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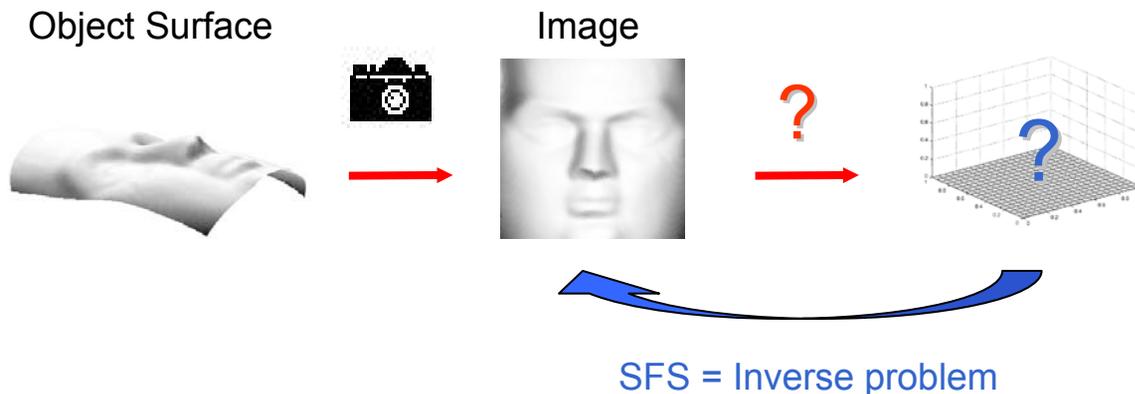
Viscosity solutions For Realistic Shape-From-Shading

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Workshop “Numerical Methods for Viscosity Solutions and Applications”
Rome, September 6-8, 2004

The Shape From Shading Problem



Problem:

- Inverse problem to image synthesis,
- Recover the surface(s) which yield(s) the same image.

Used Information:

- Shading.

Shape From Shading Data

- One image;
- Intrinsic parameters of the camera:
 - Focal length,
 - Size of the pixels
- Surface parameters:
 - Reflectance of the surface (albedo)
- Lighting parameters:
 - Flash...
- *Database of real images of faces available online:*
 - Experimental setup: scene illuminated by the flash of the camera.
 - Parameters are detailed.
 - WWW link:



<http://www-sop.inria.fr/odyssee/team/Emmanuel.Prados/>

Report:

- 1) Most of the SFS methods model the problem very basically:
Ex.: Camera=Orthographic Projection...
⇒ Not Realistic!
- 2) Lots of SFS algorithms require boundary data (generally Dirichlet BC)
⇒ Not Available with **Real Images!**



The results on real images are generally very disappointing.

Shape From Shading Contributions:

- We propose **more realistic modeling**:
Example: camera = pinhole
(perspective projection)
- We **reduce as much as possible** the requirement of **the boundary data**
(in particular on the boundary of the images).

Modeling / Mathematical formulations

■ The Basic Assumption: Lambertian Surface

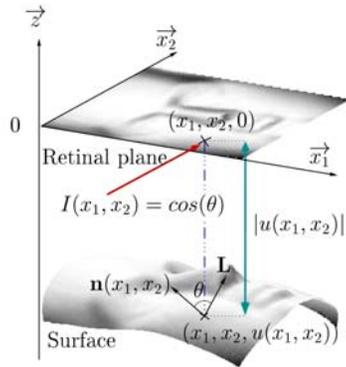
(without specularities or highlights)

⇒ Involves a connection between:

- the brightness of the image (intensity) : variable I ,
- the normal surface: \mathbf{n} , <<Depends on the parameterization>>.
- the lighting direction: \mathbf{L} <<Depends on the lighting>>.

$$I(X) = \cos(\mathbf{n}(X), \mathbf{L}(X)) = \frac{\mathbf{n}(X)}{|\mathbf{n}(X)|} \cdot \mathbf{L}(X)$$

Modeling / Mathematical formulations



■ Orthographic projection (classical modeling)

- Far light source:

$$I(x) \sqrt{1 + |\nabla u|^2} + \nabla u \cdot \mathbf{1} - \gamma = 0,$$

- Frontal far light source: (Eikonal equation)

$$|\nabla u| = f(x), \quad f(x) = \sqrt{\frac{1}{I(x)^2} - 1}.$$

■ More realistic Modeling: Perspective Projection:

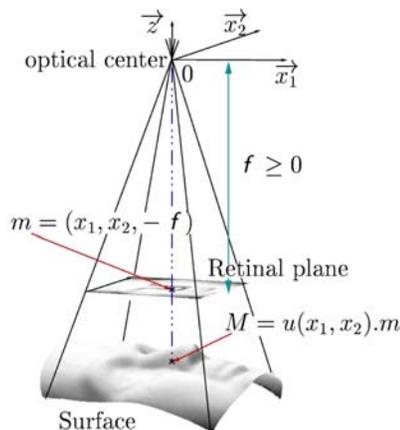
- Far light source:

$$I(x) \sqrt{f^2 |\nabla u|^2 + (x \cdot \nabla u + 1)^2} - (f \mathbf{1} + \gamma x) \cdot \nabla u - \gamma = 0,$$

- Light source located at the Optical center:

$$I(x) \sqrt{f^2 |\nabla u|^2 + (x \cdot \nabla u)^2 + Q(x)^2} - Q(x) = 0.$$

$$Q(x) = \frac{f}{\sqrt{|x|^2 + f^2}}.$$



A Generic Hamiltonian

- All these SFS equations are special cases of the “generic” equation:

$$\kappa_x \sqrt{|A_x \nabla \mathbf{u} + \mathbf{v}_x|^2 + K_x^2} + \mathbf{w}_x \cdot \nabla \mathbf{u} + c_x = 0 \quad (1)$$

Where $\kappa_x, K_x \geq 0$, $A_x \in \mathcal{M}_{2 \times 2}$,
 $\mu_x, \nu_x \in \mathbb{R}^*$, $\mathbf{v}_x, \mathbf{w}_x \in \mathbb{R}^2$ and $c_x \in \mathbb{R}$.

(1) is a HJ PDE of the form $H(x, \nabla u)$ with:

$$H(x, \mathbf{p}) = \kappa_x \sqrt{|A_x \mathbf{p} + \mathbf{v}_x|^2 + K_x^2} + \mathbf{w}_x \cdot \mathbf{p} + c_x.$$

$H(x, \mathbf{p})$ convex in \mathbf{p} .

Ill-posedness of the SFS problem

- SFS equations are of the form:

$$H(x, \nabla u) = 0, \quad x \text{ in } \Omega \quad (\text{a bounded set}).$$

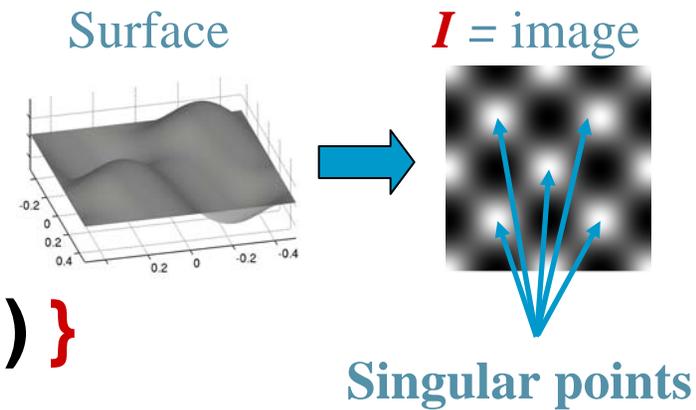
⇒ They require **Boundary Conditions** (BC) on $\partial\Omega$

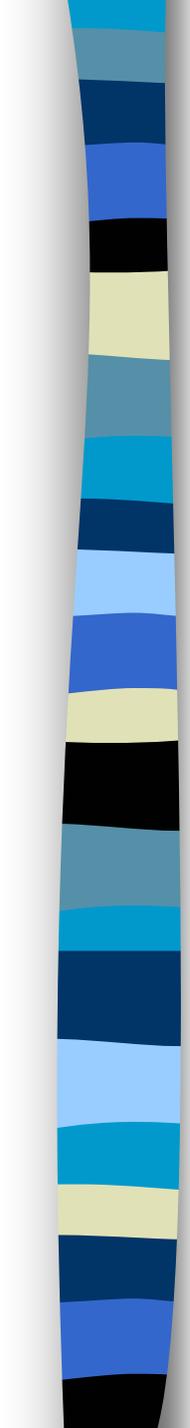
⇒ **But the BC on $\partial\Omega$ are not sufficient!**

SFS equations can have singularities:

*For all the previous SFS equations,
the set of singular points is*

$$\{ x \mid I(x) \text{ is maximal (i.e 1)} \}$$





Recall:

In Shape from Shading

Data = one image.

- ⇒ We do not have boundary data;
- ⇒ i.e. We do not know what are the adequate boundary conditions.

Solution proposed in the SFS literature

Notion	SFS Biblio.	Data (DBC)	Limitations
<p>Continuous Viscosity solutions (Crandall-Lions...)</p>	Lions/Rouy/ Tourin (92)	$S_{ing} \cup \partial\Omega$	<ul style="list-style-type: none"> Compatibility conditions. Dirich. data on $S_{ing} \cup \partial\Omega$
<p>C^1 Functions</p>	Dupuis/ Oliensis (94)	Set of local <i>Min.</i> of u	<p>Value function ? Few intuitive... But: Interest : Minimal Data are required!</p>
<p>Discontinuous Viscosity solutions (Ishii...)</p>	Prados/ Faugeras (03)	$N(S_{ing}) \cup \partial\Omega$	<ul style="list-style-type: none"> Dirichlet data on $N(S)_{ing} \cup \partial\Omega$
<p>Singular Viscosity solutions (Camilli-Siconolfi:99)</p>	Camilli/ Falcone (96)	$\partial\Omega$	<ul style="list-style-type: none"> not always adapted to SFS. still requires data on $\partial\Omega$

Contributions:

- We have slightly extended and modified the notion of singular viscosity solutions (Camilli/Siconolfi:99,02) in order to:
 - to extend the Dupuis/Oliensis' characterization of the C^1 solutions to the viscosity solutions \Rightarrow data are minimal.
 - such that compatibility conditions are not required.

Notion of SDVS

- Definition: We define a notion of solution
 - which extends the classical notions of
 - of the discontinuous viscosity solutions (Ishii...)
 - of the singular viscosity solutions (Camilli-Siconolfi:99,02).
 - Which allows to fix the values of the solutions at the singular points or on the boundary $\partial\Omega$ when we know them, and to send them at the infinity (state constraints), when we do not know them.

Properties of the SDVS:

- Existence of the SDVS
 - Dynamical programming principle,
 - value function;
 - Strong uniqueness;
 - Stability results;
- ➔ Application to the Shape From Shading problem...

Original theorems:

- Theorem: (roughly)
« A discontinuous viscosity solution without “local minima” is the SDVS».
- Corollary:
Characterization of the discontinuous viscosity solutions by their « minima ».

Numerical approximations:

- 4 stages:

- i) Management of the state constraints;

- ii) Regularization of the equations

- In SFS (we truncate the image, $I_\varepsilon = \min(I(x), 1 - \varepsilon)$);

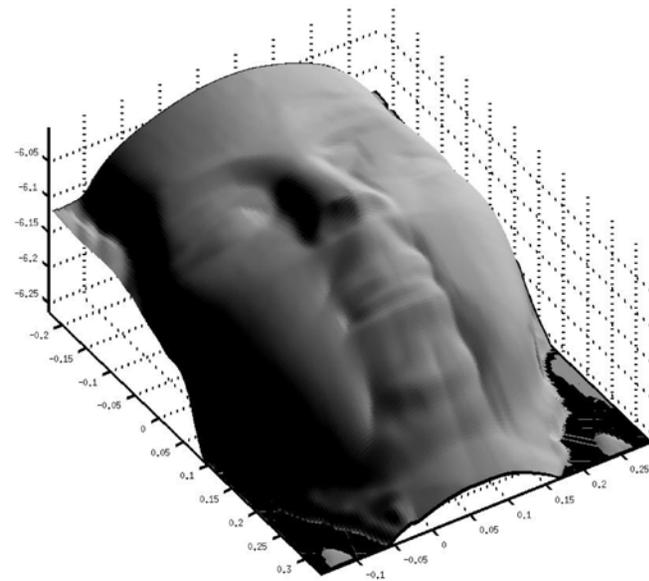
- iii) Designing of the approximation scheme;

- iv) Deduction and implementation of the associated algorithm.

A Numerical Result on a Synthetic Image (Mozart)

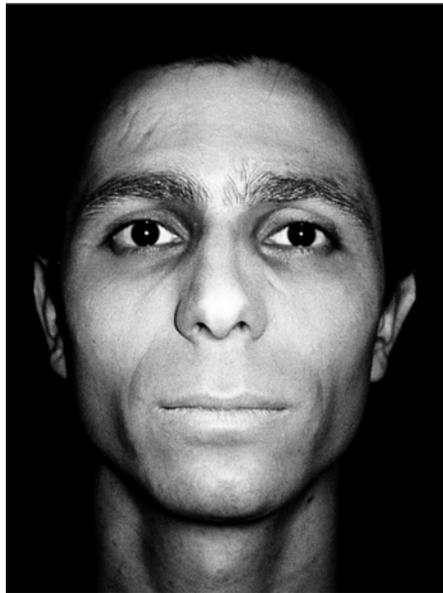


a) Synthetic image of the
classical Mozart face

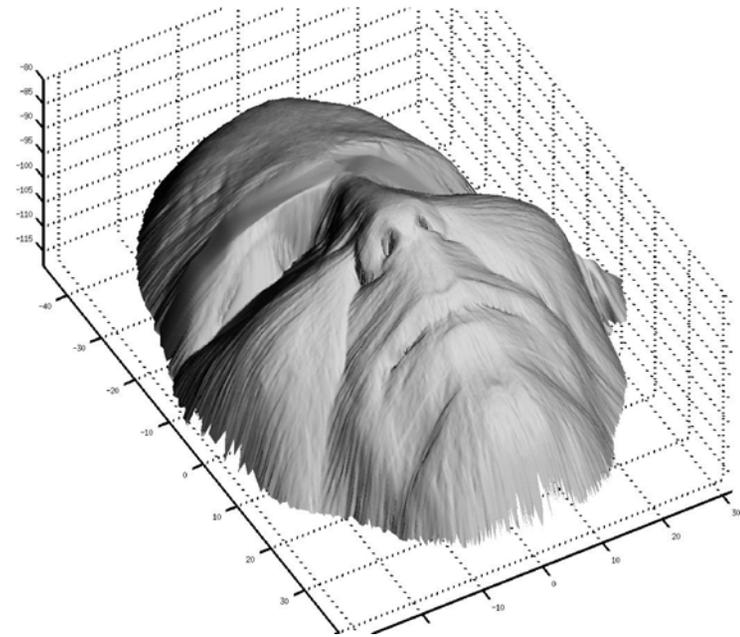


b) Reconstruction

Numerical results on Real images (Faces)...

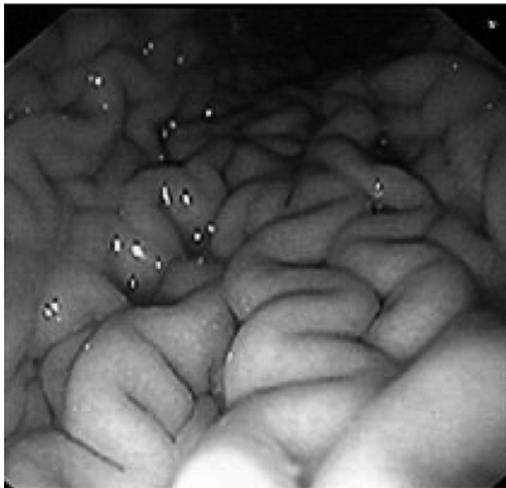


a) Real image of a face

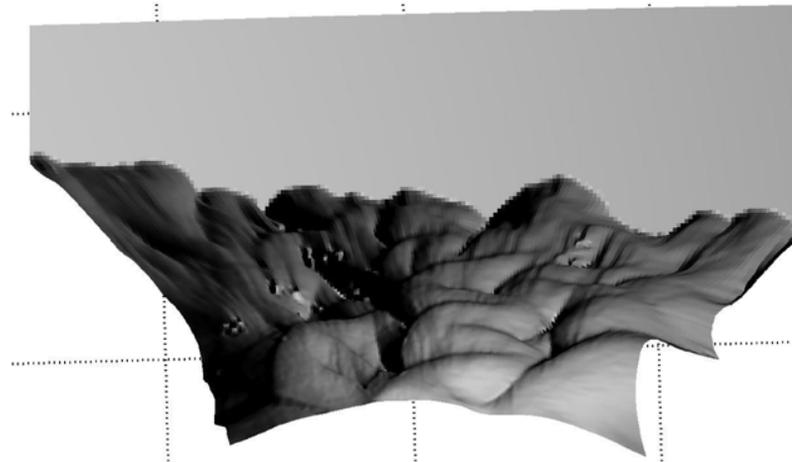


b) Reconstruction

Numerical results on Real images (Stomach)...

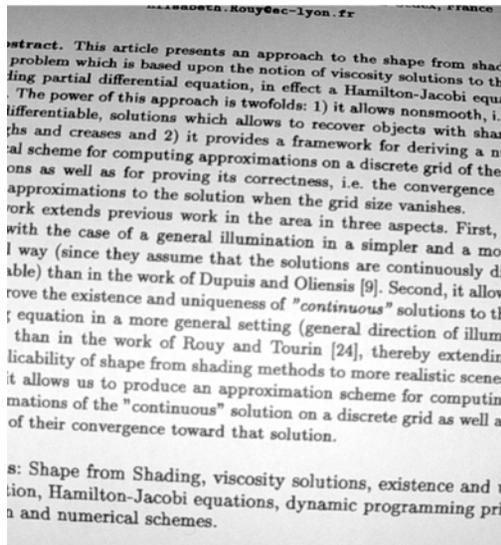


a) Real image of a stomach

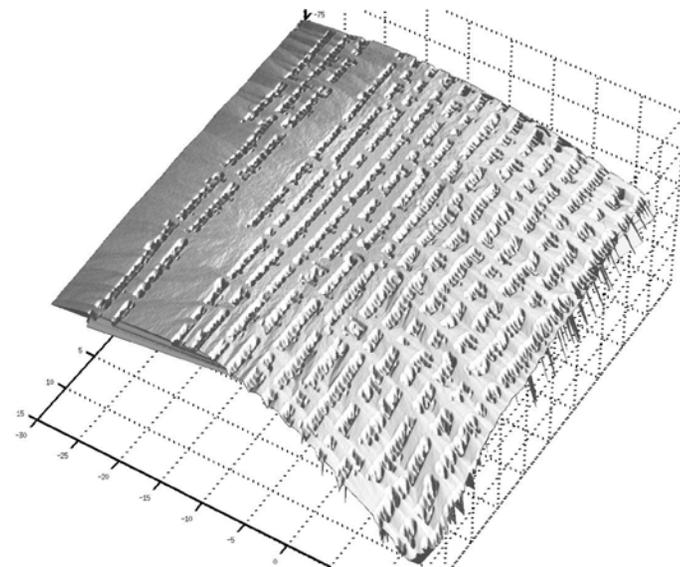


b) Reconstruction

Numerical results on Real images (Pages)



a) Real image of a page



b) Reconstruction

Conclusion:

- SFS problem \neq Eikonal equation.
 - Realistic SFS consists in solving more complex equations: for ex. the “generic SFS” equation.
- In SFS, we do not have at our disposal boundary data.
 - The notion of SDVS proposes some more adapted solutions than the other classical solutions.

For more details:

■ Theoretical part:

See the INRIA research report:

"A viscosity method for Shape From Shading without boundary data"
(*E. Prados, F. Camilli and O. Faugeras*), August 2004.

■ Numerical part: See

– the INRIA research report No RR-5005 :

"A mathematical and algorithmic study of the Lambertian SFS
problem for orthographic and pinhole cameras"
(*E. Prados and O. Faugeras*), September 2003.

– E. Prados' PhD thesis, October 2004.

■ Web page:

<http://www-sop.inria.fr/odyssee/team/Emmanuel.Prados/>

■ Thanks!