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Transport correlation functions in 1D disordered media

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Résumé. — On étudie les fonctions de corrélation $\langle T^{-1}(E) T^{-1}(E') \rangle$ et $\langle \ln T(E) \ln T(E') \rangle$ des coefficients de transmission d'un système désordonné à une dimension, pour des valeurs différentes E et E' de l'énergie. Les longueurs caractéristiques associées à la décroissance de ces fonctions sont calculées et sont différentes de l'échelle de longueur correspondante au corrélateur $\langle T(E) T(E') \rangle$. On interprète l'existence de différentes échelles associées aux différents corrélateurs comme la manifestation de distributions de probabilité anormalement larges dans ces systèmes désordonnés.

Abstract. — We study the correlation functions $\langle T^{-1}(E) T^{-1}(E') \rangle$ and $\langle \ln T(E) \ln T(E') \rangle$ of the transmission coefficients of a disordered 1D system taken for two slightly different values E and E' of the incident wave energy. Characteristic length scales associated with the decay of these correlation functions are obtained and are shown to differ from the length scale associated with the correlator $\langle T(E) T(E') \rangle$. The existence of several length scales associated with different correlators is interpreted as a manifestation of unusually broad probability distributions in such disordered systems.

1. Introduction.

One of the most exciting properties of disordered systems is their ability to exhibit coherent effects during the propagation of linear waves. Localization theory in disordered electronic systems is a striking illustration of these phenomena [1]. Recent experiments performed on submicronic conducting rings have shown magnetoresistance oscillations of the Aharonov-Bohm type, with a period corresponding to a normal flux quantum h/e [2] and a strong $h/2e$ harmonic [3]. A tentative explanation [4, 5] lies on the idea that the period of the probability distribution oscillations is $h/2e$, as predicted by weak localization theory [6] and observed in hollow cylinder experiment [7]. However, lack of self averaging occurs in 1D rings and a given sample shows mainly the h/e oscillation [4]. A cross-over from the sample behaviour to the averaged one can appear at finite temperature since the conductance in the Landauer picture involves the transmission coefficients at energy intervals of size $\sim k_B T$. Under certain conditions, this thermal superposition mechanism can have the same effect as an ensemble averaging [5]. In fact, decorrelation between transmission coefficients at two different energy values occurs when the energy separation is greater than a

correlation energy E_c . *A priori* E_c depends on the sample geometry and on the degree of disorder.

In the present paper, we calculate this quantity in 1D systems for the correlators of the logarithm and negative moments of the transmission coefficient. An estimate of E_c has been proposed in reference [4] for the two following cases :

— 1D ballistic transport : $E_c = \hbar V_G/L$, where V_G is the group velocity and L is the length of the system ;

— 2D diffusive transport : $E_c = \hbar D/L^2$, where D is the diffusion constant.

The 1D localized case is interesting because much more precise results can be derived. However, it should be noticed that in this case, there is no true diffusive weak localization regime. This is an important restriction on the validity of our results in comparison with actual experiments where the width of the wires is finite.

Correlators of positive moments : $\langle t(E) t^*(E') \rangle$ and $\langle |t(E)|^2 |t(E')|^2 \rangle$ have already been calculated by the authors of references [8] and [9], who emphasized their connection with the bandwidth [8] or the noise [9] in 1D disordered systems.

Another work [10], on the same subject was devoted to the correlators involving exact wave

functions. Is it possible to associate with these different correlators a single expression of E_c or a single characteristic decorrelation length L^* ? Our answer will be negative : L^* depends strongly on the observed quantity. This paper is organized as follows. In section 2, correlators of negative moments of the transmission coefficients are investigated. Section 3 is devoted to the correlator of logarithms. Then, in section 4, our results are compared to those obtained in previous works [8, 10].

2. Negative moments correlators.

Let us consider the following disordered tight-binding Hamiltonian, corresponding to a one dimensional geometry shown in figure 1 :

$$H = \sum_i \varepsilon_i a_i^\dagger a_i + \sum_i (a_i^\dagger a_{i-1} + \text{h.c.}) . \quad (2.1)$$

Here the ε_i 's are independent random variables, which satisfy $\langle \varepsilon_i \rangle = 0$. We suppose that disorder is limited to the region including the sites 0, 1, ..., n . Elsewhere, $\varepsilon_i = 0$, which corresponds to pure semi-infinite systems. In the absence of disorder, free modes can propagate : $\varphi_n = e^{ikn}$, with the following dispersion relation : $E = 2 \cos k$. Here E denotes the wave energy and k its wave vector.

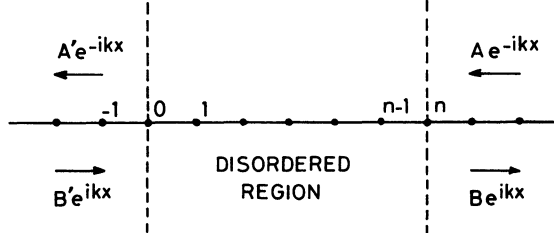


Fig. 1. — A one-dimensional disordered tight-binding model (see text).

The transmission matrix m of the disordered region can be defined by considering ingoing and outgoing waves on both sides. Our notation is explained in figure 1. We then have :

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = m \begin{bmatrix} A \\ B \end{bmatrix} .$$

Let us denote by t and r the transmission and the reflexion coefficients for incident waves coming from the right. Because the system is invariant under time-reversal, m takes the usual form :

$$m = \begin{bmatrix} 1/t^* & -r^*/t^* \\ -r/t & 1/t \end{bmatrix} .$$

As noticed in reference [11], it is useful to consider the Hermitian matrix $M = m^\dagger m$ in order to study the transmission problems. We have

$$M = \begin{bmatrix} \frac{1+R}{T} & -\frac{2r^*}{T} \\ -\frac{2r}{T} & \frac{1+R}{T} \end{bmatrix} ,$$

with

$$R = |r|^2, \quad T = |t|^2, \quad R + T = 1 .$$

The transmission matrix $m(j, \varepsilon)$ associated with a single impurity at site j where $\varepsilon_j = \varepsilon$, is given by

$$m(j, \varepsilon) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{\varepsilon}{2i \sin k} \begin{bmatrix} 1 & e^{i(2kj)} \\ -e^{-i(2kj)} & 1 \end{bmatrix} \quad (2.2)$$

The transmission matrix of the whole system is therefore given by :

$$m = m(0, \varepsilon_0) m(1, \varepsilon_1) \dots m(n, \varepsilon_n) .$$

When the length of the disordered region is increased by one unit length, this results in a modification of M as follows

$$M(n) \rightarrow m^\dagger(n, \varepsilon_n) M(n) m(n, \varepsilon_n) . \quad (2.3)$$

Let us introduce the following parametrization of $M(n)$:

$$M(n) = \begin{bmatrix} F_n & G_n e^{i\varphi_n} \\ G_n e^{-i\varphi_n} & F_n \end{bmatrix} ; \quad F_n^2 - G_n^2 = 1 . \quad (2.4)$$

Using (2.3) and (2.4), we obtain the evolution of the variables F_n and φ_n resulting from the addition of a single impurity. Up to second order in ε_n , this reads :

$$F_{n+1} = \left(1 + \frac{\varepsilon_n^2}{2 \sin^2 k} \right) F_n - \frac{\varepsilon_n}{\sin k} \sin \theta_n \cdot G_n - \frac{\varepsilon_n^2}{2 \sin^2 k} \cos \theta_n \cdot G_n \quad (2.5)$$

and

$$\varphi_{n+1} - \varphi_n = \frac{\varepsilon_n}{\sin k} \left[1 - \cos \theta_n \left(\frac{F_n}{G_n} \right) \right] + \frac{\varepsilon_n^2}{2 \sin^2 k} \sin \theta_n \left[\frac{F_n}{G_n} - \cos \theta_n \left(1 + \frac{2}{G_n^2} \right) \right] . \quad (2.6)$$

In (2.5) and (2.6), $\theta_n = \varphi_n - 2kn$. In the limit of weak disorder : $\langle \varepsilon^2 \rangle \ll 1$, after averaging over the strongly oscillating phase factor $e^{i\theta_n}$, it is possible to write a Fokker-Planck equation for the probability distribution $W(F, L)$ of the parameter F (L being the length of the disordered part) :

$$\frac{\partial W}{\partial x} = \frac{\partial}{\partial F} \left[(F^2 - 1) \frac{\partial W}{\partial F} \right] ;$$

$$x = L/\xi ; \quad \xi = \frac{4 \sin^2 k}{\langle \varepsilon^2 \rangle} .$$

Here, $x = L/\xi$ where $\xi = 4 \sin^2 k / \langle \varepsilon^2 \rangle$ is the localization length, as can be seen from :

$$\langle \ln F \rangle = x \text{ and } \langle (\ln F)^2 \rangle - \langle \ln F \rangle^2 = 2x.$$

Let us now consider two incident waves, with slightly different energies E and E' . Since the wave vectors k and k' are different, the phases θ_n and θ'_n which appear in (2.6) become uncorrelated at large L . This effect is in general larger than that resulting from the difference between group velocities : $2 \sin k \neq 2 \sin k'$. More quantitatively, the phase difference becomes significant at length scales comparable to the « optical » length $L_{\text{opt}} = \frac{1}{|\Delta k|}$.

However, the variation of the group velocity results in a variation of the localization length, which defines a characteristic length

$$\zeta = (\xi^{-1} - \xi'^{-1})^{-1} \approx \frac{\xi}{2 \cotg k |\Delta k|}.$$

In the weak-disorder limit studied here, $\xi \gg 1$ and $L_{\text{opt}} \ll \xi$.

$$\begin{aligned} \frac{\partial W}{\partial x} = & \frac{\partial}{\partial F} \left[(F^2 - 1) \frac{\partial W}{\partial F} \right] + \frac{\partial}{\partial F'} \left[(F'^2 - 1) \frac{\partial W}{\partial F'} \right] + \frac{\partial^2}{\partial u^2} \left[\left(\frac{F^2}{F^2 - 1} + \frac{F'^2}{F'^2 - 1} - \right. \right. \\ & \left. \left. - 2 \frac{FF'}{[(F^2 - 1)(F'^2 - 1)]^{1/2}} \cos(u - \beta x) \right) W \right] + 2 \frac{\partial^2}{\partial F \partial F'} \left[[(F^2 - 1)(F'^2 - 1)]^{1/2} \cos(u - \beta x) W \right] \\ & - 2 \frac{\partial^2}{\partial F \partial u} \left[\left(\frac{F^2 - 1}{F'^2 - 1} \right)^{1/2} F' \sin(u - \beta x) W \right] - 2 \frac{\partial^2}{\partial F' \partial u} \left[\left(\frac{F'^2 - 1}{F^2 - 1} \right)^{1/2} F \sin(u - \beta x) W \right] \end{aligned} \quad (2.8)$$

Here, $\beta = 2 \Delta k \cdot \xi$ and $F = (1 + R)/T = 2/T - 1$.

In the large L limit, the main contribution to the probability distribution of F, F' comes from the region $F \gg 1, F' \gg 1$. In this limit, (2.8) takes the simplified form :

$$\begin{aligned} \frac{\partial W}{\partial x} = & \frac{\partial}{\partial F} \left(F^2 \frac{\partial W}{\partial F} \right) + \frac{\partial}{\partial F'} \left(F'^2 \frac{\partial W}{\partial F'} \right) + 2 \frac{\partial^2}{\partial u^2} ((1 - \cos(u - \beta x)) W) + 2 \frac{\partial^2}{\partial F \partial F'} \times \\ & \times [FF' \cos(u - \beta x) W] - 2 \frac{\partial^2}{\partial F \partial u} (F \sin(u - \beta x) W) - 2 \frac{\partial^2}{\partial F' \partial u} (F' \sin(u - \beta x) W). \end{aligned} \quad (2.9)$$

Equation (2.9) allows us to write an evolution for the correlator $\langle FF' \rangle$ which turns out to be coupled to $\langle FF' \cos(u - \beta x) \rangle$ and $\langle FF' \sin(u - \beta x) \rangle$ in the following system :

$$\frac{d}{dx} \langle FF' \rangle = 4 \langle FF' \rangle + 2 \langle FF' \cos(u - \beta x) \rangle \quad (2.10a)$$

$$\begin{aligned} \frac{d}{dx} \langle FF' \cos(u - \beta x) \rangle = & \\ = & 4 \langle FF' \rangle + 2 \langle FF' \cos(u - \beta x) \rangle \\ & + \beta \langle FF' \sin(u - \beta x) \rangle \end{aligned} \quad (2.10b)$$

$$\begin{aligned} \frac{d}{dx} \langle FF' \sin(u - \beta x) \rangle = & \\ = & -\beta \langle FF' \cos(u - \beta x) \rangle \\ & + 2 \langle FF' \sin(u - \beta x) \rangle. \end{aligned} \quad (2.10c)$$

From this argument, we shall only consider the effect resulting from the phase difference and then set $\sin k = \sin k'$ in the relevant equations (i.e. neglect the difference of group velocities).

Let us now introduce the variable $u_n = \varphi_n - \varphi'_n$. Using (2.6) one obtains the recursion equation :

$$\begin{aligned} u_{n+1} - u_n = & -\frac{\varepsilon_n}{\sin k} \left[\frac{F_n}{G_n} \cos \theta_n - \frac{F'_n}{G'_n} \cos \theta'_n \right] + \\ & + \frac{\varepsilon_n^2}{2 \sin^2 k} \left[\left(\frac{F_n}{G_n} - \cos \theta_n \left(1 + \frac{2}{G_n^2} \right) \right) \sin \theta_n \right. \\ & \left. - \left(\frac{F'_n}{G'_n} - \cos \theta'_n \left(1 + \frac{2}{G_n'^2} \right) \sin \theta'_n \right) \right]. \end{aligned} \quad (2.7)$$

As the length L of the system increases, the probability distribution of θ_n and θ'_n spreads out over the interval $[0, 2\pi]$ after the scattering by the first impurity. However, $\theta_n - \theta'_n = u_n - 2\Delta k \cdot n$ has a slowly diffusive motion. It is then possible to obtain a Fokker-Planck equation for the probability distribution $W(F, F', u; L)$, after averaging over the phases θ_n, θ'_n and $\theta_n + \theta'_n$. Such an equation can be written as

The exponential growth rate of $\langle FF' \rangle$ is given by the largest eigenvalue of the matrix :

$$\begin{bmatrix} 4 & 2 & 0 \\ 4 & 2 & \beta \\ 0 & -\beta & 2 \end{bmatrix}.$$

This eigenvalue is equal to $6 - \frac{\beta^2}{12} + 0(\beta^3)$ up to second order in β . Consequently :

$$\begin{aligned} \frac{\langle FF' \rangle}{\langle F^2 \rangle} = & \text{Cte} \cdot \exp \left[-\frac{\beta^2}{12} x \right] \approx \\ \approx & \exp \left[-\frac{(\Delta k)^2}{3} \xi \cdot L \right]. \end{aligned}$$

This relation defines a characteristic length scale

$L_{\langle FF' \rangle}^*$ for the decay of $\langle FF' \rangle$

$$L_{\langle FF' \rangle}^* = \frac{1}{\Delta k} \cdot \frac{3}{\Delta k \cdot \xi} \quad (2.11)$$

For $\Delta k \cdot l \gg 1$, this length is much greater than the « optical length », $\frac{1}{|\Delta k|}$. Furthermore

$$\frac{\xi}{L_{\langle FF' \rangle}^*} = \frac{1}{6 \cotg |\Delta k|} \left(\frac{\xi}{a} \right) (|\Delta k| \cdot \xi)$$

(the intersite distance a has been introduced in order to exhibit the homogeneity of this ratio). From this relation, we see that it is not legitimate to neglect the variation of the group velocity when $|\Delta k|$ is very small, i.e. at $\xi \ll L_{\langle FF' \rangle}^*$.

Now, one can try to generalize this result to the case of the general correlators $\langle F^m F'^n \rangle$, m and n are integers. When $m = 1$ or $n = 1$, it is possible to generate a finite and linear differential system as above, associated with the following matrix :

$$\begin{bmatrix} m(m+1) + 2 & 2m & 0 \\ 2(m+1) & m(m+1) & \beta \\ 0 & -\beta & m(m+1) \end{bmatrix}.$$

Its largest eigenvalue λ is given by

$$\lambda = (m+1)(m+2) - \beta^2 \frac{m}{(2m+1)(m+1)}$$

and then

$$\langle F^m F' \rangle / \langle F^{m+1} \rangle = \text{Cte} \cdot \exp(-L/L_{\langle F^m F' \rangle}^*)$$

where

$$L_{\langle F^m F' \rangle}^* = \frac{(m+1/2)(m+1)}{m} \frac{1}{\Delta k} \left(\frac{1}{\Delta k \cdot \xi} \right) \quad (2.12)$$

Therefore the functional form does not depend on the exponent m which only enters as a numerical prefactor.

Another derivation of (2.11), (2.12) can be obtained using the transfer matrix method of references [8] and [9]. The corresponding calculations will not be given here.

3. Correlator $\langle \ln T^{-1}(E) \cdot \ln T^{-1}(E') \rangle$.

It is equivalent to calculate the correlator $\langle \ln F \cdot \ln F' \rangle$ where F and F' have been defined in the previous section. From the Fokker-Planck equation (2.9), we deduce :

$$\frac{d}{dx} \langle \ln F \cdot \ln F' \rangle = \langle \ln F \rangle + \langle \ln F' \rangle + 2 \langle \cos(u - \beta x) \rangle$$

and then, using the result $\langle \ln F \rangle = x :$

$$\frac{d}{dx} [\langle \ln F \cdot \ln F' \rangle - \langle \ln F \rangle \langle \ln F' \rangle] = 2 \langle \cos(u - \beta x) \rangle \quad (3.1)$$

It is then necessary to study the probability distribution of the phase difference u . From (2.9) and after integration over the variables F and F' , one obtains :

$$\frac{\partial}{\partial x} W(u, x) = 2 \frac{\partial^2}{\partial u^2} [(1 - \cos(u - \beta x)) W] \quad (3.2)$$

Let us define ψ by : $2\psi = u - \beta x$. Equation (3.2) becomes

$$\frac{\partial}{\partial x} \bar{W}(\psi, x) = \frac{\beta}{2} \frac{\partial \bar{W}(\psi, x)}{\partial \psi} + \frac{\partial^2}{\partial \psi^2} [\sin^2 \psi \bar{W}(\psi, x)] \quad (3.3)$$

where \bar{W} denotes the probability distribution of ψ .

At large L , it is possible to approximate the probability distribution of ψ by the stationary solution of (3.3), given by the solution of :

$$\frac{\beta}{2} \frac{d}{d\psi} \bar{W}(\psi) + \frac{d^2}{d\psi^2} (\sin^2 \psi \bar{W}(\psi)) = 0 \quad (3.4)$$

This differential equation has already been studied in reference [12] (Eq. (3.9)), for the calculation of the correlators involving products of exact wave functions. From the positiveness of \bar{W} and the normalization condition, this probability distribution takes the following form [12] :

$$\bar{W}(\psi) = \frac{\beta}{2\pi} \frac{1}{\sin^2 \psi} \exp\left(\frac{\beta}{2} \cotg \psi\right) \times \int_0^\psi \exp\left(-\frac{\beta}{2} \cotg \psi'\right) d\psi' \quad (3.5)$$

According to equation (3.1), the relevant quantity to evaluate is $\langle \cos(u - \beta x) \rangle = 1 - 2 \langle \sin^2 \psi \rangle$. From (3.5), one obtains :

$$\langle \sin^2 \psi \rangle = \frac{\beta}{\pi} \int_0^\infty du e^{-\beta/2u} \times \int_0^\infty \frac{dz}{(1+z^2)(1+(z+u)^2)} \quad (3.6)$$

In the limit $\beta \rightarrow \infty$, we have : $\langle \sin^2 \psi \rangle = \frac{1}{2}$ as expected, since here, the stationary distribution becomes uniform, and $\ln F$ and $\ln F'$ are uncorrelated. In the other limit $\beta \rightarrow 0$:

$$\langle \sin^2 \psi \rangle = \frac{\beta}{\pi} \int_0^\infty \int_0^\infty \frac{du dz}{(1+z^2)(1+(z+u)^2)} \equiv \lambda \beta$$

where λ denotes a numerical factor.

Using this expression and (3.1), the asymptotic value of the correlator $\langle \ln F \cdot \ln F' \rangle$ becomes

$$\langle \ln F \cdot \ln F' \rangle - \langle \ln F \rangle \langle \ln F' \rangle = 2(1 - 4\lambda |\Delta k| \xi)x + \text{Cte} \quad (3.7)$$

The correlation coefficient tends to the following limit :

$$\lim_{L \rightarrow \infty} \frac{\langle \ln F \cdot \ln F' \rangle - \langle \ln F \rangle \langle \ln F' \rangle}{\langle (\ln F)^2 \rangle - \langle \ln F \rangle^2} = 1 - 4 \lambda |\Delta k| \xi. \quad (3.8)$$

It is now interesting to study the short length regime. The initial condition is given by : $\bar{W}(\psi, x = 0) = \delta(\psi)$.

As long as ψ remains small, equation (3.3) can be written as :

$$\frac{\partial}{\partial x} \bar{W} = \frac{\beta}{2} \frac{\partial \bar{W}}{\partial \psi} + \frac{\partial^2}{\partial \psi^2} (\psi^2 \bar{W}).$$

This leads to :

$$\begin{aligned} \frac{d}{dx} \langle \psi \rangle &= -\beta/2 \\ \frac{d}{dx} \langle \psi^2 \rangle &= -\beta \langle \psi \rangle + 2 \langle \psi^2 \rangle \end{aligned}$$

and then

$$\langle \psi \rangle = -\frac{\beta}{2} x \quad \text{and} \quad \langle \psi^2 \rangle = \frac{\beta^2}{8} (e^{2x} - 2x - 1).$$

After expanding to order ψ^2 in (3.1), one gets :

$$\begin{aligned} \frac{d}{dx} [\langle \ln F \cdot \ln F' \rangle - \langle \ln F \rangle \langle \ln F' \rangle] &= \\ &= 2 \left(1 - \frac{\beta^2}{4} (e^{2x} - 2x - 1) \right). \end{aligned}$$

This leads to

$$\begin{aligned} \langle \ln F \cdot \ln F' \rangle - \langle \ln F \rangle \langle \ln F' \rangle &= \\ &= 2 \left(x - \frac{\beta^2}{8} (e^{2x} - 2x^2 - 2x - 1) \right). \quad (3.9) \end{aligned}$$

This result is valid as long as $\beta^2 e^{2x}$ remains small. Here a new characteristic length scale x^* appears as a solution of : $x^* = \frac{\beta^2}{8} e^{2x^*}$.

Then, we have :

$$L^*_{\langle \ln F \cdot \ln F' \rangle} = \xi \ln \left(\frac{\sqrt{2}}{|\Delta k| \xi} \right). \quad (3.10)$$

As can be seen from (3.1), this length scale also describes the decorrelation of the phases of the reflection coefficients.

Here we have restricted our attention to the two limiting cases of very large and very short L . The transient regime however exhibits an interesting behaviour with oscillations around the stationary distribution.

4. Comparison with other studies.

In reference [10], correlators involving exact wave functions have been calculated. For instance, the correlator $F_{\omega}^{(1)}(z)$, defined by :

$$F_{\omega}^{(1)}(z) = \left\langle \sum_{\mu\nu} \delta(E_F - \varepsilon_{\mu}) \delta(E_F + \omega - \varepsilon_{\nu}) \times \psi_{\mu}(x) \psi_{\nu}(x) \psi_{\mu}(x') \psi_{\nu}(x') \right\rangle, \quad z = |x - x'|$$

has been shown to decay with the length $z_0(\omega) = 2 \xi \ln \left(\frac{8}{\omega \tau} \right)$ where ξ denotes the localization length and τ the elastic scattering time. This length $z_0(\omega)$ is identical, up to a numerical prefactor, to $L^*_{\langle \ln F \cdot \ln F' \rangle}$ introduced in the last section. This relationship between these two lengths results from the fact that both are related to the decorrelation of the phases of the reflection coefficients. $z_0(\omega)$ is used to determine the dynamical conductivity $\sigma(\omega)$ through Kubo's formula [10]. Indeed,

$$\sigma(\omega) = -2 \pi \frac{e^2 \omega^2}{\hbar S} \int_0^{\infty} z^2 F_{\omega}^{(1)}(z) dz$$

where S denotes the cross-section area of the wire. From this relation, we recover for $\sigma(\omega)$ the following known expression [10, 12, 13] :

$$\sigma(\omega) = \frac{8}{\pi} \frac{e^2 \xi}{\hbar S} (\omega \tau)^2 \ln^2(\omega \tau).$$

The correlators

$$\langle t(E) t'^*(E) \rangle \quad \text{and} \quad \langle |t(E)|^2 |t'(E')|^2 \rangle$$

have been calculated in [8] and [9]. Here it is worth noting that the Fokker-Planck presented here is not well adapted to calculate these correlators. Since resonant events ($|t| \sim 1$) with a small probability are relevant here, the small transmission approximation used before is no longer valid. As a result, it is not possible to write down a closed system of evolution equations for a finite set of correlators, as has been done in section 2. This difficulty with the probability distribution of the transmission coefficient is well illustrated in the following known expression for the positive moments :

$$\langle |t|^{2n} \rangle = \frac{\pi^{3/2} \Gamma^2(n - 1/2) e^{-x/4}}{2 \Gamma^2(n) x^{3/2}}, \quad x = L/\xi, \quad n \gg 1.$$

The restriction of the n dependence of $\langle |t|^{2n} \rangle$ results from the contribution of these highly unprobable resonances $|t| \simeq 1$.

In [8] and [9], transfer matrices were used, and the determination of the characteristic decorrelation length was mapped into an eigenvalue problem relative to a singular non-Hermitian differential operator.

Let us just quote the results obtained in these references :

$$\begin{aligned} \langle t(E) t^*(E') \rangle &= \frac{2}{\pi^2 x^{3/2}} S(\alpha) e^{-x/4}; \\ \alpha &= \pi x^{1/2} \ln^{-1} \left(\frac{\pi}{\Delta k \cdot \xi} \right) \end{aligned}$$

$$\langle |t(E)|^2 |t(E')|^2 \rangle = \frac{\pi^2}{2x^{3/2}} S(\beta) e^{-x/4};$$

$$\beta = \pi x^{1/2} \ln^{-1} \left(\frac{2^{3/2}}{\Delta k \cdot \xi^{1/2}} \right)$$

where

$$S(\alpha) = \sum_{n=1}^{\infty} \alpha^3 n^2 e^{-n^2 \alpha^2}.$$

We were unable to understand the reason for the occurrence of β as well as the difference between α and β . We believe that α is the correct result, and this gives rise to a third length which has the following form: $L_{\langle t \rangle}^* = \xi \ln^2 \left(\frac{1}{|\Delta k| \xi} \right)$. The presence of $\ln^2 \left(\frac{1}{|\Delta k| \xi} \right)$ is reminiscent of the dynamical conductivity $\sigma(\omega) \sim \omega^2 \ln^2 \omega$. In fact, it can be shown [14] that the static conductivity of a one-dimensional system calculated with the Kubo formula is identical to the conductivity given by the simplified Landauer formula: $G = \frac{e^2}{\pi \hbar} |t|^2$. The same ideas show that the Kubo formula for the dynamical conductivity can be written in terms of correlators involving combinations of the reflection and transmission coefficients, taken at energies E_F and $E_F + \hbar\omega$. This is probably the origin of the difference between

$$\xi \ln \left(\frac{1}{|\Delta k| \xi} \right) \quad \text{and} \quad \xi \ln^2 \left(\frac{1}{|\Delta k| \xi} \right)$$

for the phases and the transmission coefficient decorrelation lengths. It is also interesting to notice that the $\ln^2 \omega$ in the Kubo dynamical conductivity comes from the squared modulus of the dipolar matrix element $\left| \int dx \psi_\mu(x) x \psi_\nu(x) \right|^2$, and the expression of $z_0(\omega)$ for the relevant correlator [10].

5. Discussion.

In this paper, we have shown that decorrelations induced by a small perturbation of the incident wave energy, are described by different characteristic length scales and this according to the considered quantity. To summarize, we have three length scales:

— for negative powers of t ,

$$L_-^* \sim \frac{1}{|\Delta k|} \cdot \frac{1}{|\Delta k| \xi}$$

— for the logarithm of $|t|^2$ or the phase difference between the reflection coefficients,

$$L_0^* \sim \xi \ln \left(\frac{1}{\Delta k \cdot \xi} \right)$$

— for positive powers of t ,

$$L_+^* \sim \xi \ln^2 \left(\frac{1}{|\Delta k| \xi} \right).$$

Note that the two lengths L_0^* and L_+^* differ only by a logarithmic factor. The absence of a single decorrelation length must be viewed as another manifestation of special features of the probability distribution in such disordered systems. For instance, the distribution of $1/T$ is very broad, but exhibits tails corresponding to resonances ($T \approx 1$).

Among these three length scales, however, L_0^* is of particular importance, since it is related to the phases of the reflection coefficients at E and E' . This length appears directly in physical pictures as those given by Mott [13] for the hopping conductivity in the strongly localized regime. Two localized states at a distance z have a mutual overlap of order $e^{-z/\xi}$. The energy separation between these levels has the same order of magnitude as the overlap: $\Delta E \sim e^{-z/\xi}$. Consequently, the typical distance between two localized states with an energy difference ΔE is given by $z^* \approx \xi \ln \left(\frac{1}{\Delta E} \right)$. At shorter length scales, the possible resonance between the two incident waves occurs on the same localized state, and the correlation remains important. At larger length scales, resonances may correspond to two different localized states and the correlation decreases.

The non-analytic dependence of L_0^* and L_+^* on Δk , as Δk goes to zero, is to be noticed. From [8] and [9], this behaviour comes from a singular perturbation in a differential equation. Physically, one can expect an analytic dependence on Δk in the limit where the wave propagation is well described by geometrical optics. Here $\Delta k \cdot L$ is the optical path difference between the two waves. However, in the localized regime, a strong interference can be built up in the system, and the waves are scattered many times by the same impurity. Consequently, when L is greater than the localization length, the expression $|\Delta k| L$ strongly underestimates the phase difference between the two waves at different energies.

Experimentally, these results are relevant to noise experiments in 1D conductors. Available experimental data on magnetoresistance oscillations of small rings seem to support the idea of a gradual crossover between h/e and $h/2e$ periods. Such a crossover occurs [4] at a temperature given by $k_B T \sim \hbar D/L^2$. However it would be interesting to investigate the other possible crossover between 1D and 2D behaviours as the number of active transmission channels (i.e. width of wires) is further decreased.

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