quad \text{if } |E| < E_c$$

and

$$\mathcal{F}(E) = \coth \left[\frac{4E}{(34E^2-3E^4-27)^{1/2}} \operatorname{Argtanh} \left(\frac{(3-E^2) - 2((1-E^2)(9-E^2))^{1/2}}{(3-E^2) + 2((1-E^2)(9-E^2))^{1/2}} \right)^{1/2} \right] \quad \text{if } |E| > E_c.$$

Here

$$E_c = \left(\frac{17-4\sqrt{13}}{3} \right)^{1/2} < 1.$$

Starting from $E = 0$, when $|E|$ becomes greater than E_c , the shape of the probability distribution of θ changes, and the asymmetry between the two channels becomes very important as $|E| \rightarrow 0$, since one of the two allowed channels becomes evanescent.

ii) $\phi = \pi, t = 1$:

The energy spectrum is defined by $1 \leq E \leq \sqrt{5}$, and we have

$$(l_1^{-1} + l_2^{-1})/2 = \frac{x \langle \varepsilon^2 \rangle}{(5-E^2)} \left(\frac{1+\lambda^2}{1-\lambda^2} \right)^2 \\ l_1^{-1} = \frac{x \langle \varepsilon^2 \rangle}{(5-E^2)(1-\lambda^2)} [1 + \lambda^4 + 4\lambda^2 - \mathcal{G}(\lambda)]$$

where

$$\mathcal{G}(\lambda) = (16\lambda^4 - (1+\lambda^4)^2)^{1/2}/2 \times \\ \times \operatorname{Arctan} \left(\frac{4\lambda^2 - 1 - \lambda^4}{4\lambda^2 + 1 + \lambda^4} \right)^{1/2} \quad \text{if } 1 < |E| < E'_c$$

and

$$\mathcal{G}(\lambda) = ((1+\lambda^4)^2 - 16\lambda^4)^{1/2}/2 \times \\ \times \operatorname{Argtanh} \left(\frac{1+\lambda^4-4\lambda^2}{1+\lambda^4+4\lambda^2} \right)^{1/2} \quad \text{if } E'_c < |E| < \sqrt{5} \\ E'_c = \sqrt{3/2}.$$

Here, we have set $\lambda = (E + (E^2 - 1)^{1/2})^{-1}$.

The difference between the cases $\phi = 0$ and $\phi = \pi$ is more striking in the limit $t \ll 1$. In what follows, the difference between the group velocities of the two channels will be neglected, because it

introduces analytic corrections of order t^2 to the localization lengths.

iii) $\phi = 0, t \ll 1$:

$$(l_1^{-1} + l_2^{-1})/2 = \frac{x \langle \varepsilon^2 \rangle}{4 - E^2} + 0(t^2)$$

$$l_1^{-1} = \frac{3}{2} \left(1 - \frac{\sqrt{3}}{\pi} \right) \frac{x \langle \varepsilon^2 \rangle}{4 - E^2}.$$

iv) $\phi = \pi, t \ll 1$:

$$(l_1^{-1} + l_2^{-1})/2 = \frac{x \langle \varepsilon^2 \rangle}{4 - E^2} + 0(t^2)$$

$$l_1^{-1} = \frac{x \langle \varepsilon^2 \rangle}{(4 - E^2)} \left(1 - \frac{1}{\ln(2E^2/t^2)} \right).$$

Note that for $\phi = \pi$, l_1 is not an analytical function of t , as t goes to zero. This is to be contrasted with the smooth behaviour at $\phi = 0$.

4.4 DISCUSSION. — From the above results, it turns out that the period of the magnetoresistance oscillations is h/e in the weak-disorder limit, both in the weak ($t \ll 1$) and isotropic ($t = 1$) coupling cases. The oscillation period of the band structure is then relevant in this weak-disorder limit. The limitations of our approach can be summarized as follows :

— In two-channel case, our calculations are restricted to time reversal invariant situations ($\phi = 0$ and $\phi = \pi$). If this symmetry is broken, the calculation of the transmission matrix for one impurity becomes tedious. Furthermore, the simple relation between the matrices u and v^* is lost and an additional parameter θ' should be introduced.

— Our weak disorder result is not valid at band edges, or when a propagation channel disappears. For instance, in 1D systems, a different scaling of the localization length occurs [25].

— The strong disorder limit has not been investigated. But the result of the necklace can be thought to be valid here. It seems reasonable to expect a cross-over from h/e to $h/2e$ when the localization length becomes of the order of a loop size ($x \langle \varepsilon^2 \rangle \geq 1$), as was numerically observed in reference [13].

5. Conclusion.

Let us briefly summarize our main results.

For a single disordered ring, the magnetoresistance of a given sample always exhibits the period h/e , in agreement with previous studies [2, 16]. The whole probability distribution of the transmission coefficient oscillates with the period $h/2e$ if the length of the ring is larger than the localization length.

For the infinite systems considered here (necklace and ladder), we have calculated the localization length which is defined from $\ln T$. Since $\ln T$ is

known to be a self-averaging quantity, the localization length is well defined for a single very long system. We have shown that

i) for weak disorder, or more precisely, when the localization length is much larger than the loop size, the main effect of the magnetic field is due to the oscillation of the band structure of the pure system in a magnetic field, with the period h/e . In this case, h/e is also the period of the magnetoresistance oscillation ;

ii) for strong disorder (localization length much smaller than the loop size), partial waves coming through the upper and lower sides of a loop have an arbitrary phase difference. After ensemble averaging, only the interference between the two time-reversed paths associated with a closed loop survives, and the period $h/2e$ is recovered. These results are in good agreement with numerical studies on disordered ladders [13]. Furthermore, the microscopic nature of disorder may have an influence on the magnetoresistance oscillation, as discussed in the case of the necklace. Such an influence has already been noticed in references [14, 15].

In order to compare these results to experiments, it should be stressed that our calculations deal with pure 1D disordered wires. In real samples, wires have always a finite width, and this leads to a quasi-diffusive behaviour (weak localization regime) in the weak disorder limit [1]. An important feature of this regime is the existence of two well separated length scales : the elastic mean free path l and the localization length ξ , which satisfy $l \ll \xi$. In the pure 1D cases investigated here, $l \approx \xi$ up to a numerical factor. A direct calculation using the same methods as those used in the present paper, on systems with wires of finite widths is difficult. However, we argue that similar conclusions can be expected for such realistic geometries, if one compares the mean free path (rather than the localization length) with the loop perimeter. Clearly, weak localization calculations [1] are no longer valid in a weak enough disorder, where the mean free path becomes larger than the loop perimeter. As in the original Aharonov-Bohm experiment, the period h/e should be expected in such a case. If the mean free path is smaller than the loop perimeter, randomness occurs in the distribution of the phases of the transmission matrices associated with different wires. Then the period $h/2e$ should be obtained for the behaviour of the whole probability distribution. This is not inconsistent with the observation of an h/e oscillation for small samples [6] (single submicronic ring) or the superposition of both h/e and $h/2e$ Fourier components [7]. In fact, as discussed in detail by Büttiker *et al.* [21], there is no self-averaging in a single ring, even for a large number of propagation channels.

It should also be noted that the self-averaging

process which is responsible for the observation of $h/2e$ oscillations in the experiments reported in references [3, 4, 5, 10] is not the same as the mechanism involved for $\langle \ln T \rangle$ of a long system. In

the latter case, only elastic scattering is involved whereas in the other case the phase coherence breaking (due to inelastic scattering) is responsible for washing out the h/e component.

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