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## High amplitude wave propagation in collapsible tube.

### I. — Relation between rheological properties and wave propagation

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**Résumé.** — La mécanique des tuyaux collabables à parois viscoélastiques prétendues longitudinalement est étudiée à la fois théoriquement et expérimentalement. Le comportement statique du tube est caractérisé par une loi reliant la pression transmural et l'aire d'une section droite du tube. Cette loi, vérifiée expérimentalement grâce à des mesures effectuées sur un banc hydrodynamique de simulation, est généralisée au cas de phénomènes instationnaires en introduisant le module dynamique mesuré directement sur des échantillons de matériau pariétal. On montre alors que l'on peut déduire de cette loi de comportement dynamique, la vitesse des ondes de pression de petite amplitude. Ces résultats sont alors confrontés aux mesures directes de vitesse de propagation des ondelettes de pressions, en imposant différentes valeurs de la pression moyenne, que le tube soit alors gonflé ou partiellement collabé.

**Abstract.** — The mechanical behaviour of collapsible tubes is theoretically and experimentally studied when the viscoelastic wall is longitudinally stretched. The dynamic rheological law is deduced from the static law by introducing the dynamic Young's modulus as experimentally obtained. It is then shown that the speed of small amplitude pressure waves is well predicted using this dynamic rheological law.

#### 1. Introduction.

Unsteady flow of an incompressible fluid inside a duct with deformable walls has been extensively studied during the past decades in order to understand the dynamics of the arterial circulatory system [1-4]. The effects of the non-linearities of the governing equations for both fluid dynamics and wall mechanics have been investigated when considering wave propagation in such a system [4-6].

It has been shown that the rheological law of the tube wall mainly controls the behaviour of both small and large amplitude waves [7, 8]. Moreover, under specific conditions a shock-like transition has been observed [9-11].

When it is applied to collapsible tubes modelling venous circulation and clinical devices of fundamental importance such as heart-lung apparatus, large amplitude wave propagation is more difficult to study and has hitherto been studied on purely elastic tubes [12, 13]. Using the method of characteristics to solve the

basic equations for the dynamics of the system requires a good knowledge of the phase velocity of small amplitude waves as a function of the pressure.

Therefore, after a brief review of the viscoelastic rheological behaviour of the tube wall, experimental and theoretical results concerning propagation of small amplitude waves on collapsible tubes will be presented (Part I). Large amplitude wave propagation shall then be studied when the tube is either collapsed or inflated (Part II). This study shows experimental evidence of forerunning waves [14, 15], whose dispersion equation will be theoretically and experimentally studied.

#### 2. Mechanical properties of collapsible tube.

##### 2.1 THEORETICAL BASIS.

**2.1.1 Static behaviour with positive transmural pressure.** — In order to characterize mechanical properties of collapsible tubes, a relation between the transmural pressure  $P$ , applied on soft tube walls and the related

wall strain, will be established. The variation of longitudinal strain will not be taken into account, since it is limited in arterial hemodynamics to 1 % to 2 % by connective tissues [16]. On the other hand, tangential wall strain variations cannot be overlooked since they come up to about 10 % [17], and thus play a basic role in the dynamics of the system.

Theoretical relationships which enable one to connect the transmural pressure  $P$  with the tube radius  $R$  have been recently reviewed by Taylor and Gerrard [18]. The most significant results they have come up with, show the fundamental role of wall thickness in tube mechanical behaviour. However, they proposed a simple approach based upon a shell theory with an adjustment coefficient, i.e., the so-called thickness factor  $\theta_0$  which depends upon wall thickness  $h$  and initial strainless tube radius  $R_0$  and thickness  $h_0$ .

Such a theoretical approach, which agrees with experimental data, enables us to write the  $P(R)$  relationship for rubber tubes as :

$$P = \frac{2}{3} \rho c_0^2 \theta_0 \left( 1 - \frac{R_0^4}{R^4} \right) \quad (1)$$

with

$$c_0^2 = \frac{E_0 h_0}{2 \rho R_0} \quad \text{and} \quad \theta_0 = \frac{1 + \frac{h_0}{2 R_0}}{\left( 1 + \frac{h_0}{R_0} \right)^2}.$$

In the expressions above,  $E_0$  is the static Young's modulus of the wall material, which is presumed to be linear isotropic and incompressible, and  $\rho$  the specific mass of the fluid within the tube.

The classical theory of elasticity [19, 20] enables us to write the azimuthal stress  $\sigma_\theta$  as  $\sigma_\theta = PR/h = G(\lambda_\theta^2 - \lambda_r^2)$ , where  $G$  is the elastic shear modulus,  $\lambda_\theta = R/R_0$ ,  $\lambda_r = h/h_0$  and (since the material is incompressible)  $\lambda_z = 1/\lambda_r \lambda_\theta$ . Eliminating  $\lambda_r$ , and introducing  $\theta_0$  and  $C_0$  this relation can be rewritten as :

$$P = \frac{2}{3} \rho c_0^2 \theta_0 \frac{1}{\lambda_z} \left( 1 - \frac{1}{\lambda_z^2} \frac{R_0^4}{R^4} \right). \quad (2)$$

**2.1.2 Static behaviour with negative transmural pressure.** — In this alternative, the relationship between pressure and section area  $S$  is made dependent on collapse pattern. So far, several relations have been put forth as far as initial collapse conditions are concerned [21]; we shall only concentrate on the collapse process which originates from a critical pressure, i.e., the lateral buckling pressure.

Such a process induces, first, a quasi-elliptical tube shape, then it gives way to a two-lobe mode, up to a stage where those lobes meet at a contact point when contact pressure is achieved ; finally, with lower transmural pressure, a contact line develops, making the main duct branch off into two distinct ducts.

The time sequence of the section shape seen as above vs. pressure, has already been dealt with by several authors [22-24]; different relationships have been established in order to relate negative transmural pressure to the section area. Among those we may single out [22] :

$$P = \frac{E_0 \left( \frac{h_i}{R_i} \right)^3}{12} \frac{1}{1 - \sigma^2} \left[ 1 - \left( \frac{S_i}{S} \right)^{3/2} \right] \quad (3)$$

if  $1 < \frac{S_i}{S} < 4$

and  $P = - \frac{E_0 \left( \frac{h_i}{R_i} \right)^3}{12} \frac{1}{1 - \sigma^2} \left( \frac{S_i}{S} \right)^{3/2}$

if  $\frac{S_i}{S} > 4$ .

In this expression  $\sigma$  is the Poisson's ratio of the wall whereas  $h_i$ ,  $R_i$ ,  $S_i$ , stand for wall thickness, tube radius and section area respectively, at zero-transmural pressure and at a given longitudinal extension  $\lambda_z$ .

If an explicit expression for pressure vs.  $\lambda_z$  is sought for an incompressible material, relationships identical to (3) can be formulated by substituting  $S_0/\lambda_z$  for  $S_i$ , where  $S_0$  is the section area at zero stress level.

**2.1.3 An approach to the dynamic rheological behaviour.** — When considering biological materials such as those constituting blood vessels and many other bioelastomers, viscoelastic rheological properties must be taken into account. The principal characteristics of these properties are revealed by a frequency-dependent and complex Young's modulus. However, in the particular case of blood vessel walls, within a low and narrow frequency range between 1 and 15 Hz, the imaginary part  $\text{Im}(E)$  of the complex elasticity coefficient is of at least one order of magnitude smaller than  $\text{Re}(E)$  its real part [25, 26]. For describing the law of the mechanical tube behaviour in such a case ( $\text{Im}(E) \ll \text{Re}(E)$ ) a method could be found by using relationships (1) to (3) where the  $E_0$ -modulus is replaced by  $E_n$ , the modulus of the complex elasticity coefficient. Moreover, it will be assumed that, with a static longitudinal extension and periodic time variation of the positive transmural pressure such as :

$$P(t) = P_0 + \sum_{n=1}^N P_n \cos(n\omega_0 t + \psi_n) \quad (4)$$

the generated strains induce a time variation of the radius which can be written as [27] :

$$R(t) = \frac{R_0}{\lambda_z^{1/2}} \left[ 1 - \frac{3 R_0 \lambda_z}{\theta_0 h_0} \left( \frac{P_0}{E_0} + \sum_{n=1}^N \frac{P_n}{E_n} \cos(n\omega_0 t + \psi_n - \phi_n) \right) \right]^{-1/4}. \quad (5)$$

In (5),  $E_0$  and  $E_n$  are the values of the modulus of the complex elasticity coefficient at  $\omega = 0$  and  $\omega = n\omega_0$  respectively, whereas  $\phi_n$  stands for its phase value at  $\omega = n\omega_0$ .

This quasi-heuristic relation has been obtained in order to have a simple expression for  $R(t)$  in the case of periodic pressure excitation with large amplitude. It gives an accurate relationship whenever either a small amplitude oscillating pressure excitation or static stresses of large amplitude are applied.

In the particular case of negative transmural pressure a similar relationship would be :

$$S(t) = \frac{S_0}{\lambda_z} \left[ 1 - 12 \left( \frac{R_0}{h_0} \right)^3 (1 - \sigma^2) \left( \frac{P_0}{E_0} + \sum_1^N \frac{P_n}{E_n} \cos(n\omega_0 t + \psi_n - \phi_n) \right) \right]^{-2/3} \quad (6)$$

if  $1 < \frac{S_0}{S\lambda_z} < 4$

and

$$S(t) = \frac{S_0}{\lambda_z} \left[ - 12 \left( \frac{R_0}{h_0} \right)^3 (1 - \sigma^2) \left( \frac{P_0}{E_0} + \sum_1^N \frac{P_n}{E_n} \cos(n\omega_0 t + \psi_n - \phi_n) \right) \right]^{-2/3}$$

when  $\frac{S_0}{S\lambda_z} > 4$ .

2.2 EXPERIMENTAL STUDIES.

2.2.1 *Direct measurements on a material sample.* — With a view to comparing various approaches as far as mechanical properties of duct walls are concerned, a rheological testing apparatus had to be devised whereby direct measurement of the characteristics of a non-linear viscoelastic material could be carried out. The apparatus we use (as shown in Fig. 1) includes a vibration electromagnetic excitor which generates sinusoidal stresses at one end of the sample, for which measurements are made by means of a strain gauge load cell. Furthermore, related extensions are monitored through an optical device.

Analog signals are digitized and stored on a data processing system which enables one to compute strains, stresses, phase and amplitude of the incremental dynamic complex Young's modulus within a frequency range from  $10^{-4}$  to  $10^2$ , as well as the static modulus.

To achieve model experiments an elastomeric silicone tube was selected; it is 10 m long and takes a cylindrical shape when inflated at slightly positive transmural pressure, with a  $10^{-2}$  m internal radius and a  $5 \times 10^{-4}$  m thickness. Tests which we have performed on both azimuthal and longitudinal excised samples, have shown a quasi-linear and isotropic static behaviour up to 30 % elongation, while

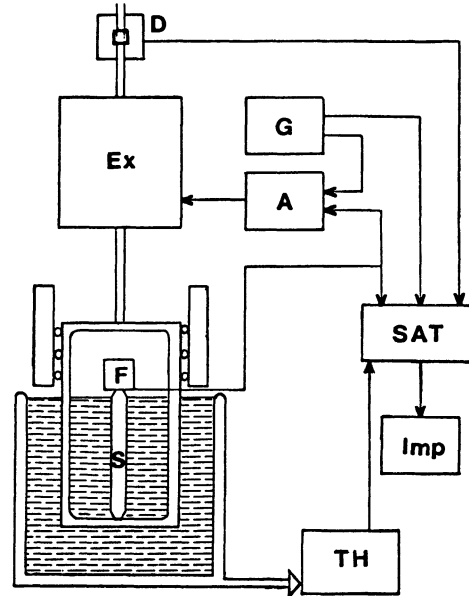


Fig. 1. — Schematic diagram of the experimental set-up. G : low frequency signal generator; A : power amplifier; Ex : vibration excitor; temperature control device (tank) : TH; SAT : signal processor; F : force transducer; D : displacement transducer; Imp : Impression of results; S : sample under test.

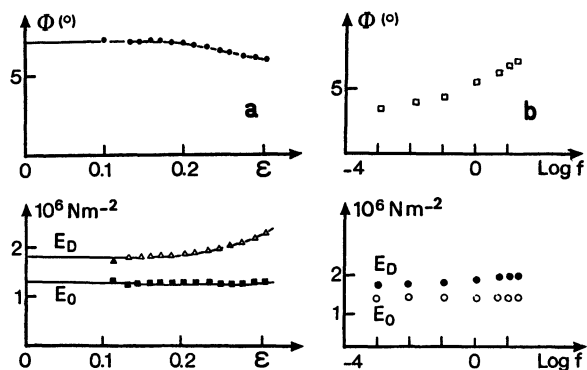


Fig. 2. — Rheological behaviour of the elastomeric silicone sample : experimental results.

a) Non-linear behaviour ( $f = 10$  Hz). The non-linear effects mainly affect the phase and the amplitude of the incremental dynamic complex Young's modulus.

b) Viscoelastic behaviour (uniaxial extension strain  $\epsilon = \lambda - 1 = 0.1$ ).

This viscoelastic behaviour is correlated with a quasi-linear dependency of the Young's modulus vs. the logarithm of the frequency.

viscoelastic effects were observed throughout every dynamic test (See Fig. 2).

From the knowledge of the Young's modulus, thus derived, and of geometrical characteristics of the tube we can theoretically infer correlations between pressure and diameter on one hand, and between

pressure and tube section area, on the other. We shall now proceed to compare such deductions with experimental data obtained as follows.

**2.2.2 Measurements of rheological static behaviour of the tube wall.** — In order to achieve experimental results of static relationships between pressure vs. section area or apparent diameter, under the same conditions as those required for propagation experiments (Part II), the tube is placed on a horizontal plane in a given longitudinal strained state. The inside of the tube, initially empty of fluid, is then filled with predetermined volume increments of water.

We can then proceed to infer the internal section area  $S$  of the wall assuming a uniform cross section, provided we have negligible end effects. Such a condition has been verified by direct measurements on a tube of identical section but different length.

Both transmural pressure  $P$  (at the lower level inside the lumen), and the apparent external duct diameter  $H$ , are measured by means of a cathetometer. The results establish relationships of the type  $P(S)$  and  $P(H)$  as shown in figure 3 (a, b),  $H$  being a prevailing parameter easily obtainable during experiments with hydrodynamic models.

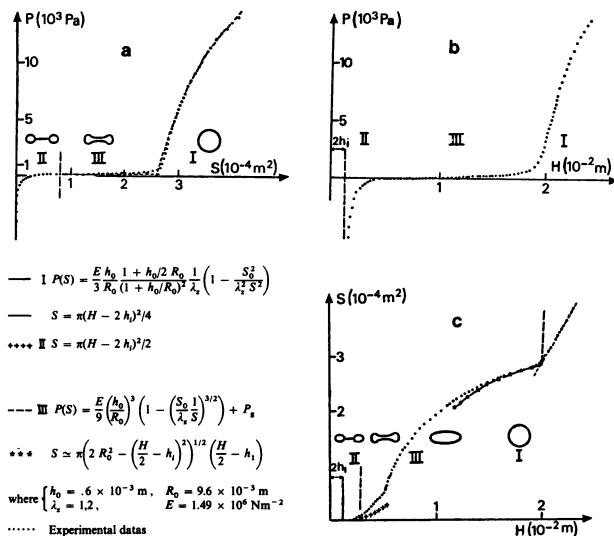


Fig. 3. — Mechanical behaviour of the collapsible tube.  
 a) Pressure-area relationship, compared with the results of a theoretical model.  
 b) Pressure-apparent diameter : under the same experimental conditions.  
 c) Area vs. apparent diameter, and theoretical relationship taking into account the shape of the section.

Experimental data concerning relation  $S(H)$  are also presented (see Fig. 3c). In such diagrams it is necessary to define chiefly three ranges :

— A part I where the tube is circular, the relation  $S(H)$  being almost parabolic :

$$S \cong \pi(H - 2 h_i)^2/4$$

and the thickness being arbitrarily fixed at its initial value,  $h_i$  ( $h_i = h_0$  if  $\lambda_z = 1$ ). Besides, the  $P(S)$  relationship can be approached by the analytic relation (2) with  $\frac{R_0^4}{R^4} = \frac{S_0^2}{S^2}$ .

— A second part where the lumen of the duct is reduced to a pair of cylindrical beams, both being parallel and quasi-circular. In this part II, thus delimited, the relation  $S(H)$  is again nearly parabolic and can be written as :

$$S = \pi(H - 2 h_i)^2/2.$$

A third area can be ascertained where the duct is also collapsed, although no contact line materialized as yet. In this part III the experimental relation  $p(H)$  is noticeably linear. Such a linear variation does not derive from the hydrostatic pressure effect but its slope can be attributed essentially to the azimuthal bending effect of the tube wall. Moreover, for the lower values of the absolute value of  $P$ , the section of the tube is quasi-elliptic and the  $S(H)$  relation can be approximated by :

$$S = \pi \left( 2 R_0^2 - \left( \frac{H}{2} - h_i \right)^2 \right)^{1/2} \left( \frac{H}{2} - h_i \right).$$

Agreement between experimental results, on one hand, and theoretical curves, on the other, is outstanding (Fig. 3a). It must be remembered that in order to draw such theoretical curves, the Young's modulus value used has been selected in accordance with results of direct rheological measurements on a wall material sample. Also, it will be noted that the relation (3) has been modified, by introducing a  $P_g$  constant factor which takes into account the location of the pressure measurements on the lower part of the tube section. It must be pointed out that analytic relations account inaccurately (or very little) for the part that connects the previous ones ; for instance the part corresponding to the critical lateral buckling pressure (the value of which has been shown [26] to be  $- Eh/(4 R^3(1 - \sigma^2))$  in disagreement with the previous relationship (3)).

**2.2.3 Measurements of dynamic rheological behaviour.** — It has been emphasized that the dynamical behaviour of such a structure is, to a large extent, dependent on frequency insofar as Young's modulus is frequency dependent. This dependence, similar to that of arterial wall material, must not be overlooked. Therefore, it is worth trying to experimentally assess how such dependence can be translated to dynamical pressure-radius relationships.

The experimental apparatus that was set up included a harmonic pressure generator linked to a sufficiently small length section (with regard to the wavelengths of the pressure wave) of the distensible tested tube, so that propagation effects are negligible. Simultaneous measurements of pressure and diameters were made,

for different positive values of mean transmural pressure. The experiments were carried out with a harmonic small pressure signal of 1 Hz. One can verify the validity of equation (5) by computing the derived dynamic Young's modulus of the material as follows :

$$P(t) = P_0 + P_1 \cos(\omega t + \phi)$$

and

$$R(t) = R_1 + R_2 \cos \omega t \tag{7}$$

$$E_1 = \frac{P_1}{R_2} \frac{R_1^5}{R_0^5} \frac{3 R_0^2}{4 \theta_0 h_0} (\cos \phi) \lambda_z^3.$$

Experimental values thus obtained have been compared with those directly derived from rheological tests on material samples (see Fig. 4). We can point out the quite fair agreement between the results of both methods, which indirectly validates the above-mentioned expression (5).

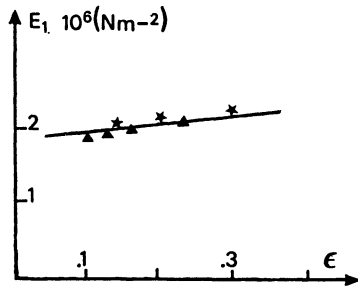


Fig. 4. — Example of data for direct (▲) or indirect (★) measurements of dynamic Young's modulus (1 Hz) as a function of the uniaxial extension strain  $\epsilon = \lambda - 1$  (rubber-like silicone tube, cf. characteristic Fig. 3).

In conclusion, the dynamic behaviour of a viscoelastic tube with reasonably small viscous effect can be approximately described with an equation similar to that applied to static behaviour, provided that the static Young's modulus  $E_0$  is to be superseded by the dynamic quantity  $E_1$ .

### 3. Propagation of small amplitude waves.

3.1 THEORETICAL CONSIDERATIONS. — Since a number of authors have already extensively studied low amplitude pressure wave propagation in distensible tubes, we shall only briefly review the subject here. Whenever the transmural pressure is positive, the simple geometry of the system enables us to obtain precise analytical solutions in the case of a thin wall, and wavelength large compared with the tube radius [29, 30].

Moreover, theories have been put forth in the case of thick wall tubes, for which longitudinal bending resistance and shearing effects can be neglected [31].

A significant result emerging from these diverse theories can be written as follows : for a given small amplitude pressure wave (oscillation frequency  $\omega$ , fluid kinematic viscosity  $\nu$ ) associated with a frequency parameter value  $\alpha = R \left( \frac{\omega}{\nu} \right)^{1/2} > 10$ , the thickness

$\left( \frac{\nu}{\omega} \right)^{1/2}$  of the oscillating viscous boundary layer becomes negligible with respect to the tube radius  $R$ , so that the induced flow can be described as a unidimensional potential flow of an inviscid fluid.

With a partially collapsed tube, the complex geometry of the lumen no longer yields simple analytic answers. However, if the hypotheses of unidimensional flow and an inviscid fluid are made, and longitudinal tapering effects neglected, then the phase velocity  $C$  of the pressure wave can be deduced from knowledge of the pressure section  $P(S)$  relationship, through the classical equation of wave phenomena in continuum media (of specific mass) :

$$C^2 = \frac{S}{\rho} \frac{dP}{dS} \tag{8}$$

Such an equation, valid whether with positive or negative transmural pressures, raises issues that hinge on the use of the  $P(S)$  relationship when considering dynamic effects. Indeed, in (8) the dynamic  $\frac{dP}{dS}$  expression rather than the static one should be used. In the case of positive transmural pressure, that leads to the classical Moens-Korteweg expression for the wave velocity i.e. :

$$C^2 = \frac{E_d h}{2 \rho R} \tag{9}$$

where  $E_d$  is the dynamic Young's modulus value for the wave frequency  $\omega$  considered, while  $h$  stands for the thickness of the wall. In other words, we assume that for small dissipative viscoelastic effects (imaginary part of the viscoelastic Young's modulus negligible compared with its real part) one may write :

$$\left( \frac{dP}{dS} \right)_\omega = \frac{E_d(\omega)}{E_0} \left( \frac{dP}{dS} \right)_0 \tag{10}$$

Therefore, the relationship (8) will be written as :

$$C^2 = \frac{S}{\rho} \frac{E_d(\omega)}{E_0} \left( \frac{dP}{dS} \right)_0 \tag{11}$$

Eventually a further point should be raised concerning the previously-mentioned analytical relationship used to represent static  $P(S)$  relationships. Taking into account the remarks concerning dynamic effects (11) it is possible from such relations (2, 3) to infer the phase velocity  $C$  of pressure waves :

$$\frac{C}{C_1} = \left(\frac{\theta_0}{3}\right)^{1/2} \frac{S_0}{S\lambda_z} \quad \text{for } P > 0$$

$$\frac{C}{C_1} = \frac{h_0}{R_0} \left(\frac{S_0}{S\lambda_z}\right)^{3/4} \quad \text{for } P < 0 \text{ and } 4S > S_0 \quad (12)$$

where

$$C_1^2 = \frac{E_d h_0}{2 \rho R_0}$$

It will be noted that, on the diagram representing  $\frac{C}{C_1}$  versus  $\frac{S}{S_0}$  as drawn on figure 5 (for  $\lambda_z = 1$ ), there is a theoretical discontinuity corresponding to a change of the tube law : in the vicinity of  $\frac{S}{S_0} \simeq 1$ . In particular, noticeable in this domain is the inadequate fit of the analytical relations (2, 4) with experimental  $P(S)$  data. It is hence necessary in this case around  $P = 0$  to calculate  $C(S)$  from (11) using an experimental polynomial expression for  $P(S)$  obtained by fitting a curve to the data.

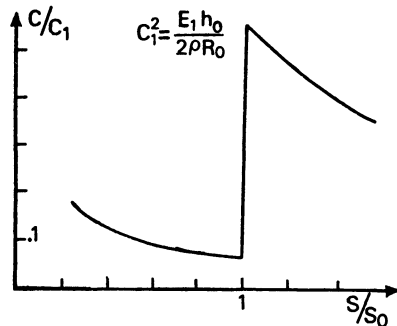


Fig. 5. —  $C(S)$  curve inferred from theoretical relationships (12) for  $\lambda_z = 1$ .

**3.2 EXPERIMENTAL RESULTS.** — When studying wave propagation along distensible tubes with positive transmural pressures where the lumen presents a circular section, the experimental methods that are used are generally straightforward. They usually gave accurate data which were easy to interpret and a fair agreement was generally found between such data and theories previously mentioned [32-35]. On the contrary, as far as partially collapsed tubes are concerned, experimental processes are far more difficult to put into effect, since the highly non-linear properties of the system force the experimenter to characterize propagation phenomena of pressure waves using extremely low amplitude ( $< 10^2$  Pa). For instance Bonis and Ribreau [36] have measured wave speed in initially elliptical collapsible tubes. The method they have used (fluid anemometry techniques), the material properties (purely elastic) of the tube they have tested and the geometry of the apparatus (annular external

duct) make difficult a comparison between the results they have obtained and the results obtained in the present work. Three different experimental methods we used will be now reviewed.

**3.2.1 Propagation of sinusoidal pressure waves of low amplitude.** — An electromagnetically driven pump was used to generate pressure waves of small enough amplitude that the unsteady induced stress in the wall can be assumed as resulting from a locally linear  $P(R)$  relation. If the system can be considered as free from reflection, wavelengths of the one-way propagating pressure waves can be measured by taking photographs of the instantaneous apparent profiles of the tube. Knowing the frequency  $\omega$  of the wave, the phase velocity is easily obtained for such a medium of sufficiently slight dispersion.

Nevertheless, it is necessary in such a method to get rid of reflected waves by a large increase of the test section length and the addition of a matched section downstream (with characteristic impedance). Moreover, the wall displacements (mostly radial due to the longitudinal friction between the tube and the support) being extremely small, they cannot be accurately measured, since the static profile of the tube is not exactly uniform.

**3.2.2 Propagation of a pressure impulse of low amplitude.** — Two displacement transducers (optical ones, for instance) were located on either side of an arbitrary position where the impulse wave was generated (Fig. 6a), at the respective distances of  $l$  and  $2l$ . Assuming a non (or a slightly) dispersive medium, and no fluid motion initially, it is then possible to compute the wave speed knowing the time delay  $\tau$  between the responses  $H_1(t)$ ,  $H_2(t)$  of the two transducers. However  $\tau$  can be small compared to the time duration of the impulses, and can be difficult to evaluate since

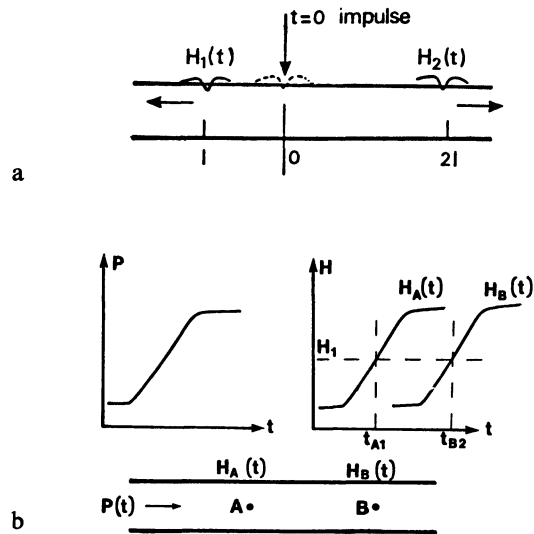


Fig. 6. — Principle of the experimental set-up used to measure the wave velocities.

$H_2(t - \tau)$  is different from  $H_1(t)$ , depending on the nature of the impulse (symmetry) and the dispersive properties of the medium. It may then be necessary to correlate  $H_1$  and  $H_2$  in order to evaluate  $\tau$ . Such a method means that we do not need to know exactly the initial time of the impulse.

**3.2.3 Finite amplitude wave propagation as a superposition of small amplitude wavelets.** — When considering a finite amplitude pressure wave propagating and passing through two points A and B, with  $\overline{AB} = l$ , (Fig. 6b) and assuming this wave to be a superposition of small amplitude waves, the velocity of each wavelet can be computed as :

$$C_1 = l/(t_{B1} - t_{A1}) = C(H_1).$$

The related experimental results are shown figure 7. From the experimental data (Fig. 3b), we have computed with (11) the wave speed as a function of the pressure and hence of  $H$ . The best agreement is found for positive transmural pressures. For lower values of the pressure, the inhomogeneity of the shape of the tube makes it difficult to obtain a very precise measurement of  $H$  and hence of the wave speed. The lack of data for the lowest values is related to the experimental inaccuracy of the wave speed measurements in this range of pressure where the tube is collapsed in two separate ducts. Due to the relatively large ratio between the thickness and the radius of the tube, and the scale of the inhomogeneities, and due to the damping of the waves, the inaccuracy of the measurement of the wavelengths is too large to allow any interpretation of the results in this range of pressure.

#### 4. Conclusion.

In the past, a great deal of progress had been made in the quantitative modelling and understanding of wave phenomena with fluid inside inflated elastic tubes. However, a generalization of such studies to the case of collapsed viscoelastic tubes was missing.

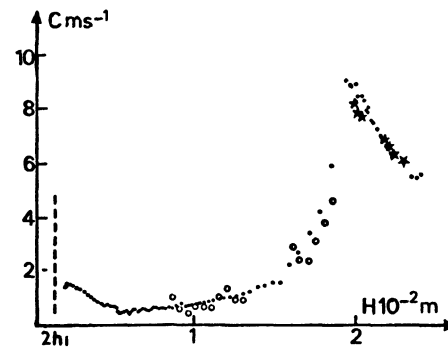


Fig. 7. — Wave speed as a function of apparent diameter  $H$ . ● Values obtained using the theoretical relationship  $C^2 = \frac{S}{\rho} \frac{E_1(\omega)}{E_0} \frac{\Delta P}{\Delta S}$ ; ★ experimental results (small amplitude wave propagation); ○ experimental results (finite amplitude wave propagation).

In order to take into account the viscoelastic properties of the tube wall material, and to give an interpretation to the mechanical behaviour of the vessel, a dynamic Young's modulus had to be introduced in place of the static modulus. Moreover when dealing with a tube in its collapsed state, we can suggest using such a treatment, which is based on the notion of dynamic distensibility of the tube and its relation to the speed of propagation of pressure waves. Nevertheless, the basic hypothesis all along the development of such a theory is the linearization of the incremental dynamic rheological behaviour which implies application to the wall material of small, time-dependent strains and stresses. In other words, we were necessarily placed under the conditions of small amplitude wavelet propagation. We have experimentally shown then that a fairly good agreement arises from direct rheological tests, and indirect wave propagation velocity measurements.

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