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THE SHELL MODEL FORM FACTORS WITH EFFECTIVE INTERACTIONS

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Résumé. — L'étude du facteur de forme de la diffusion inélastique de nucléons est faite avec deux types d'interactions effectives.

Abstract. — The form factors for inelastic scattering of nucleons by a target nucleus were studied using two different types of effective nucleon-nucleon interaction and the results were compared with those for the ordinary Gaussian potential.

1. Introduction. — One of the recent developments in the last few years has been the successful calculation of the effective interaction between nucleons in the nucleus in terms of the interaction between nucleons in the free space. It turns out that the effective interaction is density dependent [1], two nucleons interact differently in the presence of other nucleons than in the free space. The effective density dependent interaction was recently used in the Hartree-Fock calculations and a very satisfactory agreement with experiment was obtained [2], [3], [4].

It is natural to extend these studies to the case where one nucleon is unbound, namely elastic and inelastic scattering, and some steps in this direction have been taken [4].

The present work deals with the shell model form factors for inelastic scattering of nucleons by a target nucleus when effective nucleon-nucleon interactions are used. For this purpose the radial form factors (which are the only factors affected by the choice of the interaction in both the distorted wave Born approximation and the coupled equation technique) for the inelastic scattering process $O^{17}(n, n')O^{17}$ were calculated using two types of effective interactions. The effective nucleon-nucleon interactions used in the present work are Brink and Bocker effective interaction [5] and Nemeth and Bethe density dependent effective interaction [6].

2. Radial form factors for inelastic scattering. — The distorted wave amplitude for inelastic scattering from an initial channel $|i\rangle$ to a final channel $|f\rangle$ includes as a factor the nuclear matrix element $\langle f|v|i\rangle$. The same factor appears in the coupling terms when the coupled equation technique [7] is used. In the present model $V = \sum_i v_{in}$ where i refers to a

target nucleon and n to the incident nucleon. The effective nucleon-nucleon interaction is reasonably expected to be nonlocal and dependent upon the relative angular momentum of the interacting pair. Further, it may well depend upon the positions of the two nucleons within the nucleus as well as their relative separation; the effective interaction is different when both are imbedded in the nuclear matter from what it is when both are outside the bulk of the nucleus. In the present work we take the effective nucleon-nucleon interaction in the form

$$v_{in} = g_{00}(\mathbf{r}_i, \mathbf{r}_n) + g_{01}(\mathbf{r}_i, \mathbf{r}_n) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_n \\ + g_{10}(\mathbf{r}_i, \mathbf{r}_n) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_n \\ + g_{11}(\mathbf{r}_i, \mathbf{r}_n) (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_n) (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_n) \quad (1)$$

where \mathbf{r}_i and \mathbf{r}_n are the position vectors for the target nucleon i and the incident nucleon n respectively.

In calculating the nuclear matrix element, $\langle f|v|i\rangle$, one usually makes the following multipole expansion of the function $g_{ST}(\mathbf{r}_i, \mathbf{r}_n)$

$$g_{ST}(\mathbf{r}_i, \mathbf{r}_n) = \sum_{LM} g_{ST}^L(r_i, r_n) Y_L^M(\hat{r}_i) Y_L^M(\hat{r}_n)^* \quad (2)$$

Each term in this expansion can give rise to the transfer of orbital angular momentum L to the nucleus and corresponds to a parity change of $(-)^L$. In addition, the second and last terms of eq. (1) can give an additional transfer of $S = 1$ unit. The total angular momentum transfer J is then the vector sum of these

$$\mathbf{J} = \mathbf{L} + \mathbf{S}.$$

When $g_{ST}(\mathbf{r}_i, \mathbf{r}_n)$ contains truncated function

$$f(|\mathbf{r}_i - \mathbf{r}_n|),$$

which is zero for $|\mathbf{r}_i - \mathbf{r}_n| < d$ [4]; the multipole

expansion given by eq. (2) is not convenient. In this case it is more convenient to make the Fourier expansion of $g_{ST}(\mathbf{r}_i, \mathbf{r}_n)$ and to express the radial part $g_{ST}^L(r_i, r_n)$ in terms of the Fourier transform $\tilde{g}_{ST}(k)$ which may be given by

$$\tilde{g}_{ST}(k) = 4\pi \int j_0(kX) g_{ST}(X) X^2 dX. \quad (3)$$

In eq. (3), it is assumed that $g(\mathbf{r}_i, \mathbf{r}_n)$ contains only truncated functions of $|\mathbf{r}_i - \mathbf{r}_n|$ or functions of $\frac{1}{2}|\mathbf{r}_i + \mathbf{r}_n|$. The generalisation of $g(\mathbf{r}_i, \mathbf{r}_n)$ to include

both functions can be easily made, but we shall limit ourselves here to functions of the form $g_{ST}(|\mathbf{r}_i - \mathbf{r}_n|)$ or $g_{ST}(\frac{1}{2}|\mathbf{r}_i + \mathbf{r}_n|)$.

Now we define the spherical spin-orbit tensors [8]

$$T_{LSJ,\mu}(\hat{r}, \sigma) = \sum_{MM_S} i^L Y_L^M(\hat{r}) S_{SM_S}(\sigma) \langle LM, SM_S | J\mu \rangle \quad (4)$$

with $S_{00} = 1$ and $S_{1\lambda} = \sigma_\lambda$. The effective nucleon-nucleon interaction may be written in terms of the spherical tensors, given by eq. (4), as

$$v_{in} = \frac{2}{\pi} \sum_{\mu} (-)^{J+S-\mu-N} T_{LSJ,-\mu}(\hat{r}_i, \sigma_i) T_{LSJ,\mu}(\hat{r}_n, \sigma_n) O_{T,N}(\tau_i) O_{T,-N}(\tau_n) \int dk k^2 \tilde{g}_{ST}(k) j_L(kr_i) j_L(kr_n) \quad (5)$$

where $O_{00} = 1$ and $O_{1\lambda} = \tau_\lambda$. Using eq. (5), the nuclear matrix element for the transition $A(n, n')A'$ may be expanded as

$$\begin{aligned} \langle f | V | i \rangle &= \left\langle S_{n'} \mu_{n'}, t_{n'} N_{n'}, S_{A'} \mu_{A'}, t_{A'} N_{A'} \left| \sum_i v_{in} \right| S_n \mu_n, t_n N_n, S_A \mu_A, t_A N_A \right\rangle \\ &= \frac{2}{\pi} \sum_{LSTJ\mu N} (-)^{J+S-\mu-N} \langle S_{n'} \mu_{n'} | T_{LSJ,-\mu}(\hat{r}_n, \sigma_n) | S_n \mu_n \rangle \langle t_{n'} N_{n'} | O_{T,-N}(\tau_n) | t_n N_n \rangle \times \\ &\quad \times \int dk \left\{ k^2 j_L(kr_n) g_{ST}(k) \left\langle S_{A'} \mu_{A'}, t_{A'} N_{A'} \left| \sum_i j_L(kr_i) T_{LSJ,\mu}(\hat{r}_i, \sigma_i) \right| S_A \mu_A, t_A N_A \right\rangle \right\} \quad (6) \end{aligned}$$

where t_i, N_i, S_i and μ_i stand for the isospin, its projection, the spin and its projection, respectively, for the nucleon (i). The matrix elements appearing in the integrand of eq. (6) contain single particle operators which only connect configurations differing by at most one nucleon. If only one configuration in both initial and final states contributes to the transition, one can define a radial part [4], [8]

$$F_L^{ST}(r_n) = \int dk k^2 j_L(kr_n) \tilde{g}_{ST}(k) I_L(k) \quad (7)$$

which is the only factor affected by the choice of the interaction. $I_L(k)$ is given by

$$I_L(k) = \int dX X^2 U_2(X) U_1(X) j_L(kX)$$

where $U_i(r)$ are the radial parts of the single particle wave functions involved in the transition; the subscript (i) stands for the set of quantum numbers n_i, l_i, j_i .

3. Density dependent interaction. — We use a density dependent interaction [6] derived from Reid's soft core potential in the local density approximation. The interaction can be written as

$$v_{in} = f[\rho(\mathbf{R})] \delta(\mathbf{r}_i - \mathbf{r}_n) + v(r_{in}, \rho) \quad (8)$$

where \mathbf{R} denotes the centre of mass coordinates $\frac{1}{2}(\mathbf{r}_i + \mathbf{r}_n)$. In this expression v is a long range central force which depends on density in the triplet even

states only and it is mainly due to the second order tensor contribution. The form of v is given in reference [9].

The function f in eq. (8) was adjusted in such a way that the binding energy per nucleon in the nuclear matter is -16 MeV for the equilibrium density $\rho_0 = 10.8 \text{ fm}^{-3}$. The explicit form of the function f is given in reference [10] and the compressibility coefficient of the nuclear matter was taken to be

$$\rho^2 \frac{\partial^2 (E/A)}{\partial \rho^2} = 32 \text{ MeV}.$$

4. Results. — In order to examine the shell model form factors, calculated using effective interactions, on inelastic scattering of nucleons by a target nucleus, the reaction $O^{17}(n, n')O^{17}$ was considered. Two types of effective interactions, namely Brink and Boeker effective interaction and Nemeth and Bethe density dependent effective interaction were used to examine $F_L^{01}(r_n)$ and $F_L^{11}(r_n)$ for $L = 0, 2, 4$. To show the effect of the effective interaction on the radial form factors the results for the attractive Gaussian part of Brink and Boeker interaction were presented on figure 1.

It is known that $F_L^{ST}(r_n)$ are sensitive to the range of the interaction. With a zero range form of the interaction potential, F_L^{ST} are independent on L . As the range increases, the low multipole form factors for the attractive Gaussian potential becomes larger than those for the high multipoles while high multipoles

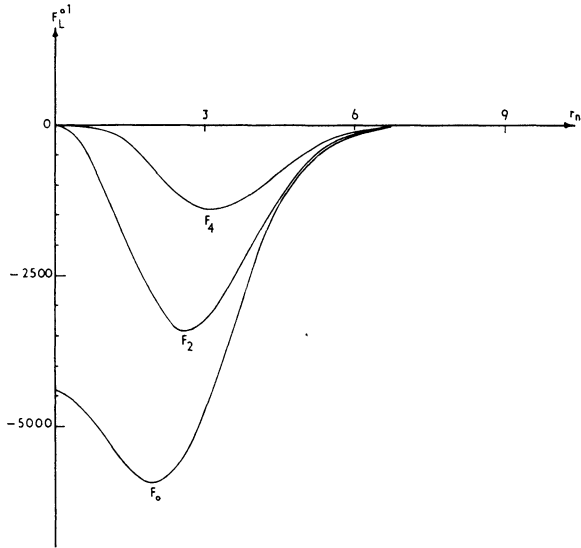


FIG. 1. — Radial form factors for a 1 $d_{3/2}$ neutron in O^{17} . The attractive Gaussian potential was used to calculate $F_L(r)$.

tend to peak at larger radii as shown in figure 1. These differences in both the shape and magnitude between the different multipoles, $F_L^{01}(r_n)$, are reduced when Brink and Boeker effective interaction was used but still the zero multipole form factor $F_0^{01}(r_n)$ has large value at $r_n = 0$ as shown in figure 2.

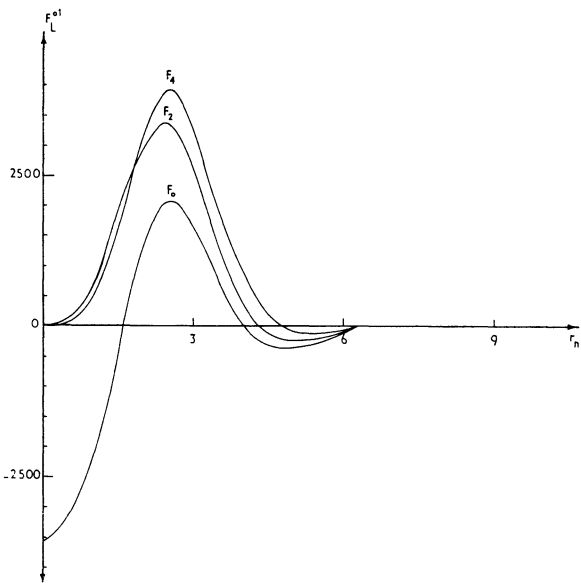


FIG. 2. — Radial form factors for a 1 $d_{3/2}$ neutron in O^{17} . Brink and Boeker effective interaction was used to calculate $F_L(r)$.

To calculate the effect of the density dependent effective interaction on the reaction $O^{17}(n, n')O^{17}$, the O^{17} nucleus was considered as an inert core O^{16} and a single nucleon outside a closed shell. The density $\rho(R)$ in eq. (8) is given by

$$\rho(R) = \sum_{\lambda=1}^A |U_\lambda(R)|^2$$

where $U_\lambda(R)$ are the radial parts of the single particle wave functions in O^{16} nucleus. The single particle wave functions were assumed to be oscillator wave functions whose oscillator constant chosen to produce the measured root mean square radius of the target nucleus [11].

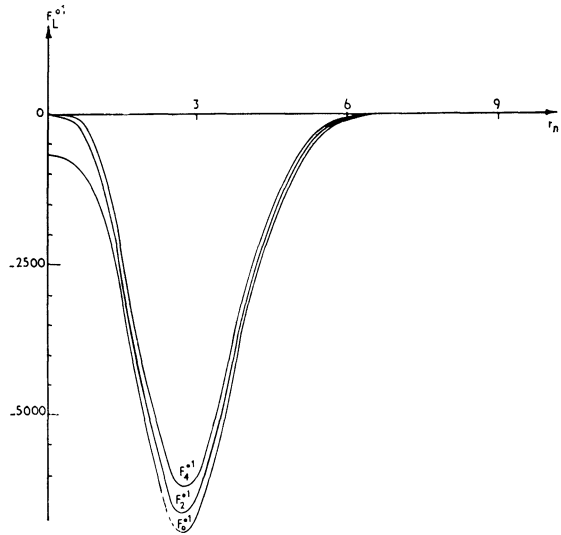


FIG. 3. — Radial form factors for a 1 $d_{3/2}$ neutron in O^{17} . Bethe and Nemeth density dependent effective interaction was used to calculate $F_L^{01}(r)$.

The density dependent effective interaction given by eq. (8) reduces strongly the contribution of $F_0^{01}(r_n)$ from the nuclear interior. This is expected since the density function, f , is very small inside the nucleus. For the two other multipoles F_2^{01} and F_4^{01} the density dependent interaction predicts very similar shapes as Brink and Boeker interaction except that the former tends to peak at slightly larger radius as shown in figure 3. The $F_L^{11}(r_n)$ components have the same features as the ordinary Gaussian potential (Fig. 4).

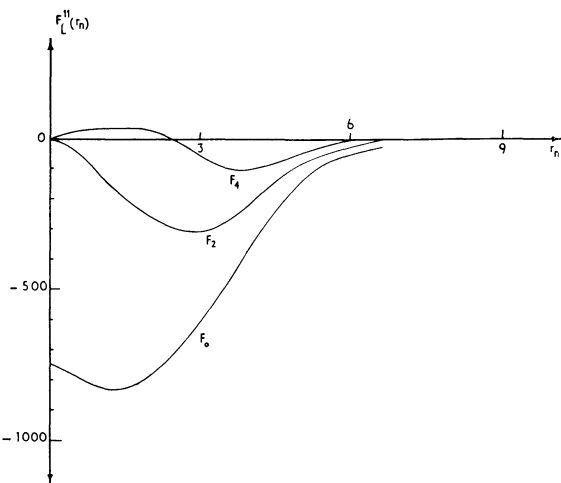


FIG. 4. — Radial form factors for a 1 $d_{3/2}$ neutron in O^{17} . Bethe and Nemeth density dependent interaction was used to calculate $F_L^{11}(r_n)$.

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