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The non-measurability of nuclear energy level shifts due to pion diamagnetism in the Mössbauer effect

Y. Avron

Department of Physics, Technion-Israel Institute of Technology, Haifa, Israel

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Résumé. — Le potentiel neutron-proton devient plus faible dans un champ magnétique (diamagnétisme du pion). Dans un noyau, ceci produit un déplacement des niveaux d'énergies qui diffère de l'effet Zeeman. Nous montrons que la dépendance de ce déplacement est quadratique dans le champ magnétique et qu'elle est ainsi trop faible pour être observée dans l'effet Mössbauer.

Abstract. — The neutron-proton potential weakens in external magnetic fields (diamagnetism of the pion). This leads to shifts in nuclear levels distinct from the Zeeman splitting. It is shown that the dependence on magnetic fields B is quadratic and so is too weak to be observable in the Mössbauer effect.

Interactions mediated by charged spinless particles weakens in external gauge fields. This is a consequence of the diamagnetic property of bosons [1, 2] : their propagator in an external gauge field is bounded by the free propagator [2, 3].

Does this general principle have observable experimental effects in the abelian (i.e. magnetic fields) case ? The purpose here is to answer the above question in the negative. Such diamagnetic effects are too weak to be measurable for any reasonable field strength.

The scale for magnetic phenomena is given by the square of the mass of the exchanged particle. Since π^\pm are the lightest spin zero charged particles it is natural to consider the weakening of the neutron-proton effective potential with magnetic fields. With m_π the pion mass the scale for B is

$$B_\pi^* = m_\pi^2 c^3 / \hbar e \sim 3.3 \times 10^{18} \text{ G} .$$

For B fields small on this scale one expects shifts of nuclear levels energies, while for B fields large on this scale dramatic phenomena may occur such as the disintegration of certain nuclei.

We shall not consider the regime of large B for the following reasons : (a) The largest fields believed to occur in nature are those in certain neutron stars where $B \leq 10^{14}$ G. (b) Although there have been theoretical speculation regarding fields of 10^{20} G fields larger than 10^{14} G are expected to be unstable to photoproduction [4]. (c) Large fields may cause many additional complications [5].

In spite of the largeness of B_π^* , the Mössbauer effect is sufficiently sensitive to linear changes in B/B_π^* , even for magnetic fields as small as a few tens of gauss. This is readily seen by recalling that the pion is roughly 7 times lighter than the proton, and so the magnetic field effects on the pions are comparable to the nuclear Zeeman effect. Since the nuclear Zeeman splitting is easily observed in the Mössbauer spectrum [6] energy shifts due to the pion's interaction with the magnetic fields that are linear in $|B|/B_\pi^*$ should also be observable as a kind of an isomer shift ⁽¹⁾. On the other hand, energy shifts that are quadratic in B/B_π^* are too small to be observable in the Mössbauer effect.

We shall first give an argument suggesting a linear dependence on $|B|$ and then explain why the argument is false. We shall then consider a caricature of a Yukawa theory where the B dependence of the potential is explicitly calculated and for which a B^2 dependence of the energy levels shifts can be shown.

In the naive Yukawa theory the heavier the pion the weaker the nuclear force. Roughly, this is what the magnetic field does for the charged pions. (This is one intuition behind diamagnetism.) A naive way to estimate an effective pion mass depending on B is as follows : the pion in a magnetic field gains the zero point energy due to the lowest Landau levels which is linear in $|B|$. This suggests $m_\pi^2(B) \simeq m_\pi^2 + e\hbar |B|/c^3$.

⁽¹⁾ We shall disregard the possible dependence of the gyromagnetic ratio on B .

The nonanalytic behaviour at $B = 0$ is a consequence of the fact that a homogeneous magnetic field is a singular perturbation (with x^2 dependence) of the free Hamiltonian. If this argument were correct, there would be energy levels shifts linear in $|B|$.

This argument is, however, false. The reason is that m_π^{-1} is much smaller than the Larmour radius and so the pion never goes sufficiently far for the zero point energy to build up. To see this in more details consider a gauge invariant, one pion exchange theory [7]

$$\mathcal{H}_1 = -\lambda\delta(\mathbf{x})|\varphi(\mathbf{x})|. \quad (1)$$

The nucleon potential in the static limit [7] is

$$\begin{aligned} U_B(\mathbf{x}) &= -\frac{\lambda^2}{2} \left| \frac{1}{(i\nabla + \mathbf{A})^2 + m_\pi^2}(\mathbf{x} + \mathbf{y}, \mathbf{y}) \right| \\ &= -\frac{\lambda^2}{2} \left[-\Delta + \frac{B^2}{4}(x^2 + y^2) + m_\pi^2 \right]_{L_z=0}^{-1}(\mathbf{x}, 0) \\ &\leq 0. \end{aligned} \quad (2)$$

Note that the first line in eq. (2) is gauge invariant for arbitrary A and gives a translation invariant potential for homogeneous magnetic fields. The second line in (2) for homogeneous fields B follows from gauge invariance and the vanishing of the Landau orbits for $L_z \neq 0$ at the origin. Eq. (2) shows explicitly (e.g. by the Wiener integral) that $-U_B(\mathbf{x})$ is a pointwise decreasing function of B which for $B = 0$ reduces

to the usual Yukawa potential. To estimate the change in $U_B(\mathbf{x})$ write :

$$\begin{aligned} U_B(\mathbf{x}) - U_0(\mathbf{x}) &= \frac{\lambda^2 B^2}{2} \times \\ &\times \left(\frac{1}{-\Delta + m_\pi^2}(x^2 + y^2) - \frac{1}{-\Delta + \frac{B^2}{4}(x^2 + y^2) + m_\pi^2} \right)(\mathbf{x}, 0). \end{aligned} \quad (3)$$

Using the pointwise domination of the resolvent with $B \neq 0$ by the resolvent with

$$B = 0 \quad \text{and} \quad x^2 + y^2 \leq x^2 + y^2 + z^2$$

one finds

$$|U_B(\mathbf{x}) - U_0(\mathbf{x})| \leq \lambda^2 B^2 / (8 \pi m_\pi^2). \quad (4)$$

By first order perturbation theory, the right hand side of eq. (4), multiplied by the number of neutron-proton pairs is an upper bound on the energy shifts of nuclear levels. This shows that the diamagnetism of the pion does not produce a shift of nuclear levels which is linear in B .

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