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► **To cite this version:**

L.P. Kubin, Y. Estrin. A nonlinear aspect of crystal plasticity : the Portevin-Le Chatelier effect. Journal de Physique, 1986, 47 (3), pp.497-505. 10.1051/jphys:01986004703049700 . jpa-00210230

HAL Id: jpa-00210230

<https://hal.science/jpa-00210230>

Submitted on 4 Feb 2008

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Classification

Physics Abstracts

81.40L — 62.20F — 03.40K

A nonlinear aspect of crystal plasticity : the Portevin-Le Chatelier effect

L. P. Kubin (*) and Y. Estrin (**)

(*) Laboratoire de Métallurgie Physique (+), Faculté des Sciences, 40, av. du Recteur Pineau, 86022 Poitiers Cedex, France

(**) Technical University Hamburg-Harburg, Harburger Schloßstraße 20, 2100 Hamburg 90, F.R.G.

(Reçu le 27 juin 1985, révisé le 18 novembre, accepté le 21 novembre 1985)

Résumé. — Afin d'illustrer l'intérêt de la dynamique des systèmes non linéaires appliquée à la plasticité des solides cristallins, nous considérons une description simplifiée de l'effet Portevin-Le Chatelier. Les propriétés dynamiques des dislocations, moyennées à petite échelle, sont exprimées en termes d'un oscillateur non linéaire du premier ordre. Les solutions obtenues font apparaître une périodicité temporelle sous la forme d'oscillations de relaxation, et une organisation spatiale sous forme d'une onde solitaire. Diverses vérifications expérimentales sont décrites ou suggérées, et plusieurs systèmes dynamiques analogues sont discutés.

Abstract. — In order to illustrate the potentialities of application of the methods used in the dynamics of nonlinear systems to crystal plasticity, we consider a simple description of the Portevin-Le Chatelier effect. The dynamic properties of dislocations are modelled through a small scale averaging procedure, in terms of a first-order nonlinear oscillator. Solutions are obtained which exhibit time periodicities of the kind of relaxation oscillations and spatial organization in form of a solitary band. Several proposals for further experimental investigation are outlined, and various analogue systems are discussed.

1. Introduction.

Microscopic models of crystal plasticity usually postulate that the macroscopic mechanical behaviour can be described in terms of dynamic properties of individual dislocations, the crystal lattice defects which carry plastic deformation on the atomic scale. This is furnished through the Orowan transport equation [1] :

$$\dot{\epsilon} = \rho_m b v_D \quad (1)$$

which, with the accuracy of a geometric factor omitted here, relates the plastic strain rate, $\dot{\epsilon}$, to the density of mobile dislocations, ρ_m , moving with a velocity v_D . The quantity b represents the magnitude of the Burgers vector that characterizes the « strength » of a dislocation. The velocity v_D is, of course, a result of averaging of the individual dislocation velocity over various dislocation paths within the dislocation ensemble. Accordingly, it depends not only on the applied stress σ but also on microstructural parameters which vary in the course of deformation. As a result, both ρ_m

and v_D depend, in a nonlinear manner, on various external and internal quantities.

Strictly speaking, the Orowan relation (1) is only valid on the local scale. In view of the fact that microstructures developing during plastic deformation exhibit a marked nonuniformity [2, 3], identifying the macroscopic averages with the local quantities appears severely wanting. Structural « self-organization » is present on various scales. Within an elementary slipped volume (referred to as a « cross-section » henceforth ⁽¹⁾), dislocations appear to be distributed more or less regularly or exhibit collective behaviour owing to mutual interactions. In a crystal subjected to uniaxial deformation, these slipped volumes have, as a result of predominantly planar dislocation motion, a thickness of only 10 to 100 Å and fill the cross-sections to a large extent.

In what follows we assume that averaging is possible on that scale and we focus on nonuniformities on

⁽¹⁾ This description refers, in a strict sense, to single crystal deformation but can be applied, for our purposes, to polycrystals as well. A « cross-section » is generally oblique to the specimen axis.

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micrometer or millimeter scale as observed, e.g., by optical methods. The quantities entering equation (1) are then understood as averages over a cross-section. The cross-sections never deform simultaneously in the same manner, and only a small fraction of the crystal volume (1/20 to 1/50, say) is active at a given moment [2, 3]. Plastic deformation is usually confined to discrete bands with sharp boundaries (e.g., the slip bands [2, 3]). Under certain conditions, deformation bands propagate along the specimen in an orderly fashion, plastic activity being continuously extinguished at the rear side by virtue of strain hardening and ignited in the virgin material in front of the band, in a kind of « relay race ».

Nonuniformity of plastic deformation is closely related to nonlinearity of dislocation properties. It is the aim of the present paper to demonstrate this relationship. As a practically important example we consider the Portevin-Le Chatelier (PLC) effect well-known to metallurgists. Microstructurally, it consists in repeated propagation of deformation bands along the specimen; this is accompanied by the appearance of abrupt stress drops or steps on the deformation curves. An insight into this phenomenon can be gained with the help of far-reaching analogies with the properties of nonlinear oscillators. These analogies will be discussed in the following, along with such a specific feature of the PLC effect as the existence of propagating solutions.

In the next section, the essential features of the PLC effect are briefly described. Section 3 deals with the identification of the type of instability associated with this phenomenon within a simple model. Section 4 discusses analogues of the PLC effect in various physical systems.

2. The Portevin-Le Chatelier effect. Influence of the mode of testing.

Review articles (e.g., [4] to which the reader is referred to for details) mention that the effect was first discovered by Le Chatelier in 1909, on mild steels at slightly elevated temperature. It was later found by Portevin and Le Chatelier on duraluminium alloys at room temperature. To date, the PLC effect has been recorded in a number of dilute interstitial and substitutional alloys on Al, Cu, Ni, Fe, etc. basis, usually around room temperature.

Since the mode of testing strongly affects both the observed behaviour and the ease of modelling, we discuss it first. In uniaxial straining, a constraint is placed upon such macroscopic quantities as the rate of variation of the applied stress, $\dot{\sigma}$, or the total plastic strain rate, \dot{E} , which is, by definition, an average over the (generally nonuniform) local plastic strain rate $\dot{\epsilon}$:

$$\dot{E} = (1/L) \int_0^L \dot{\epsilon}(x) dx. \quad (2)$$

Here x is the coordinate along the axis of the specimen with the gauge length L . The testing conditions are specified through

$$\dot{\sigma} + M\dot{E} = \dot{\sigma}_0 \quad (3)$$

where the constants $\dot{\sigma}_0$ and M have the following meaning in the different modes of testing of interest here :

- (i) $\dot{\sigma}_0 = M\dot{\epsilon}_a = \text{Const.}$ under *constant imposed strain rate* $\dot{\epsilon}_a$, M denoting the combined elastic modulus of the system « specimen + testing machine »;
- (ii) $\dot{\sigma}_0 = \text{Const.}$, $M = 0$ (i.e. $\dot{\sigma} = \dot{\sigma}_0$) under *constant stress rate* conditions; equation (2) is still valid here, but no external constraint is placed on E in this case;
- (iii) *creep* conditions are obtained as a limiting case of (ii) by setting $\dot{\sigma}_0 = 0$ [5].

The distinctive macroscopic feature of the PLC effect is the occurrence, within a definite range of temperature and strain- or stress-rate, of pronounced periodicities or quasiperiodicities in the deformation curve $\sigma(E)$. These are illustrated by figure 1 obtained for a polycrystalline Al-5 % Mg alloy at room temperature. The samples were solution-treated (460 °C, 3 h, argon atmosphere) and tested immediately afterwards to avoid decomposition. Serrated stress-strain curves (« jerky flow ») appearing at constant strain rate result from relaxation of the elastic strain when the total plastic strain rate suddenly exceeds the imposed strain rate : following equation (3) $\dot{\sigma}$ is then negative. At constant stress rate, the stress remains practically unchanged within a short time interval ($\ll 1$ s) during which a sudden change of plastic strain rate occurs, and a stair-case deformation curve is recorded. As already emphasized in previous publications [5, 6], constant stress rate testing, although not so widespread in practice, provides the simplest experimental conditions for studying the PLC effect. The reason is that no constraint is placed on the total, macroscopic strain rate, and the macroscopic behaviour is determined by the local behaviour in an individual cross-section.

On the optical scale, each jerk or step on the deformation curve appears to be connected with the formation and motion of a deformation band (whose thickness is typically a few millimeters) with well-marked edges. Slip is concentrated within such bands, as schematically illustrated by figure 4. Band initiation usually takes place at one end of the specimen where stress concentrations are available. The band then

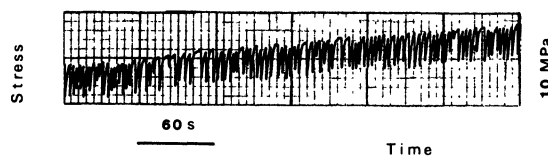


Fig. 1. — Serrated deformation curve associated with the PLC effect in Al-5 % Mg deformed with constant strain rate (room temperature; $\dot{\epsilon}_a = 2.5 \times 10^{-4} \text{ s}^{-1}$).

propagates continuously or by successive steps (hopping bands) until it reaches the opposite end of the specimen where it disappears. This is a recurrent process, new bands being usually initiated at the same end of the specimen. Passage of each band gives rise to a jerk or to a step on the macroscopic stress-strain curve.

Microscopic mechanisms of the PLC effect are now well-understood, at least qualitatively [7-10]; they involve a dynamic interaction between mobile dislocations and diffusing point defects (see, e.g., the review [4]). Even in the absence of a self-consistent quantitative theory, one reaches a conclusion, supported by experiment, that in the domain of the PLC effect the glide resistance should decrease with increasing strain rate.

As discussed by Penning [11] (see also [5]), this qualitative picture should be sufficient to account for most of the macroscopic features described above. The dynamic properties of dislocations averaged over a cross-section can be expressed, with some approximation, through the state or constitutive equation [11] :

$$\sigma = h\varepsilon + F(\dot{\varepsilon}). \quad (4)$$

Here the strain hardening rate $h = (\partial\sigma/\partial\varepsilon)_{\dot{\varepsilon}}$ that characterizes the build-up of athermal stresses in the course of deformation can be considered constant. The function $F(\dot{\varepsilon})$ takes account of the interaction between mobile dislocations and localized obstacles which is strain rate dependent. The strain rate sensitivity (SRS) of the flow stress is defined as

$$S = (\partial\sigma/\partial \log \dot{\varepsilon})_{\varepsilon} = \dot{\varepsilon}(dF/d\dot{\varepsilon}). \quad (5)$$

In normal conditions, plastic deformation is thermally activated which is often expressed by an Arrhenius-type rate equation, meaning that S is a positive quantity and, consequently, that F is a monotonically increasing function of $\dot{\varepsilon}$. As mentioned above, the PLC effect is associated with the conditions when $F(\dot{\varepsilon})$ has a negative slope [9-11], implying negative strain rate sensitivity (NSRS) of the flow stress.

The results of the measurements for Al-5% Mg carried out at constant strain rate at 300 K are depicted in figure 2. The function $F(\dot{\varepsilon})$ was extracted from the strain rate dependence of the flow stress at a fixed strain ($\varepsilon = 8 \times 10^{-2}$). As a matter of fact, the result has to be independent of the choice of this « reference strain » if equation (4) that contains no cross-over terms in $\dot{\varepsilon}$ and ε is valid. This condition was found to hold only in an approximate sense. The approximately constant strain hardening rate h obtained is equal to 950 MPa at 300 K. As further discussed below, the stress-strain curves are serrated in the domain of negative apparent strain rate sensitivity drawn dashed. The data points corresponding to this domain represent some average stress between the upper and the lower stress at a given strain rate.

The results of figure 2 indicate a correlation between

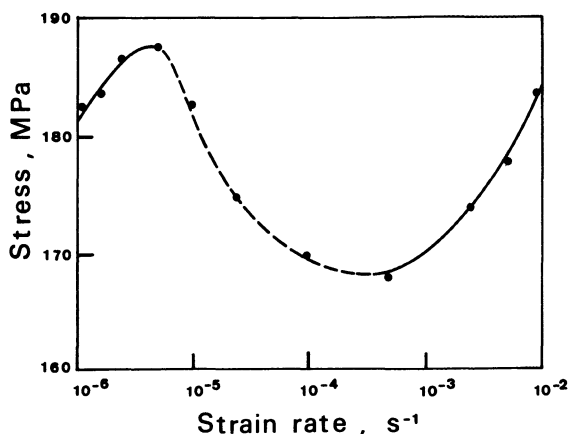


Fig. 2. — The flow stress at $\varepsilon = 8 \times 10^{-2}$ for Al-5% Mg tested at 300 K under constant strain rate. The function $F(\dot{\varepsilon})$ is obtained by subtracting from the flow stress a constant hardening term $h\varepsilon$.

the occurrence of the PLC effect and the condition that the slope of $F(\dot{\varepsilon})$ be negative (cf., e.g., [10]). At small strain rates (i.e., at low dislocation velocities, cf. Eq. (1)), point defects are dragged along with moving dislocations which process requires high stresses. At large velocities, i.e., beyond the minimum of $F(\dot{\varepsilon})$, dislocations are liberated from their point defect atmospheres and can move under comparatively small stresses. Between the two extrema, dynamic interaction responsible for the occurrence of an instability is effective. The dislocation ensemble is in an intermediate state, while a single dislocation is either in the pinned or in the unpinned state.

The model discussed below is based on this typical nonlinear, S-shaped form of $F(\dot{\varepsilon})$ found experimentally in the temperature range where the PLC effect occurs. We are not concerned here with a quantitative microstructural theory, still to be elaborated, that can give the detailed shape of this curve. Similarly, we do not discuss such features as an incubation strain for the onset of the PLC effect or mechanical aspects dictating the deformation band orientation in polycrystals. In this brief survey, only a schematic view of the PLC effect is presented. The model used relies on a few simplifying assumptions, and, thus, is subject to some limitations already discussed in [5, 6].

Finally, several other side effects which are not considered here require further sophistication of the model : for instance, at room temperature, the domain of existence of the PLC effect does not coincide exactly with the strain rate range of negative slope of F . The strain hardening rate h is not exactly constant as mentioned above. In some cases situations can be found where nonuniformity arises from anomalies both in the strain hardening rate (Lüders bands) and in the SRS. In order to avoid a rather common confusion between the Lüders bands and the PLC-bands we adopt here a strict definition given by Kocks [10] according to which the PLC-bands originate exclu-

sively from SRS anomalies while the Lüders bands are associated with strain hardening anomalies.

Some roughness of the present model notwithstanding, it contains all the components to account for the basic features of the PLC effect and provides a suitable frame for further discussion.

3. Nonlinear aspects of the PLC effect.

Combining equation (3) with equation (2) and (4) one obtains an integrodifferential nonlinear system which, when solved for $\varepsilon = \varepsilon(x)$, yields the distribution of strains, that may be uniform or nonuniform. The knowledge of the corresponding integral quantity, E , makes it possible to calculate the deformation curve, $\sigma(E)$, for the bulk specimen.

Despite several attempts initiated in [11], no explicit solutions could be found to date for the constant strain rate conditions because of the coupling between $\dot{\sigma}$ and \dot{E} introduced by the non-zero term $M\dot{E}$ in equation (3). By contrast, we have shown [5, 6] that simple solutions can be obtained under constant stress rate conditions.

We do not reproduce here in full detail the results pertaining to the case of constant stress rate. We rather consider the basic properties of the systems underlying the PLC effect and establish their relationship with well-known properties of certain classical nonlinear oscillators.

The behaviour of an individual cross-section, at a fixed position x [or, alternatively, of a hypothetical uniform solution $E \equiv \varepsilon = \varepsilon(t)$] can be studied by considering equation (4) differentiated with respect to time t :

$$\dot{\sigma} = h\dot{\varepsilon} + [dF(\dot{\varepsilon})/d\dot{\varepsilon}] \ddot{\varepsilon}. \quad (6)$$

3.1 STEADY STATE AND ITS STABILITY. — Four following properties of the model under consideration are obtained readily.

(i) Due to the constancy of h there exists a uniform steady state solution of equation (6) for the strain rate (i.e., $\ddot{\varepsilon} = 0$). This reads

$$\dot{\varepsilon}_s = \dot{\sigma}/h = \dot{\sigma}_0/(h + M), \quad (7)$$

where the quantities $\dot{\sigma}_0$ and M are specified by the testing conditions, cf. equation (3).

(ii) We introduce in one cross-section a fluctuation

from steady-state in the form $\delta\dot{\varepsilon} = \dot{\varepsilon} - \dot{\varepsilon}_s = (\delta\dot{\varepsilon})_0 \exp \lambda t$, where $(\delta\dot{\varepsilon})_0$ is a constant. The associated fluctuation in the second derivative of strain rate with respect to time reads $\delta\ddot{\varepsilon} = \lambda\delta\dot{\varepsilon}$.

The system which globally controls the deformation conditions is not able to respond to fluctuations. Therefore, the latter will obey the constitutive equation, equation (4) or (6), but not equation (3) which is related to the testing conditions.

Taking into account that $h\dot{\varepsilon} - \dot{\sigma} = h(\dot{\varepsilon} - \dot{\varepsilon}_s)$, linearizing equation (6) with respect to these small deviations from steady-state yields:

$$\lambda = -h/(dF/d\dot{\varepsilon})_{\dot{\varepsilon}=\dot{\varepsilon}_s}. \quad (8)$$

The strain hardening rate, h , is normally positive. Hence the steady state solution is unstable with respect to fluctuations in the strain rate when $dF/d\dot{\varepsilon} < 0$ (because then $\lambda > 0$ leading to the growth of the fluctuations with time), i.e. in the interval

$$\dot{\varepsilon}_1 < \dot{\varepsilon}_s < \dot{\varepsilon}_2, \quad (9)$$

where $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ denote the positions of the maximum and the minimum of $F(\dot{\varepsilon})$, respectively.

The condition that λ given by equation (8) be positive, and the inequalities (9) following from it, express the criterion for the onset of nonuniformity.

(iii) Since the instability condition (9) holds both for constant strain rate and constant stress rate testing, equation (7) provides a means of checking on the equivalence of deformation conditions in the two modes of testing with respect to the occurrence of the PLC effect. This equivalence was confirmed experimentally on Al-5 % Mg alloys at 360 K, as illustrated by table I.

(iv) We notice from equation (8) that when $\dot{\varepsilon}_s$ reaches the boundaries of the interval $(\dot{\varepsilon}_1, \dot{\varepsilon}_2)$, i.e., when it takes the values $\dot{\varepsilon}_1$ or $\dot{\varepsilon}_2$ so that $dF/d\dot{\varepsilon}$ becomes zero, the transition between uniform and nonuniform deformation is connected with λ turning infinite. This is distinct from the situation when the vanishing hardening rate gives rise to a change of sign of λ , which case can be referred to as bifurcation. This is in keeping with the observation by Kocks [10] that instabilities stemming from NSRS exhibit a more abrupt behaviour than those originating from strain hardening features.

Table I. — *Experimental steady state strain rates, $\dot{\varepsilon}_s$, obtained at 360 K during constant strain rate and constant stress rate tests. Values at which the PLC effect occurred are underlined. The domain of the PLC effect is determined by $9 \times 10^{-4} \text{ s}^{-1} \lesssim \dot{\varepsilon}_s \lesssim 10^{-2} \text{ s}^{-1}$. Broken underlining indicates an intermediate situation where the PLC effect occurred occasionally or was not well-pronounced.*

	Constant Strain Rate Test						
$\dot{\varepsilon}_s (\text{s}^{-1})$	2.4×10^{-4}	4.8×10^{-4}	<u>9.6×10^{-4}</u>	<u>2.4×10^{-3}</u>	<u>4.8×10^{-3}</u>	<u>9.6×10^{-3}</u>	<u>2.4×10^{-2}</u>
	Constant Stress Rate Test						
$\dot{\varepsilon}_s (\text{s}^{-1})$	1.96×10^{-4}	3.37×10^{-4}	<u>9.42×10^{-4}</u>	<u>2.06×10^{-3}</u>	<u>3.72×10^{-3}</u>	<u>9.85×10^{-3}</u>	

3.2 RELAXATION OSCILLATIONS. — We now consider the behaviour of an individual cross-section in non-uniform conditions with the help of equation (6) expressed in the form

$$\ddot{\epsilon} = h(\dot{\epsilon}_s - \dot{\epsilon}) / (dF/d\dot{\epsilon}). \tag{10}$$

When $\dot{\epsilon} = \dot{\epsilon}_1$ or $\dot{\epsilon} = \dot{\epsilon}_2$, the derivative $dF/d\dot{\epsilon}$ vanishes implying that $\dot{\epsilon}$ undergoes an instantaneous jump. Such a jump necessarily takes place at constant strain, but also at practically constant stress, for stress changes cannot be transmitted faster than the sound waves in the material. As a result, the trajectory followed by the system in the F vs. $\dot{\epsilon}$ diagram involves two jumps, as illustrated by figure 3. This trajectory is necessarily periodic if the (unstable) steady state strain rate, $\dot{\epsilon}_s$, lies within the « forbidden gap », $(\dot{\epsilon}_1, \dot{\epsilon}_2)$. The system permanently tends to reach steady state [note that beyond the interval $(\dot{\epsilon}_1, \dot{\epsilon}_2)$ the sign of $\ddot{\epsilon}$ is opposite to that of $\dot{\epsilon} - \dot{\epsilon}_s$], but it finds itself thrown onto the opposite ascending branch of the curve $F(\dot{\epsilon})$, over this interval, each time it reaches one of the boundaries.

The mathematical expression for the cyclic trajectory can be written in the form

$$\left[(\dot{\epsilon}_s t - \epsilon) - \frac{F(\dot{\epsilon})}{h} \right] \left[\frac{\dot{\epsilon}_s - \dot{\epsilon}}{\ddot{\epsilon}} \right] = 0 \tag{11}$$

where the first term originates from a regular integration of equation (6) and describes the parts of the orbit which are covered with finite velocity, i.e. correspond to the ascending branches of $F(\dot{\epsilon})$, while the second term corresponds to the « jumps ». Changing variables, $\dot{\epsilon}_s t - \epsilon = z$, $y = \dot{\epsilon}$, and introducing $F(\dot{\epsilon}_s - \dot{\epsilon}) = hf(y)$ we rewrite equation (11) as :

$$[z - f(y)].dz/dy = 0. \tag{12}$$

Equation (12) is formally identical with the equation

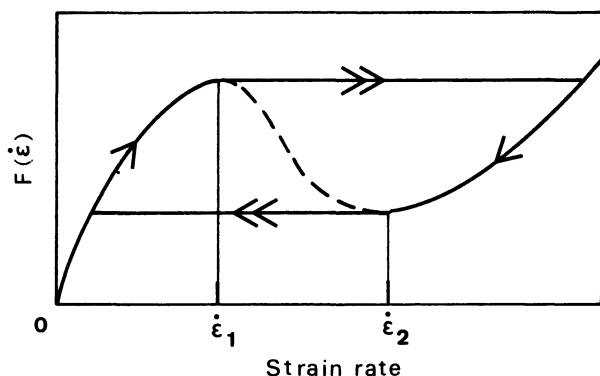


Fig. 3. — Relaxation oscillations associated with the PLC effect. The closed trajectory indicated by the arrows is periodically followed by the strain rate in a specimen cross-section; the double arrows denote the fast jumps occurring when the boundaries of the « forbidden gap », $\dot{\epsilon}_1$ or $\dot{\epsilon}_2$, are reached.

describing the motion of the Van der Pol oscillator in the limiting case of very large coefficient in the damping term. The typical cycles of slow-fast-slow-fast motion along a closed orbit characteristic of this limiting case are referred to as relaxation oscillations [12, 13]. Furthermore, we notice that the graph $\gamma(z)$, or $\dot{\epsilon}(\epsilon)$, represents the familiar cusp catastrophe [12, 14] as recently suggested by Blanc and Strudel [15].

Summarizing the behaviour of an individual cross-section, we can state that it can be represented by a nonlinear equation of the first order in $\dot{\epsilon}$ which can be transformed into an autonomous one. There exists one steady state which acts as an attractor when it is situated outside the interval $(\dot{\epsilon}_1, \dot{\epsilon}_2)$ or as a repeller otherwise. In the latter case a periodic orbit exhibiting relaxation oscillations is obtained. An important property of such a system is that all of its solutions differ by a phase shift only : if $\dot{\epsilon}(t)$ is a solution then $\dot{\epsilon}(t - t_0)$ is a solution either (t_0 is an arbitrary shift in time) [16]. This implies that once the origin of time is fixed (e.g. at the moment when the system enters the domain of oscillatory behaviour) the solution is unique.

3.3 SPATIAL ORGANIZATION. — The fact that the spatial coordinate, x , does not explicitly enter the differential equation (4) implies that properly speaking this constitutive equation is not a local equation. This is why the growth of spatial fluctuations with preferential wavelengths could not be investigated. However, the system discussed has a simple kind of self-organization. Since all cross-sections follow the same orbit, with the accuracy of a time shift $t_0 = t_0(x)$, one is left to search for the form of $t_0(x)$. This is easily done for the case of constant stress rate testing. Differentiating both sides of equation (4) with respect to x and combining with the corresponding form of equation (6) we get, with the use of the condition $d\sigma/dx = 0$,

$$\epsilon' + [(\dot{\epsilon}_s - \dot{\epsilon})/\dot{\epsilon}] \dot{\epsilon}' = 0 \tag{13}$$

where the prime means differentiation with respect to x . The solution of equation (13) reads

$$\dot{\epsilon} - \dot{\epsilon}_s = -v\epsilon' \tag{14}$$

where v is an arbitrary constant with the dimension of velocity. Differentiating with respect to time yields

$$\ddot{\epsilon} = -v\dot{\epsilon}' \tag{15}$$

meaning that the strain rate has the form of a propagating solution $\dot{\epsilon} = \dot{\epsilon}(x - vt)$. The spatial behaviour can be thus obtained from the temporal behaviour of an individual cross-section by setting $t_0 = x/v$. Accordingly, the strain rate profile has the form of a succession of propagating bands with spatial periodicity reflecting the periodicity in time found for a single cross-section. Each band has two abrupt edges characterized by the strain rate jumps and traverses the specimen with a constant velocity v (cf. Fig. 4).

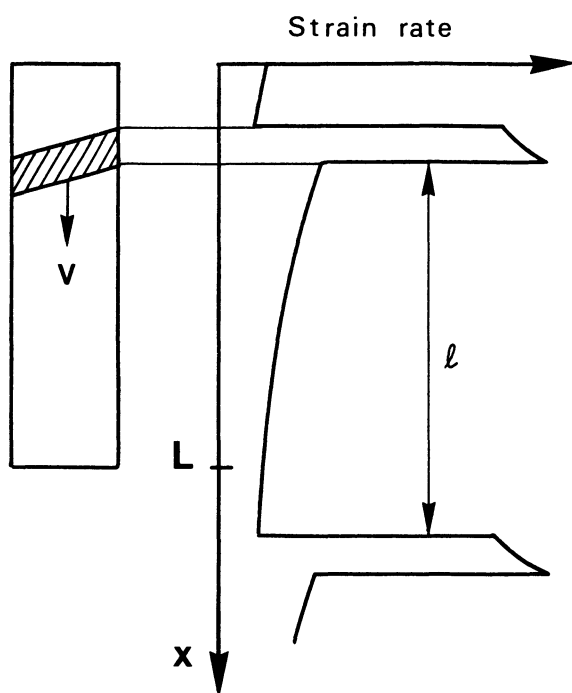


Fig. 4. — A PLC band and the corresponding strain rate profile reflecting the time periodic behaviour shown in figure 3. Depending on the ratio L/l , no band, one band or several bands can be active in the specimen at a given time.

3.4 PHYSICAL INTERPRETATION AND EXPERIMENTAL CONSEQUENCES. — In the above considerations, the propagation velocity v appears as an arbitrary constant, since no interaction between the cross-sections was introduced. In practice, v is determined by the material and the testing conditions and has a typical order of magnitude of cm/s [17]. To our knowledge, there exists no model to date that would be able to predict the magnitude of v [5, 10], and one can only rely on experimental evaluation of it.

The occurrence of a gap in plastic strain rate stems from the fact that a single dislocation cannot stay, even for a limited time, in a situation between being pinned or being free. If the plastic strain rate averaged over a cross-section or over the entire specimen happens to have a steady state value that lies within the gap, the local values on the dislocation scale are either much lower (dislocations dragging along their point defect atmosphere) or much larger (free dislocations within a band). The macroscopic behaviour thus appears as a complex composition of two kinds of behaviour obtained in uniform conditions, i.e. for $\dot{\epsilon} < \dot{\epsilon}_1$ and $\dot{\epsilon} > \dot{\epsilon}_2$.

A practical consequence is that, contrary to a common belief, there is absolutely no point in attempting to extract values of simple relevant microscopic properties from the data pertinent to the instability domain. What is measured in such cases is rather a complex mixture of a large number of parameters. The quantities having the real significance in con-

nection with the PLC effect are the values of $\dot{\epsilon}_1$ and $\dot{\epsilon}_2$, the boundaries of the forbidden gap which are often interpreted in terms of activation energies or diffusion coefficients [8].

The present considerations point some new directions for experimental investigations.

(i) To take advantage of the simple solutions obtained here, constant stress rate tests should be employed preferentially. It is worth mentioning in this connection that no simple solutions of the discussed type that would propagate with a constant velocity are associated with the constant strain rate mode of testing [5].

(ii) Periodicity with respect to time and to the coordinate can be fully characterized by experimentally determining the band velocity. As an example, figure 5 shows a preliminary comparison of the calculated and the experimental time intervals between two successive steps (i.e., in practice, the period of the PLC effect) for Al-5% Mg. The tests were performed with constant stress rate at 360 K. The values of the stress rate were chosen in such a way as to correspond to steady state strain rates falling within the forbidden gap. The comparison demonstrates that, despite the roughness of the model, it appears to have

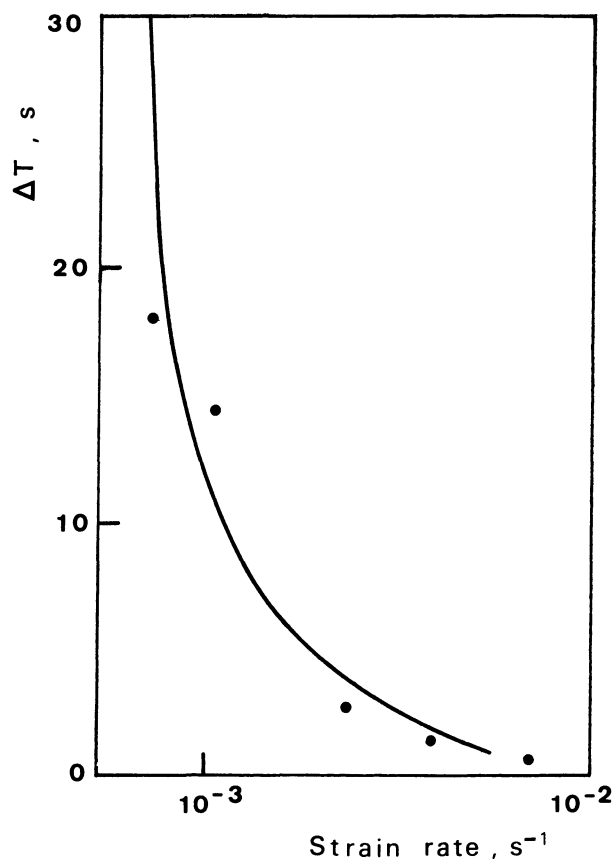


Fig. 5. — Time (ΔT) between successive strain bursts in Al-5% Mg deformed with constant stress rate as a function of $\dot{\epsilon}_s$, the steady state strain rate, within the « forbidden gap » ($T = 360$ K).

a remarkable predictive capability, which could be further improved by relaxing restrictive approximations mentioned above.

(iii) From the results presented in figure 5 one cannot infer whether the period remains finite or becomes infinite when $\dot{\epsilon}_s$ approaches the lower boundary of the PLC domain, $\dot{\epsilon}_s \cong \dot{\epsilon}_1 \cong 5 \times 10^{-4} \text{ s}^{-1}$. More generally, it would be interesting to investigate the critical points where $\dot{\epsilon}_s = \dot{\epsilon}_1$ or $\dot{\epsilon}_s = \dot{\epsilon}_2$, in order to determine whether or not PLC bands appear with finite width and in finite number, in particular, at $\dot{\epsilon}_1 \rightarrow \dot{\epsilon}_2$.

(iv) Depending on the relation between the specimen length L and the spatial period of the propagating strain rate profile, no band, one band or several bands may be present in the specimen at a given moment. A very particular situation would be found if the distance l between the rear edge of a band and the front edge of the next one is commensurate with L , cf. figure 4. In this case, the discontinuous character of the deformation curve would disappear because, within a translation, the strain rate profile in the specimen undergoes no change with time. It appears of interest to design an experiment for checking this feature.

Concluding this section, we would like to stress again the two basic properties of the PLC effect which in combination provide a clue to the understanding of this phenomenon. First, it is the periodic behaviour, identified as relaxation oscillations, within a single cross-section. Second, it is the existence of propagating solutions unfolding the relaxation oscillations into a spatially periodic band pattern moving with a constant velocity. A large number of physical systems can exhibit a similar behaviour, and no wonder that there exists an extensive literature about analogues of the PLC effect. In the next section a brief critical review of this literature is given.

4. Analogues of the PLC effect.

Relaxation oscillations belong to a particular class of nonlinear oscillatory phenomena, distinct from bifurcational type of oscillations. Many examples of this class are treated in classical textbooks, related mainly to mechanics and electrical circuitry [18-21]. Here we only collect examples of systems which have been considered explicitly as analogous to the PLC effect.

A more general discussion, in terms of feed-back processes, is found in [22], showing examples where spatial periodicity of a property is connected with propagation processes. However, contrary to the statement by Frank [22], oscillations including electrical transport processes are not strictly bound to heterogeneous interfacial structure as demonstrated in the next section.

4.1 ELECTRICAL ANALOGUES : THE GUNN EFFECT. — The basic concept is the notion of negative differential

resistivity (NDR) analogous to the negative strain rate sensitivity (NSRS) [21, 23]. This point can be illustrated by various electrical circuits [18, 21] or interfacial devices (see, e.g., [24]) such as the tunnel diode. An electrical circuit has recently been developed to simulate the PLC effect; its properties have been investigated with regard to similarities between the NDR and NSRS [25].

In most of these examples (except, perhaps, in [25]) the analogy with the PLC effect appears somewhat superficial since such a basic feature as band propagation is not considered. A much closer analogy can be found with the electrical behaviour of bulk semiconducting devices exhibiting the NDR associated with the Gunn effect.

According to Ohm's law, charge carriers drifting with a velocity \bar{v} under the action of the electric field ϵ produce a current density

$$j = qn\bar{v} \tag{16}$$

where q is the charge and n the density of the carriers. In certain bulk semiconductors, such as, e.g., n -type GaAs or InP, the electric characteristic, j vs. ϵ , exhibits a range of NDR, cf. figure 6. The conditions for this shape of the characteristic to occur as well as a theoretical explanation are given by the RWH (after Ridley, Watkins, and Hilsum) theory [26, 27] which relates this effect to the field dependence of the drift velocity.

From a comparison of figure 2 and figure 6, one can easily establish a correspondence $j \leftrightarrow \sigma$ and $\epsilon \leftrightarrow \dot{\epsilon}$ (²). In the regime of NDR (e.g., at a point A in Fig. 6), any local charge fluctuation is bound to grow because of the feed-back higher charge (higher field) \rightarrow smaller

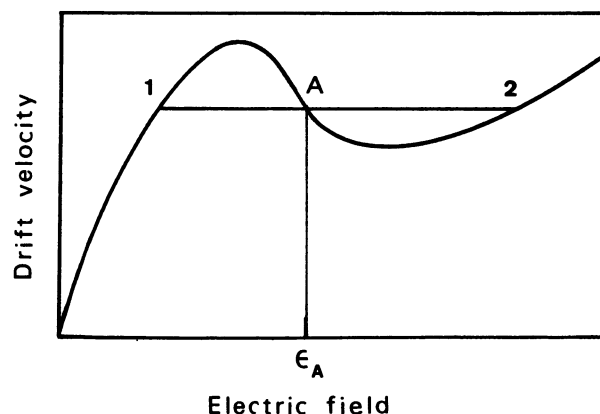


Fig. 6. — Negative differential resistance and the Gunn effect. Electron drift velocity under an electric field (compare with Fig. 3).

(²) Note that this is not the kind of correspondence that might be expected from a direct consideration of equation (1) and (16), since one of the effects (Gunn) is force-driven and the other (PLC) is flux-driven (cf. Ref. [22] for terminology).

current \rightarrow higher charge (higher field). Growth stops when stable states on the ascending branches are reached (e.g., points 1 and 2). Such devices can be operated in various modes which would correspond to various modes of mechanical testing in the PLC case. In the so called dipole layer mode, there exists propagating solutions for the field ϵ which behave quite similarly to the PLC bands under constant stress rate conditions, as illustrated by figure 7 based on the results of reference [28]. Band propagation is the essence of the Gunn effect. Due to a diffusion term incorporating interaction between the « cross-sections » the propagating bands in the Gunn case have smooth edges, however, which distinguishes them from the sharply edged PLC bands.

A band forms at places where doping fluctuations or space charge accumulation facilitate nucleation, very often near the cathode ohmic contact. Under the bias voltage the band moves with a constant velocity keeping its shape until it disappears at the anode. The field then begins to rise uniformly until it reaches a threshold value at which a new band can be formed. Under other operation conditions, e.g., when the bias voltage drops below the threshold value while a band is moving, hopping bands can be obtained. The latter conditions are analogous to constant strain rate testing in the PLC case.

As in the PLC effect, the major properties of the Gunn effect are the oscillatory behaviour within an individual cross-section and propagation of this behaviour into the adjacent bulk. In both phenomena, the oscillatory output results from the periodic formation of solitary bands. Nonuniformity arises from the drive to achieve an average located within the forbidden (unstable) part of the characteristic, through a distribution of states in the stable part.

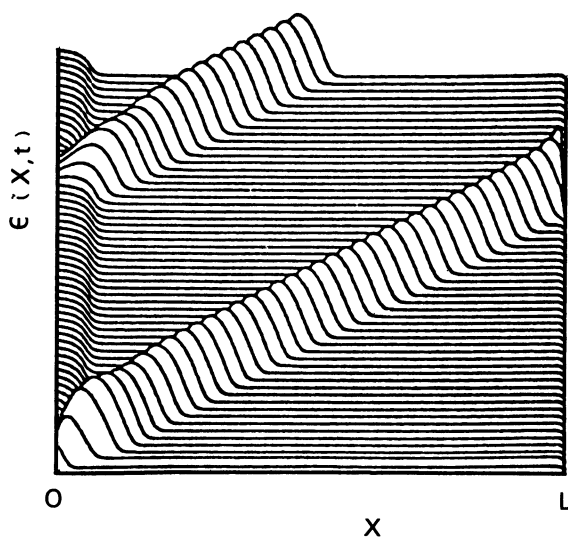


Fig. 7. — Propagating electric field (ϵ) waves in the Gunn effect. The specimen length is L ; the cathode ohmic contact is located at $x = 0$. Successive profiles are shifted along the vertical axis for clarity (after [28]).

4.2 CHEMICAL AND MECHANICAL ANALOGUES. — Following Nicolis and Prigogine [13], an oscillatory system is associated with autocatalytic reactions of the form $A \rightarrow B$, with hidden intermediate products X, Y, \dots not entering the global balance. Interesting nonlinear features only appear with cubic forms, i.e., the model should be at least trimolecular. The analogy with the PLC effect worked out recently [29] is based on identifying A, B, X, Y, \dots with certain dislocation states (e.g., free, pinned, multiplying, sessile) connected by simplified reaction constants. Although stepped deformation curves could be simulated (without any spatial organization), the physical relevance of the underlying dislocation reaction mechanisms appears questionable.

Usually quoted [30, 31] is the system consisting of a mass connected to a spring and placed on a rolling belt where a friction force is exerted. Another model [32] introduces, in addition, a normal force increasing with the displacement to account for strain hardening. All of these systems exhibit relaxation oscillations, and these instability mechanisms are known in the field of mechanics under the name of « stick-slip ».

5. Concluding remarks.

In the present work, the PLC effect has been described in terms of dynamics of a nonlinear system. A comparison with analogous physical systems has been given. A simple frame chosen made it possible to use as a basis the properties of a nonlinear first-order differential equation to obtain the time periodic behaviour. This treatment is complemented with a demonstration of the existence of moving deformation bands.

A more elaborate description of experimental behaviour in a wide range of control parameters (temperature, composition, etc.) can be given using this frame, provided that a reliable microstructural model is available. Presently existing micromodels do not appear to possess the required reliability, however.

It should be desirable to elaborate more sophisticated constitutive equations by including the interactions between the deforming cross-sections. This would yield local equations, the response of which to spatial fluctuations could then be examined. On the experimental side, simple tests proposed in section 3 to check on the predictions of the present model would be useful for judging whether it can be used as a basis for future development.

Acknowledgments.

The authors are indebted to Professor J. Friedel who indicated the analogy between the Portevin-Le Chatelier effect and the Gunn effect. The contribution of Drs. K. Chihab and J. Vergnol to the experimental part of the present work is gratefully appreciated.

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