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Some precursors of current theories of syllogistic reasoning

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Syllogisms have a long logical and philosophical history. At first sight, one might wonder whether this history may be of any interest to experimental psychologists. Respect for the autonomy of psychology and the fear of logicism might explain such a negative attitude. Still, as will be shown in this chapter, investigators of human reasoning have a great deal to learn from an examination of logical and philosophical theories of the syllogism because, through the exposition of their principles and methods of resolution, these often make use of, and disclose, processes that are psychological in nature.

The work of the German psychologist, G. Störring, published in 1908, which in all likelihood contains the first series of psychological experiments on syllogistic reasoning ever published, will be taken as a point of departure and reference. Indeed, it is interesting, if only for the sake of curiosity, to go back to the origins, but this is only a secondary motivation. The main reason to comment on the Störring study is that it contains a few ideas of capital importance, which either passed unnoticed or were forgotten. And it turns out that the main current approaches to syllogistic reasoning are based on a resurrection of the same ideas which

can, in turn, be traced back to the writings of philosophers and logicians of the past, including Aristotle. In brief, the logico-philosophical and psychological histories of syllogistic reasoning and its psychological explanation are intertwined.

The Störring study

In 1908, Störring published a one-hundred-and-twenty-seven-page article devoted to the experimental study of two areas in deductive reasoning: relational and syllogistic. We will consider only the latter (note 1). The paper has a very short introduction showing psychologism to be the author's implicit frame of reference; he presents his experimental study as an attempt to provide an answer to two questions debated *between logicians:* Is every conclusion inferred on the basis of spatial representations? Does the conclusion result from a synthesis of the terms in the premises, or does it result from a process of comparison?

The method is succinctly described: the administration was individual; premises were visually presented and remained visible until the participants had responded. They were asked to answer with absolute certainty, an instruction repeated on every trial. The response times were measured and then a clinical interview started, whose aim was to make the participants specify, from a descriptive point of view, how they understood each premise and generated the conclusion. The propositions had an abstract content: letters of the alphabet stood for the three classes and the formulation adopted was such that a syllogism like EI-1 would be: *no A belong [German Gehören] to class D; some T belong to class A* ∴ *?* (note 2).

The paper has no explicit hypotheses, no experimental design, no statistical treatment and there were only four participants. Although this seems irredeemable to our current standards, after closer examination, it appears that the paper contains a number of essential observations, reviewed below, that certainly make it abreast with modern studies.

The figural effect

First, the author noticed longer response times when the major premise is in the first position as compared with the case where it is in the second position. This inversion amounts to comparing the first and the fourth figures (note 3). Three quarters of a century later, this difference in difficulty was observed by Johnson-Laird & Bara (1984) who showed that fewer errors and shorter latencies occurred on the fourth figure than on the others. They also observed that most of the problems in the first and fourth figures produce a response bias, namely all the subjects tended to give a response following the S P order on the first figure and the P S order on the fourth figure. Störring's explanation for the increase in response time was that

> when the major premise is in the first position, the identification of the elements that function as a middle term can occur only after reading the premises. When the major premise occupies the second place, it occurs while reading and comprehending.

In brief, there is a difference in difficulty to integrate the premises and this explanation foreshadows that of the mental model theory and its interpretation of the figural effect in terms of the ease of integration of the premises in working memory.

Strategies

The next discovery made by Störring is far more important. Throughout his paper he emphasizes that people adopt *strategies* to solve syllogisms, and he distinguishes two of them. The first one is a visual representation of the premises: often people imagine circles standing for classes, sometimes they imagine clusters of letters standing for members of classes (note 4); then, the conclusion is read off these configurations. The other strategy is verbal: it concerns the reformulation of the premises and efforts to identify two terms with each other, on which more will be commented below.

Obviously, Störring described the two types of strategy rediscovered by Ford (1995) with a similar method (interview and verbal report). She observed two kinds of participants: some made drawings such as circles or squares to represent the classes; some others made transformations on the terms, such as replacing a term by another, rewriting a syllogism in the form of an equation, or drawing arrows to relate terms. Nowadays it seems very likely that not all individuals tackle syllogisms in the same manner: some rely on a semantic or analogical strategy of the type hypothesized by mental model theory; some rely on a syntactic treatment. One moral of the Störring experiments is that investigators have much to gain from looking for possible individual processes and differences; from this point of view, the administration of paper-and-pencil questionnaires seems of little use. Indeed, Newell (1981) criticized what he called the "fixed-method fallacy" but this was avoided in only a few studies such as Ford's and Bucciarelli &Johnson-Laird's (1999).

The process of "insertion"

Unlike the two previous observations, which by now can be considered well-known established phenomena, the third one has been recognized only recently by a few authors. It will be argued later that this observation might well be the most important ever made on the psychology of syllogisms, because it reveals the essence of syllogisms and possibly the way they are solved, according to one major theoretical approach.

Let us follow the reports of Störring's subjects who had to solve the IA-4 syllogism: *some P belong to class M; all the M belong to class S* ∴ *?* One participant said: "all the M, including some P, belong to class S." Another one commented: "these M, to which some P belong, belong to class S". Or take a syllogism in the first figure with a negative premise, EA-1: *no M belong to class P; all the S belong to class M* ∴ *?* One participant commented: "One can say the same thing about S as about M" and he went on saying that he "inserted" (his own words) S in the place of M, hence the conclusion "no S belong to class P". The same participant reported for IE-4 *(some P belong to class M; no M belong to class S* ∴ *?):* "What is said of M can also be said of some P" to conclude correctly "some P do not belong to class S".

These are instances of the various ways of expressing a common process by which subjects select the end term of one premise and insert it by the side of the middle term in the other premise. And to reach the conclusion it will suffice to extract it from the composite expression: "the concluding sentence is obtained by abstraction of a part of what has been asserted as a result of the insertion", that is, in the first example, from *all the M, including some P, belong to class S,* one extracts the conclusion *some P belong to class S*.

Thus, Störring observed the process of insertion in all his four subjects and it was the only clearly identifiable strategy when it was at all possible for participants to report in some detail. Only recently has this process been identified by other investigators: As we will see later, Ford (1995), Stenning & Yule (1997), and Braine (1998) essentially described the same process, each of them formulating it within his or her own framework.

We leave the Störring investigation only to note that the most important psychological phenomena associated with syllogism solving were discovered nearly one hundred years ago by a careful investigator but, sadly enough, were soon forgotten. It has already been mentioned that they have been rediscovered, the last one only very recently. We will engage in an enquiry that will take us much farther back in time, in search of the origins of the concepts underlying Störring's three main observations, which we will consider in turn.

The figural effect

Compare the two syllogisms in the first and fourth figure, respectively, AI-1: *all M are P; some S are M* ∴ *some S are P,* and IA-4: *some P are M; all M are S* ∴ *some P are S.* Although they are logically equivalent (as a change in the order of the premises shows) there is general agreement that the conclusion of the latter follows more "naturally", or more fluently, than the conclusion of the former. As we have seen, greater ease of the fourth figure has nothing mysterious: it is linked with the contiguity of the two occurrences of the middle term.

Could this have influenced logicians of the past? One would expect a negative answer because logic is a formal matter, and logicians are not

expected to be influenced by performance factors. However, most surprisingly, the question has a positive answer, to be found in the father of the syllogisms himself. Before this, a brief historical foray into the formulation of syllogisms is in order. Lukaciewicz (1958) points out that the current formulation of syllogisms, which follows a tradition fixed a long time ago, (note 5) differs sharply from Aristotles'. Only one difference will concern us: our current use of the copula *IS A/ ARE* in categorical sentences such as in *some S are M,* differs from Aristotle's formulation, who used either *M is predicated of some S,* or *M belongs to some S.* Consequently, we state the terms in an order that is the reverse of Aristotle's formulation. The same obtains for the three other categorical sentences. This inversion has a remarkable consequence. Take for instance the AI syllogism in the first figure. While it is nowadays expressed as *all M are P; some S are M,* the original syllogism was *P is predicated of all M; M is predicated of some S,* so that the occurrences of the middle term appeared in contiguity in the first figure, not in the fourth figure as is the case in the tradition. Notice that the first figure is the only one in which such a contiguity occurs.

We are now in a position to answer our question and will concern ourselves with Aristotle's notion of perfect syllogism. As we will see later, Aristotle's main method of proof is to transform the syllogism under study into one belonging to a small set, the perfect syllogisms, whose truth is regarded as evident. The perfect syllogisms are essential because, in modern terms, they play the role of axioms of the system. Patzig (1968) discusses at length Aristotle's reasons for considering the truth of some syllogisms as evident; in other words, the question is, What are the formal

properties that qualify a syllogism to be regarded as evident by Aristotle? Now, it turns out that all the perfect syllogisms are in the first figure but logically they need not be in the first figure because there are alternative possible axiomatic choices. Patzig's answer is that the formal property consists in the immediate connection between the premises offered by the middle term, a feature possessed only by the first figure.

So, we reach this remarkable conclusion: in all likelihood, Aristotle based the perfect-imperfect partition of syllogisms on a formal property, namely the contiguity of the occurrences of the middle term, which has a psychological import rather than a logical one: it is this contiguity which provides the syllogism with greater evidence.

In summary, the identification of the role of the figure as a source of difficulty (or rather, of easiness) is to be found in Aristotle himself, a fact that has been concealed because of the choice made after him to formulate the relational term that links the subject and the predicate of categorical sentences: this formulation resulted in removing the characteristic feature of the first figure and in transferring it to the fourth figure instead (a figure which, in addition, was discarded by Aristotle for reasons that are beyond the scope of this paper).

The analogical strategy and the representation of categorical propositions

Many people, starting with Störring's subjects, associate diagrams with syllogism solving. Since the 17th century, various kinds of diagram have been proposed by logicians to represent syllogisms and how to solve them. Although Euler is often credited with the invention of the circles

that are named after him, he was in fact preceded by Leibniz, who developed not only the circles, but also a straight lines representation. Bochenski (1970) mentions that both kinds of diagram can even be traced back to earlier logicians in the 17th century (note 6).

Later, there have been various kinds of diagrams to represent and solve syllogisms (for a survey of some methods developed in the nineteenth century, see Gardner, 1958). The most famous, and no doubt the most practical method, is that of Venn diagrams (which, it is reminded, are made of three mutually intersecting circles whose overlaping parts and common borders are shaded or marked by a cross in order to encode the mood of the syllogism). But Venn diagrams are an automatized method to encode the premises, work out a solution, and decode it. They have two characteristics: they are external representations, and they support an algorithmic method of resolution (and in this sense are operative). No claim of psychological validity (as internal representation) has been made for Venn diagrams. Such a claim does not seem tenable because it is hard to see how the eight areas defined by the three overlaping circles and the areas obtained by addition or subtraction of these could ever be kept in working memory; it is precisely the power of the graphical method to help visualize so many areas that makes the method efficacious.

Leibniz's straight line diagrams

Leibniz's diagrams are described in an undated 18-page opuscule.
In the line diagrams, a class is represented by a straight line, and the two classes of a categorical sentence by two parallel lines whose ends are determined by the relation that the two classes entertain. The four basic categorical sentences are represented in Figure 1. The vertical dotted lines

indicate that a statement is made relating the two classes.

Figure 1 near here

To represent a syllogism, the major premise is placed on lines one and two (with the middle term on line two) and the minor premise on lines two and three. By considering lines one and three, which represent the relation between the end terms, one produces the vertical dotted lines if necessary, and read off the solution. An example is given in Figure 2, in which it can be noticed how easy it is to read the solution.

Figure 2 near here

There is a remarkable feature in Leibniz's lines: whereas a horizontal line represents the extension of a class, a vertical line represents a single member; consequently, the intersecting points of a vertical line with the three horizontal lines describes one single entity that can be considered from the point of view of its belonging to either one of the classes or to more than one class (when there is more than one intersection). The same obtains of course with circle diagrams: one point in the common part of, say, S and M can be viewed as an S individual or an M individual. The important idea underlying Leibniz's representation, whether in lines or in circles, is that an individual (e.g. a point in A) that is considered *qua* A can also be considered *qua* B whenever the two classes A and B intersect each other: it enables a multiple characterization, and a subsequent change in perspective (the topic, or subject shifting from one class to another one).

What use did Leibniz make of his diagrams? The answer is the same for the straight lines and for the circles, as they are isomorphic: they were

used for illustration purposes, each concludent syllogism being accompanied by one single line diagram together with its unique circle diagram counterpart. To some extent, Leibniz chose for each sylllogism one prototypical diagram (often the only possible one, of course); the point is that he did not envisage the possible multiple configurations for each syllogism.

The current psychological theory that makes extensive use of diagrams is the mental model theory. Leibniz's lines share two important features with diagrams used by mental model theory: (i) the extension of a class is represented along a line (a graphical line in the former case, an alignment of a number of tokens in the latter case); and (ii) the relation between two classes is captured by the relative position or shift of the two lines along their common direction, allowing for the presence or the absence of members common to the two classes on a perpendicular line.

Euler's circles

The definition of Euler's circles and how he used them can be found in a few of his *Lettres à une Princesse d'Allemagne,* written in 1761. The author's aim is essentially a didactical exposition of the theory of the syllogism (and indeed it is a remarkable achievement from this point of view). It is doubtful that the author had a stronger objective in mind, such as offering a method of proof. Referring to his way of representing propositions, he says: "this way will disclose [French *nous découvrira*] the correct forms of all the syllogisms", an ambiguous expression which, as we will see, should be interpreted in its loose sense: Euler's aim is more to illustrate than to demonstrate. Another consequence of this *genre* of exposition (letters

written by a tutor do not constitute a treatise) is that what is gained in clarity is often lost in conceptual depth: Euler seldom elaborates to justify his method.

After remarking that

> since a general notion contains an infinity of individual objects, it is regarded as a space in which all these individuals are enclosed

Euler gives the representation of the four propositions as in Figure 3.

Figure 3 near here

Notice that the A proposition is given a strict inclusion representation. For the I proposition,

> a part of space A will lie inside space B, as it is well visible that some thing that is contained in notion A is also contained in notion B.

Accordingly, the letters A and B in the diagram (Figure 3c) denote two spaces which do not have the same status: while B labels the B class (or notion), A marks the area which captures the concept of a common part to two notions (and foreshadows the concept of set intersection). Similarly, the representation of the O proposition is defined as follows:

> a part of space A must lie out of space B [. . .]; here it will be noticed chiefly that there is something in notion A which is not comprised in notion B, or which lies out of this notion.

and while the letter B in the diagram (Figure 3d) just labels the B class, the letter A marks the area crucial to represent the concept of the part of A that is not B (the difference between A and B in set-theoretical terms) which may be all or only part of the A class. This distinction is confirmed

by the fact that in Figure 3d, A and B are symmetrical, which by parity would invite the unfortunate interpretation that some B are not A, were the notational difference just mentioned not kept in mind. It is with these four diagrams only that Euler tackles the task of identifying the valid syllogisms (and showing in passing how to identify the invalid ones). A century earlier, Leibniz's had made exactly the same choice for the representation of the four basic sentences.

Euler's method is the following. First, make the single diagram that represents the major premise (so representing the relation between M and P). Second, integrate the minor premise by (i) placing S with respect to M, which is easy because there is only one way of representing a proposition, while (ii) considering at the same time whether S can have different positions in relation to P. These relative positions are among the following three: the notion S is either entirely contained in the notion P (inclusion), or partly contained (overlaping), or outside (exclusion). In brief, for any syllogism, it will be necessary to draw one, or two, or at most three diagrams.

Let us illustrate with the EI-1 syllogism: *no M are P; some S are M.* Given that the notion M is entirely out of the notion P,

> if the notion S has a part contained in the notion M, this part will certainly lie out of the notion P, like in Figure 4a; or in that way (Figure 4b), or yet in (Figure 4c):

Figure 4 near here

Although this explanation is clear and convincing (so much that, in fact, one could nearly dispense with the diagrams) the usage of the diagrams is

somehow incoherent. In effect, we could expect Euler to look for the conclusion by trying to identify in each diagram, in terms of S and P, one of the four basic propositional relations which he has defined. For instance in diagram 4c, the definitional pattern of an O sentence appears between S and P. However, Euler does not exploit this. Instead, he makes a non-definitional, intuitive use of the diagrams to support his demonstration (based on a container-content interpretation of the sentences). In the present case, the part common to M and S (in which S is aptly written) is considered in isolation and shown to lie necessarily out of P because it is part of M; but being as well part of S it can be designated as *some S,* hence the conclusion *some S is not P.* This applies to all three diagrams (4a, b, and c). There is one good reason for this usage of the diagrams, which appears clearly on diagrams 4a and 4b: these exhibit respectively the two interpretations of the O sentence which Euler has not considered in his definitional diagram; as a consequence he is prevented from reading the O relation between S and P off each diagram; rather, as just seen, he argues in terms of the relation between the intersection labelled S, and P.

Notice that in Euler's usage of the diagrams, it is not even necessary to draw more than one diagram. For instance, even an undetermined diagram such as the one given in Figure 5 demonstrates that P and M lying entirely out of each other, any part of M cut out by S will remain out of P (assuming the border of S to be closed and convex) (note 7). Interestingly, Leibniz gives exactly the same diagram as Euler's 4a for EI-1, and he always restricts himself to a single diagram whatever the syllogism under consideration.

Figure 5 near here

Thus, one can wonder why Euler took the trouble to represent multiple diagrams of syllogisms. This is necessary in order to be exhaustive, only if diagrams are used as a search procedure to identify the conclusion in terms of each basic proposition. For example, for the EI-1 syllogism, one can interpret the relation between the S and P terms and notice that the only solution compatible with the three diagrams is the O sentence. This is very much reminiscent of the mental model theory, and indeed the three diagrams are identical with those produced by this theory. However, this agreement does not always obtain. There are several reasons for that. One is just some carelessness on the part of the author; for example, for the OA-2 syllogism, he gave two diagrams (the same ones as mental model theory) but in the preceding letter he gave only one diagram for the AO-2 syllogism even though he recognized that it was the same as OA-2 (the latter being non standard), which he discarded as redundant for that reason. Another reason is that there are optional parts in the diagrams of mental model theory, which on some occasions yields fewer diagrams than Euler's method (e.g. one diagram instead of two for IA-4). Yet another, more fundamental reason, is due to the restriction imposed by having a single diagram for each sentence, which cuts the combinatorial analysis. Interestingly, on only four occasions does this lead to an ambiguous conclusion: on AO-2 for the reason noticed earlier, and on OA-3, IA-3, and IA-4 the two diagrams support both an O and an I solution. The unwarranted solution would be eliminated if the O (resp. I) premise was allowed a possible exclusion (resp. inclusion) interpretation.

Back to the question raised, we can speculate that although Euler made a figurative usage of his diagrams, he must have recognized the potential operative usage of them *as a method of proof*. The necessity of the combinatorial exploitation which such a usage implies must have been so compelling that he could not refrain from applying it even though his figurative usage of the diagrams did not require it.

There is, however, one domain where it was appropriate to look for alternative diagrams, namely the identification of the invalid forms. Here the method is straightforward: try to show that the relation of S to P can be either an exclusion, or an overlap, or an inclusion. For this, three diagrams are required. Euler provided the demonstration for IA-1 and IA-3 together, for II-1 and II-3, and also for the non standard IE-1 and IE-3 considered from a standard point of view. He did not review the other cases, including the cases (the EO and OE syllogisms) where the method fails (yielding an O solution) again for lack of considering the exclusive interpretation of the O sentence.

To summarize, Euler did not develop an algorithmic use of the diagrams that would enable one to identify the valid syllogisms and provide their conclusion. This would have required different definitions of the basic diagrams (about which more below). To such an operative usage of diagrams, he preferred a figurative usage supporting the interpretation of the various syllogisms. This interpretation in turn reflects the interpretation of the *IS A* relation in terms of container-content which he chose for didactical reasons:

> the foundation of all these forms [of syllogisms] consists, in short, in these two principles on the nature of container and content: (i) all

> that is in the content is also in the container. (ii) all that is out of the container is also out of the content.

The extraordinary ease with which syllogisms seem to be understood, and even most of the time solved, under a container-content construal of them is certainly of great psychological interest. Consider for instance the OA-3 syllogism which is very seldom passed, and let us follow the royal tutor:

> Assume [. . .] that a part of the notion A lies out of the notion B. In that case, if the third notion C contains the notion A entirely, it will certainly also have a part out of notion B [. . .], hence this syllogism: some A are not B; all A are C; therefore some C are not B.

Euler's explanations consist in reformulating the argument, replacing all ARE, no ARE, some ARE, some ARE NOT with *is entirely contained in, is entirely out of, is partly in, is partly out* respectively, and this sems enough to deeply modify the task. Apparently, the human being (who, as shown by nearly a century of experiments and over two millennia of painful exposition to logic treatises), is not cognitively equiped to solve the majority of the syllogisms worded with the underdetermined IS A relation), seems to become an expert (admittedly, experimental data are missing) at solving the same syllogisms worded in container-content terms. A possible reason for this will be considered later.

Gergonne's circles

We have noticed Euler's choice of one and only one diagrammatic representation for each proposition (and the unfortunate consequences of this choice from a proof-oriented point of view). The consideration of the five possible relative positions of two circles in the same plane (the circles

representing two "ideas", one being a subject and the other a predicate) is due to Gergonne (1816-1817) (note 8). He expounded the correspondence between the four propositions of the natural language and the five logical relations, which by now has become familiar, not by giving the mapping of the former onto the latter, but in the form of two tables. The mapping between the four propositions and the five diagrams is given in Figure 6 using the symbols proposed by Gergonne for each of the five relations (note 9).

Figure 6 near here

The relations can be paraphrased as follows (notice the explicit quantification of the predicate):

- P **X** Q: a part of P is part of Q;
- P ⊃ Q: a part of P is the whole of Q
- P **c** Q: the whole of P is a part of Q;
- P **I** Q: the whole of P is the whole of Q;
- P **H** Q: no part of P is a part of Q.

The Gergonne correspondence displays the semantics of quantified sentences and it is far more complicated than a one to one correspondence. It poses the following questions: (i) the semantic question: why is the mapping not one to one? (ii) the pragmatic questions: how does one restrict the number of choices from diagram to propositions and from proposition to diagrams? (iii) the psycholinguistic question: given a proposition, are all its related diagrams psychologically equivalent?

The first question is why natural languages have not evolved to produce

one and only one expression for each relation. One putative answer, based on the general assumption that language reflects cognitive constraints, is the following. Given any two concepts, in the absence of a priori knowledge about the world, an individual is not in a position to express a judgment coinciding with any of the five basic states or relations, but this state of ignorance changes with experience. Take some of our ancestors who may have had no a priori judgment about the relative extension of mushrooms and poisonous things. One of them has a bad experience and it may be important to communicate belief in this newly discovered state of the world where there are poisonous mushrooms; this can be communicated by a unique expression, namely, in our evolved language, *some.* At this stage, the fifth relation is eliminated but there is not enough information to know which of the four diagrams for *some* obtains: this is matter of further experience or of systematic enquiry. Only after more data have been gathered will it be possible to know (or at least start knowing) which is the state of the world (note 10). In brief, because there is not enough information, the new belief, *some mushrooms are poisonous,* has to be communicated without qualification. Now, some more experience may reveal two different states of the world:

- One, that there are occasions when no poisoning occurs after eating mushrooms. Communicating this revised belief may have survival interest if mushrooms is the only food available. This can be done by qualifying *some* (i) either by *only:* *<u>only</u> some mushrooms are poisonous;* (ii) or by using a negation: *some mushrooms are poisonous <u>but some are not</u>.* Either of diagrams 1 and 2 refer to this state. Which one is the case again may still not be decidable for lack of information but nevertheless the judgment which both represent can be communicated as indicated.

- Two, that there is always poisoning after eating mushrooms, in which case there is a need to extend *some;* the linguistic device *all* does just that: *all mushrooms are poisonous* communicates this new belief compatible only with diagrams 3 and 4 (and again this will suffice pending new information). It seems that there is no lexical device in English to further distinguish the two diagrams for *all* and also (in the backgroung knowledge that *some* is the case) the two diagrams for *only some* (or *some are not).*

To summarize: after the very first experience, in a language lexicalized with the five relations, one would have to state: $P\boldsymbol{X}Q$ *or* $P \supset Q$ *or* $P\,\boldsymbol{c}\,Q$ *or* $P\,\boldsymbol{I}\,Q$ in order to express the important belief that there are P that are Q: *some P are Q* fulfills this purpose much more economically. Similarly, by iteration of the same principle, in a situation of emergency it is more appropriate to state *only some P are Q* than $P\boldsymbol{X}Q$ *or* $P \supset Q$, and also *all P are Q* than $P\,\boldsymbol{c}\,Q$ *or* $P\,\boldsymbol{I}\,Q$. In brief, the mapping between relative extensions of properties in the world and natural language seems optimal from the viewpoints of ease and efficiency in communicating information: natural language is well adapted as it is.

The second question concerns the pragmatics of quantifiers and it will be treated succintly. Consider first the mapping from right to left: on each of the five cases it is one to two. The problem is to decide which of the two verbal expressions to choose and we shall see that this choice is determined by considerations of relevance (Sperber & Wilson, 1986), which in turn depend on the background assumptions about the hearer's beliefs. We assume each time that the speaker is knowledgeable, i. e. , that she has obtained enough evidence to support the relation under

consideration.

The first two relations, P **X** Q and P ⊃ Q, are mapped onto I and O and will be treated together. The relevant information is the one that helps the hearer to eliminate states of the world. The speaker choses a *some* sentence or a *some are not* sentence on the basis of her assumptions about the hearer's beliefs regarding the states of the world that obtain. Suppose an initial background where P **H** Q is already known to be false (i. e. , where, by elimination, the background is made of the first four relations); then the relevant contrast is between the first two and the next two relations, so that a *some not* sentence is appropriate because it eliminates the third and fourth states. On the contrary, in a initial background of uncertainty with regard to P **H** Q, the relevant contrast is between all first four relations on the one hand and P **H** Q on the other hand, and a *some* sentence is appropriate because it eliminates P **H** Q. (The speaker may even use the awkward *some are and some are not* sentence if both pieces of information are estimated useful to the hearer as an additional contrast against P **c** Q and P **I** Q when P **H** **Q** is uncertain). Logically, the choice amounts to a detachment from I&O: detach I or detach O, whichever is relevant. The same applies to the P ⊃ Q relation.

Suppose now the speaker believes one of the next two relations (P **c** Q or P **I** Q); there is a similar choice to be made, but this time between *all* and *some:* in an initial background where P **H** Q is known to be false, an *all* sentence is appropriate because it enables the hearer to eliminate the first two relations; whereas in an initial background where P **H** Q is uncertain, both *all* and *some* are *logically* appropriate: *some* eliminates P **H** Q, and *all* eliminates P **H** Q and also P **X** Q and P ⊃ Q.

For instance, suppose the wizzard has made many trials and concludes that the local mushrooms and poisonous food are related as per P **c** Q or P **I** Q. In order to convey this discovery to someone who believes that some mushrooms are poisonous, he will say that *all mushrooms are poisonous,* so contrasting P **I** Q (or P **c** Q) with the other three relations, thereby eliminating them. But if the hearer's background knowledge is ignorance, there are logically two possible utterances: *all mushrooms are poisonous,* because it contrasts with P **H** Q and also with P **X** Q and P $\supset$ Q, and *some mushrooms are poisonous,* because it contrasts with P **H** Q. However, while the *all* sentence, which eliminates the most states of the world, is optimally relevant to express the P **c** Q (or the P **I** Q) relation, the *some* sentence is not, because it eliminates fewer states (only the P **H** Q state). This means that, given a presumption of optimal relevance, a *some* sentence is not appropriate to express the P **c** Q (or the P **I** Q) relation. Therefore, if a *some* sentence is used, the hearer can infer that among the relations mapped onto *some,* only the other relations than P **c** Q (or the P **I** Q), namely P **X** Q (or P $\supset$ Q) are being expressed. This inference is the source of the well-known conversational implicature (Grice, 1975, Horn, 1989) which gives to *some* its interpretation *some but not all* and countermands its use to express an *all* state of the world (note 11). The same situation obtains, *mutatis mutandis,* for the *no/some not* opposition as for the *all/some* opposition, for the same reasons and need no specific treatment; there is only a change in the background assumption which is (P **c** Q or P **I** Q) instead of P **H** Q.

Before taking up the psycholinguistic question, it will be useful to complement the correspondence with the *converse* propositions. The

mathematical formalism, with P in the first position and Q in the second one, conceals that in ordinary language the converses are very natural: for instance, the **X** relation can be characterized as naturally by s*ome Q are not P* as by *some P are not Q;* still such converses do not appear in Gergonne's first two tables. (They appear later in another form). So, at this stage, the mapping fails to exhaust the meaning of the diagrams. This is why a more complete mapping is presented in Figure 7, in which *some Q are not P* (noted O') and *all Q are P* (noted A') have been introduced.

Figure 7 near here

Now, on the right of Table 7 there appears a formula for each relation, which shows that they all are a conjunction of three propositions. For example, an exhaustive description of the first diagram P **X** Q is: *some P are Q <u>an</u>d some P are not Q <u>an</u>d some Q are P.* It will be noticed that across the first four relations, I is an invariant, as expected since they all are mapped onto I; similarly, O is an invariant across relations **X**, $\supset$, and **H**; and A is invariant across relations **c** and **I**; while E is of course invariant for its single relation **H**; similar invariance obtains for A' and O'.

For each proposition type in turn, we can now ask the question, Is there a diagram, that is, a formula, more fundamental than the others? The answer is easy, and it is affirmative. It results from the observation that for each formula except the first, one or two of the components are logically implied by another one: I is implied by A as well as by A', and O and O' are implied by E. The definitional sentential components of each relation have been underlined accordingly.

The simplest case is the E proposition for which there is nothing to add: it

has only one diagram. Next, the A proposition has two equally fundamental diagrams: in both the **c** and the **I** diagrams, A is a definitional component. Contrary to what is generally claimed, there is not one single logically "correct" diagram to represent the A sentence (supposed to be the strict inclusion **c**, while the identity **I** would result from a conversion error): there are two equally correct representations differring by their third components, which negate each other: in one case it is A', the converse of A, and in the other case it is O'. A testable consequence of this claim is that people should be willing to identify A sentences with both diagrams equally (provided, of course, that the sentence does not refer to known states of the world in which one, and only one, of the diagrams obtains).

The O proposition has three diagrams but in the last one (**H**) the O component is obtained by inference, so that for *some not* there are two diagrams which do not require an inference and are fundamental for that reason, viz. **X** and ⊃. Again, a testable consequence is that people should be reluctant to identify the **H** diagram with *some not*, whereas they should equally accept **X** and ⊃ as representations for it.

Finally, the I proposition has four diagrams but only in the **X** relation is the I component not inferred, so that although the four diagrams are logically correct, one of them is psychologically more fundamental than the others because it does not require an inference for the concept of "common part" to be identified in the diagram. The testable consequence is that people should show a preference for the overlap relation (note 12).

Studies of the comprehension of quantified sentences confirm that the

overlaping position plays a more fundamental role, similar to that of a prototype. For example, in Begg & Harris' (1982) study, when people were asked to distribute 100 points over the five Gergonne diagrams, for the I sentence the overlaping position received the absolute majority of the points (63%) while the next choice (P **c** Q) received only 21% of the points. Interestingly, for the O sentence, the points were equally shared by the overlaping (45%) and the B included in A (44%) diagrams, as predicted by the inferential analysis just proposed. And similarly for the A sentence, people's choices were not far from equality between identity (**I** : 57%) and inclusion (**c** : 43%).

Coming back to Euler's choice of diagrams to represent propositions, one is better able to explain what must have motivated it. His didactical obligations may have recommended a single diagram for the sake of simplicity, but why precisely the overlaping configuration for the I proposition? Our analysis answers this question: the I proposition has one diagram provided with the proprerty of greater ease for judgment of representation, namely the overlaping position. For the O and A propositions, Euler's choice coincides with one of the diagrams that we have identified as equally easy. Notice that Leibniz's choice for representing the four basic sentences was exactly the same; this is interesting because in all likelihood Euler was not aware of Leibnz's diagrams, which were discovered and published only in the first years of the twenteeth century.

Finally, what is the psychological plausibility that the Gergonne diagrams be the basis for a mental representation of quantified propositions? The one-to-several mapping between linguistic quantifiers and diagrams is a

common argument against this plausibility (Johnson-laird & Bara, 1984) because of a "combinatorial explosion" when integrating two propositions. However, the argument is not compelling: people might limit their usage of the correspondence to the preferred diagrams (just as Euler did), or they could have a more flexible representation in which there is, for each proposition, one basic diagram marked with compulsory and optional parts; this proposal was made by Whetherick (1993) and applied by Stenning & Oberlander (1995). For instance, in Whetherick's representation, a *some* sentence is based on the basic overlapping position; but only the central "lens" is drawn with a continuous line, while the left and right arcs are drawn with a dotted line whose limits indicate optional areas. With reference to Figure 7, this diagram shows an **X** relation underlining the invariance of I (the common part); suppressing the left dotted line, or the right one, or both, amounts to negating O, or O', or both in the formula I&O&O', so changing the diagram into I&A&O' (P **c** Q), or I&O&A' (P ⊃ Q), or I&A&A' (P **I** Q), respectively.

The verbal strategy

Recent investigations

We will consider first the modern versions of Störring's insertion strategy and begin with Ford (1995). About one half of her subjects exhibited on the majority of the syllogisms what she calls a "substitution behavior", that is, replacing one term in a premise with another, as if solving an algebraic problem. The substitution also appeared in the form of arrows linking terms in the premises, or in the form of terms crossed and replaced with another term. Using the IA-4 syllogism as an example *(some P are M; all M are S),* Ford says that the second premise allows one to give the value of S

to M; the value of S can be substituted for M in the first premise, giving the conclusion. This amounts to collapsing Störring's insertion and abstraction processes (note 13).

Ford formalizes the substitution procedure as follows. The premise that provides the replacement term plays the role of a rule relating membership of class C and property P, while the premise that contains the term to be replaced provides specific objects whose status with regard to C or P is known. The following rules guide the process of substitution:

> A. If a rule exists affirming of every member of the class C the property P then (1) whenever a specific object, O, that is a member of C is encountered it can be inferred that O has the property P, and (2) whenever a specific object, O, that lacks property P is encountered it can be inferred that O is not a member of C.
> B. If a rule exists denying of every member of the class C the property P then (1) whenever a specific object, O, that is a member of C is encountered it can be inferred that O does not have the property P, and (2) whenever a specific object, O, that possesses the property P is encountered it can be inferred that O is not a member of C. (Ford, 1995, p 21).

Notice that individual objects are introduced, an important point about which more will be said later. Apart from this novelty, there is formally nothing new in these rules. They were already spelled out (in their universal formulation) three hundred years ago in the Logic of Port-Royal. In the chapter on syllogisms, Arnauld and Nicole formulate the *principle of the affirmative moods* for the first figure as follows: "that which applies to an idea taken universally, also applies to all that of which this idea is affirmed." Using Ford's names for classes, the property P which applies to all the C also applies to the object O of which C is affirmed: this is rule A1. Similarly, the *principle of the negative moods* (still for the first

figure) says that "that which is denied of an idea taken universally is denied of all that of which this idea is affirmed." In other words, the property P denied of all the C is also denied of the object O of which C is affirmed: this is rule B1. In fact, these two principles of the first figure have a much longer history: they were referred to by the Scholastic logicians, in a more synthetic formulation including both of them, as the *dictum de omni et nullo.*

Finally, the *principle of the AEE and AOO moods* for the second figure given by the Port-Royal logic expresses rule A2: "whatever is included in the extension of a universal idea does not apply to any of the subjects of which it is denied." That is, being part or whole of those C which have the property P does not apply to those O of which P is denied (note 14). In brief, Ford claims that her rules, which are the singular counterparts of formal principles identified by logicians of the past, have psychological reality, and she relates her claim to the fact that A1 and B1 are equivalent to *modus ponens,* and A2 and B2 are equivalent to *modus tollens.*

We now turn to Braine's (1998) contribution. In his essay on mental-predicate logic, he deals very shortly with categorical syllogisms. He only considers the reasons for individual differences in performance and identifies one of these reasons with possessing or not a strategy for choosing a *secondary topic:*

> The strategy is to choose as secondary topic the subset of the subject of which the middle term can or cannot be predicated (the S that are, or are not, M, as determined by the premise relating S and M). (p. 321)

Once a secondary topic is chosen, it is transferred into the other premise:

this executes Störring's insertion process. The resolution then proceeds in two steps: one is the application of either a generalization of *modus ponens* for predicate logic (equivalent to Ford's rules A1 and B1), or, by a *reductio ad absurdum,* of a *modus tollens* (equivalent to Ford's rules A2 and B2); the other is a schema of universal or existential generalization. Braine offers this example (EA-3): *none of the M are P; all of the M are S.* The secondary topic, *the S that are M,* is provided by the second premise; according to the first premise, *the S that are M are not P* (by modus ponens), hence: *some S are not P* (by existential generalization).

The notion that the verbal strategy hinges upon the insertion process, followed by the operation of *modus ponens* and *modus tollens,* appears also in one of the main hypotheses of Stenning and Yule's (1997) theoretical approach. They propose that syllogisms exist and are soluble owing to one of their structural properties, namely the *identification of individual cases.* An individual can be characterized by the fact that it possesses, or does not possess, the properties defined by three categories, S, M, and P which constitute the premises, so that there are eight types of individual: $S^+M^+P^+$; $S^+M^+P^-$; $S^+M^-P^+$; $S^+M^-P^-$; $S^-M^+P^+$; $S^-M^+P^-$; $S^-M^-P^+$; $S^-M^-P^-$. For each syllogism, the joint premises warrant or do not warrant the existence of such individuals: in the affirmative, the syllogism has a conclusion. The authors describe two procedures for identifying individual cases, one analogical by models, the other by rules, which constitute two different implementations of a common underlying abstract individual identification algorithm.

For the analogical procedure, the authors apply the graphical algorithm defined by Stenning & Oberlander (1995). As mentioned above, a

"combibatorial explosion" ensueing the use of Gergonne diagrams is not obliged: each diagram has a minimal representation defined by the "critical regions", that is, those which must exist if the premise is true (these correspond to the invariants in the Gergonne formulas mentioned above). A simple procedure follows to integrate the premises. If there remains a critical region in the diagram restricted to the end terms, this is the diagram of the conclusion; if not, there is no conclusion.

The verbal procedure is decribed in the form of a three-step algorithm. At the first step, a "source premise" is selected: this is the premise which will provide the first two terms of the individual description. One of these is necessarily the middle term M. At the next step, it is compared with its occurrence in the other premise from the viewpoint of quality. There are three possible cases: (i) if the qualities match and M is subject of the other premise, a *modus ponens* is applied whose conclusion (which is the predicate of the other premise) provides the third term of the individual description; (ii) if the qualities do not match, and M is the predicate of the other premise, this means that there is a M^+ and a M^- , which allows a *modus tollens* whose conclusion (which is the subject of the other premise) provides again the third term of the individual description; (iii) if none of the two previous cases occurs, there is no conclusion. At the third step, the M term is eliminated and a quantifier is introduced. We exemplify with AO-2: *all the P are M; some S are not M.* The source premise is the second premise, which contains a negated M (M^-); this provides S^+M^- for the first two terms of the individual description. Since in the first premise M is predicate and affirmative, *modus tollens* applies: *all the P are M; not-M* yields *not-P,* that is P^-, which completes the individual description to yield

$S^{+}M^{-}P^{-}$, hence the conclusion: some S are not P.

In summary, Stenning & Oberlander's verbal algorithm specifies in detail Störring's insertion and abstraction process. It highlights the commonality in the approaches that we have considered: the first step is equivalent to, and specifies, Braine's choice of a secondary topic; the second and third steps coincide with, and specify Ford's substitution process and application of rules, as well as Braine's application of inference schemas.

It has been mentioned that Stenning and Yule claim that both of their algorithms are implementations of a more abstract algorithm for individual case identification. Thus, this concept would be at the heart of syllogism solving, and in particular it would account for the process of insertion in the verbal startegy. The rest of this paper will be devoted to show that Störring's insertion process and its more recent variants appear explicitly in Aristotle's writings and that Stenning and Yule's concept of individual case identification, which seems to capture the essence of the syllogism, is implicitly present in one of Aristotle's method of proof.

Aristotle's methods of proof and Ecthesis

As is well-known, in order to identify the concludent modes, Aristotle distinguished two methods of proof, and a subsidiary one. Since the first two involve a "reduction" in two different senses, it is wise to follow Bochenski (1970) and call them the *direct,* and the *indirect method,* respectively.The *direct method* consists of the transformation of the syllogism into a perfect one by the conversion of one or both premises; in addition, depending on the syllogism, it may be necessary to apply one or both of the following operations: the transposition of the premises, the

conversion of the conclusion. Thus, this method (often calld *method of conversion* for that reason) requires knowledge of the conversion of I and E propositions: *some A are B* to *some B are A; no A are B* to *no B are A;* and *all A are B* to *some B are A* (called *conversion by limitation).*

Not all syllogisms can be proved by conversion, and a complementary method is needed. This second method, the *indirect method* applies a *reductio ad impossibile.* Given a syllogism *P, Q / C,* it consists of conjoining the negation of the conclusion *(not-C)* to one of the premises, say *P,* and showing that *P, not-C / not-Q* is a perfect syllogism. It is a fairly sophisticated method for non logicians, and one should not expect to find participants in psychological experiments to apply it (especially when the conclusion is not provided). In contrast, the direct method, in the simple cases where only one or two conversions are involved, is easy to execute. Since the rules of I and E conversion are mastered by a majority of people (Begg and Harris, 1982; Newstead & Griggs, 1983; Politzer, 1990), one could expect that some individuals have a strategic use of the direct method. Unluckily, usual methodology does not allow its identification; think aloud protocols might be useful to look for this possibility.

All valid syllogisms can be solved by the direct or the indirect method. However, Aristotle mentions a third method, *ecthesis* (also called by classical authors the *method of exposition)* which he applies only a few times, and always in a rather allusive manner. This lack of precision, and also the fact that such an alternative method of proof was not necessary, has aroused many comments and speculations about Aristotle's motivation (Kneale & Kneale, 1978). Here is the passage with the most explicit use of ecthesis, aiming to give the proof of AAI-3 *(all M are P; all M are S* ∴

some S are P):

> . . . if both P and S belong to every M, should one of the Ms, e.g. N, be taken, both P and S will belong to this, and thus P will belong to some S. (Analytica Priora, 6, 28a; transl. Ross, names of classes changed to fit this chapter's notations).

Here Aristotle executes the extraction (which is the meaning of *ecthesis)* of "one of the Ms", calls it N, of which P and S are still predicated, and states that from this it follows that P is predicated of S.
Lukaciewicz points out that, as objected in the third century A.D. by Alexander of Aphrodisias (one of Aristotle's greatest commentators), in order for the conclusion to follow, one would have to assume N to be a sub-class of M and apply AAI-3 to *all N are P; all N are S,* again an AAI-3 syllogism, which is entirely circular. Rather, Alexander proposed a non-logical interpretation of the passage, according to which N is an *individual* of which it is easy, *through perception,* to predicate both P and S, and so realize that *some P is S.*

In brief, this is a *psychological* interpretation of Aristotle's obscure passage, which is of great interest to us because it shows how the intuition of a logician in the antiquity meets the heuristic followed by logically naïve individuals in the twentieth century, captured by the first step of Storring's insertion process: *to solve syllogisms, try first to extract an* individual *out of one premise.*

But is there not a *logical* interpretation of Aristotle's passsage above? Modern analysts of Aristotle's syllogistic agree to answer affirmatively (Lear, 1980; Lukaciewicz, 1958; Patzig, 1968; Thom, 1981). A logical interpretation can be given on the assumption that in the present case, as

in many other occasions, Aristotle limited himself to giving the sketch of a proof. The analyses diverge in that for Lukaciewicz and Patzig the logical proof requires an existential quantification over a term; Patzig proposes the following law for ecthesis:

some B are A $\Leftrightarrow$ ($\exists$C) [(all C are A) & (all C are B)]

(and a similar one, mutatis mutandis, for the *some not* case) from which the proof for AAI-3 follows easily. However, formally, there are difficulties linked with the fact that such laws insert syllogistic in propositional logic and second-order predicate logic.

The other interpretation of ecthesis, based on the extraction of an individual-variable rather than of a class-variable, does not have these shortcomings and Thom (1981) shows that it results in a simple system which contains singular and universal syllogisms. The only two singular syllogisms in the first figure are axioms. They are (**a** being an arbitrary instance of A, **b** of B, etc.) *all B are A;* ***c*** *is a B* $\therefore$ ***c*** *is an A,* and *no B are A;* ***c*** *is a B* $\therefore$ ***c*** *is not an A.* From the rule of proof *per impossibile,* taken as an axiom, and the previous two syllogisms, the only two singular syllogisms in the second figure follow: *all B are A;* ***c*** *is not an A* $\therefore$ ***c*** *is not a B,* and *no B are A;* ***c*** *is an A* $\therefore$ *c is not a B.*

It will be noticed that these four syllogisms are identical with Ford's rules A1, B1, A2, B2, respectively. Adding the axiom ***a*** *is an A,* there follows from the two singular syllogisms in the first figure the following theses: *all A are B* $\therefore$ ***a*** *is a B* and its negative counterpart *no A are B* $\therefore$ ***a*** *is not a B* which both capture the concept of extraction of the individual **a** out of A. By an application of the first one to ***b*** *is an A;* ***b*** *is a C* $\therefore$ *some C are A*

(which is provable *per impossibile* applied to the second singular syllogism in the first figure) one reproduces Aristotle's proof of AAI-3. It can be seen that this proof explains Aristotle's by providing the missing step (viz. the latter syllogism) that made it obscure, while justifying Alexander's intuitive rendering of it.

In brief, ecthesis is a logically correct method of proof, and comments such as Alexander's reflect, besides the lack of logical tools to justify it, the intuitive appeal of the logical principles on which it is based. In didactical situations, proof by ecthesis is very convincing, a sufficient indication that it captures something psychologically essential to solve syllogisms. Nevertheless, classical logicians lost interest and sight of it, and this had the unfortunate consequence that it has escaped psychologists' attention, at least consciously.

In agreement with Stenning & Yule's theory, the operative use of diagrams (whether Stenning & Oberlander's algorithm or Venn diagram) and the container-content analogy enable one to solve syllogisms because they all enable the reasoner to catch *simultaneously* the three properties affirmed or denied of an individual (or of a set of individuals) and this probably captures the essence of syllogism solving. The container-content analogy is powerful in that it exploits our capacity to view simultaneously an individual as an A, a B, a C or their negation through a spatial interpretation of the abstract relation *IS A.* Now, with respect to this conceptualization, ecthesis can be viewed as a crucial step in that it primes the process of identification by providing an individual together with the first term of the description (a role played by Stenning &Yule's source premise). Störring's description of his participants' insertion process is the

counterpart of this logical process observable among those participants who possess the strategy.

What about ecthesis within Aristotle's writings? It is fascinating that what seems to be at the heart of the *psychology* of syllogisms, namely the capability of extracting an entity in order to make a subsequent multiple affirmative or negative attribution, surfaces in the writings of the founder of the syllogistic. The fact that ecthesis was logically unnecessary (that is, unnecessary to prove the validity of syllogisms in his system), that Aristotle knew that (having proved all the syllogisms by the two main methods), but nevertheless used it, attests to its importance from a point of view different than logical. We can surmise that Aristotle must have recognized by introspection its role in reasoning, that is, in the mental process by which he himself, without any doubt a particularly skilled individual capable of using such a strategy, probably worked out the solution of some of them. Admittedly, this is pure speculation, but a psychologist involved in the study of syllogistic reasoning cannot refrain from considering such a hypothesis.

Appendix: A traditional description of categorical propositions and of syllogisms

Categorical propositions

From an extensional point of view, they affirm or deny that a class S is included in a class P in whole or in part, which gives four types of proposition:

all S are P (universal affirmative, called an A proposition)

no S are P (universal negative, called an E proposition)

some S are P (particular affirmative, called an I proposition)

some S are not P (particular negative called an O proposition)

Categorical syllogisms

They are deductive arguments made of three categorical propositions: the two premises and the conclusion. The three propositions taken together involve three classes, each of which occurs in two propositions. The three classes are labelled S, P, and M, (called the *terms* of the syllogism). P (the *major* term) occurs in the first premise (called the *major premise)* and as the predicate of the conclusion. S (the *minor* term) occurs in the second premise (called the *minor premise)* and as the subject of the conclusion, and M (the *middle term)* occurs in each premise but not in the conclusion. These constraints determine the following four dispositions, called *figures:*

	1	2	3	4
major premise	M P	P M	M P	P M
minor premise	S M	S M	M S	M S
conclusion	S P	S P	S P	S P

For each figure, there are 4(proposition types)$^{3(\text{propositions})}$ = 64 manners of

constituting a syllogism, called *moods.* Combining with the four figures, this yields 256 syllogisms designated by their abbreviated propositions and their figure number. For instance, EIO-1 designates: no M are P; some S are M ∴ some S are not P.

In psychology, the term *mood* is often used to refer only to the two premises presented to the participant. In this special sense, there are only $4^2 = 16$ moods. Investigators select their problems among 4 (figures) X 16 (moods) = 64 problems called *syllogisms* although they are only pairs of premises. (In this sense, the previous example will be called an EI-1 syllogism, omitting the conclusion O).

With the conventional constraint that the conclusion should have the S term as a subject and the P term as a predicate (S P conclusion) there are only 19 valid syllogisms. But this constraint is hard to maintain when participants are required to produce their own response: they are free to give conclusions with P as a subject and S as a predicate as well (P S conclusions). After relaxing this constraint, there are 27 valid syllogisms, that is, syllogisms whose conclusion cannot be false if the premises are true.

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Footnotes

1. The major part of the article is occupied by relational reasoning. It includes different kinds of two-premise arguments: spatial (e. g. S is to the left of D, R is to the right of D ∴ ?); temporal (e. g. action A is posterior to action C, action D is anterior to action C ∴ ?); linear (F is longer than K; L is shorter than K ∴ ?); and of equality (e. g. A = K; P = K ∴ ?).
2. A description of syllogisms is given in the Appendix.
3. The first figure being MP; SM, the inversion of the premises yields SM; MP which coincides with the fourth figure, as shown by the position of the middle term (the labels P and S are immaterial).
4. There is also, while solving an AA-4 syllogism *(all the P are M; all the M are S)* a report of a three-step stair configuration, with S, M, and P terms on the top, middle, and bottom steps, respectively.
5. Possibly by Boethius (6th century A.D.) according to Patzig (1968).
6. Straight lines can be found in Alstedius' (1614) work and circles in Sturm (1661).
7. Remarkably, one of Ford's participants did just that for IE-3 (drawing page 61). This supports the hypothesis that people who use diagrams may have a figurative, rather than operative, use of them: in that case, the diagram comes in support of their verbal strategy to illustrate the principles which they apply at a metacognitive level.
8. In view of this, the expression "Euler circles" often used to refer to Gergonne's five diagrams is a clear historical mistake. Although Gergonne himself explicitly refers to Euler, he correctly remarks that Euler failed to fully exploit his own idea.

9. Gergonne's paper actually does not contain any diagram!

10. In terms of causality (but this point of view is in no way obligatory) P **c** Q is a sufficient condition, P ⊃ Q, a necessary one, P **I** Q a necessary and sufficient condition, and P **X** Q a statistical correlation whose value depends on the extent of overlaping.

11. Horn (1989) notes that the Gricean view was anticipated by philosophers and logicians, starting with J.S. Mill who pointed out the *not all* inference made in natural language when interpreting I sentences.

12. Another prediction concerns reaction times: it should take them equal times to make *all* judgments on **c** and **I** diagrams; reaction times for *some not* judgments should be equal on **X** and ⊃ diagrams, and shorter than on **H** diagrams; and they should be shorter for *some* judgments on the **X** diagram than in ⊃ , **c**, and **I**.

13. Ford notes that this strategy must be used with care because equating two terms should not imply their universal equivalence: for example, in EA-4 *(no P are M; all M are S),* substituting S for M without precaution would yield *no P are S,* which is erroneous). She describes a "sophisticated substitution" applicable to similar cases.

14. Arnauld and Nicole did not give the counterpart of this principle for the moods EAE-2 and EIO-2 (the equivalent of B2) for reasons of economy: they prefer to justify them by appeal to the principle of the negative moods for the first figure applied after the conversion of the E premise (which turns these two syllogisms into the first figure).

Figure 1. The representation of the four quantified sentences in Leibniz's straight lines system.

```
all A is B          A   ._______.
                        .       .
                        .       .
                    B   ._______._____________.

no A is B           A   _______.       .
                               .       .
                               .       .
                    B          .       ._______.

some A is B         A   _______._______.
                               .       .
                               .       .
                    B          ._______._______.

some A is not B     A   ._______._____________.
                        .       .
                        .       .
                    B   .       ._____________.
```

Figure 2. Leibniz line diagram for the AA-3 syllogism.

```
1.          P     _____.__________.
                        .          .
                        .          .
2.          M           .__________.
                        .          .
                        .          .
3.          S           .__________._____.
```

lines 1 & 2: all M is P
lines 2 & 3: all M is S
lines 1 & 3: some S is P

Figure 3. Euler's diagrammatic representation of the four categorical propositions.

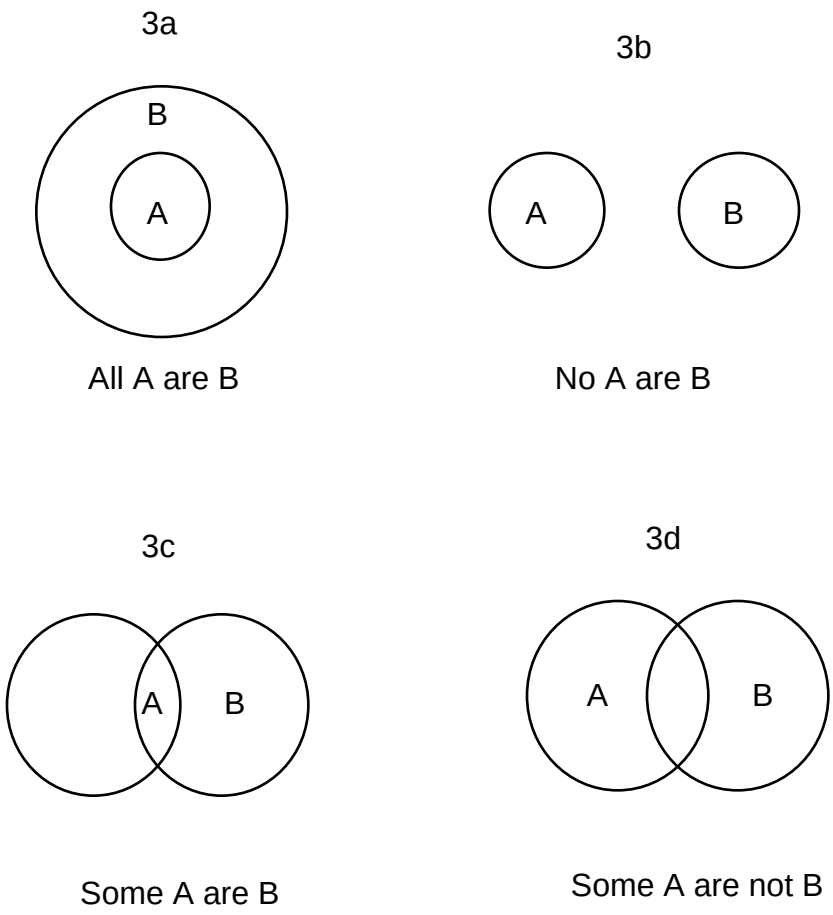
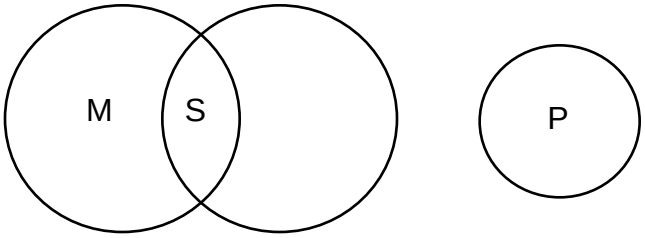
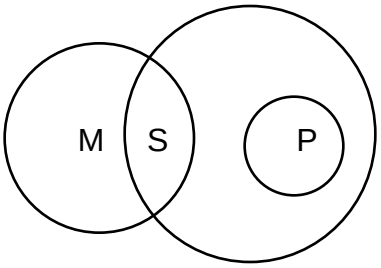


Figure 4. Euler's diagrams for the EI-1 syllogism.

4a



4b



4c

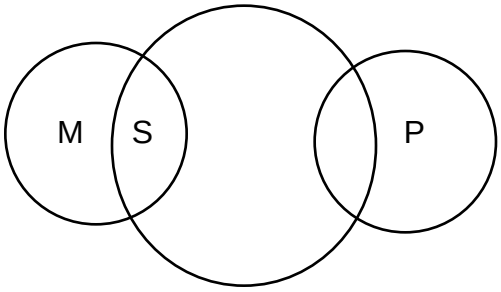


Figure 6. Gergonne's diagrams and their mapping onto the four categorical propositions.

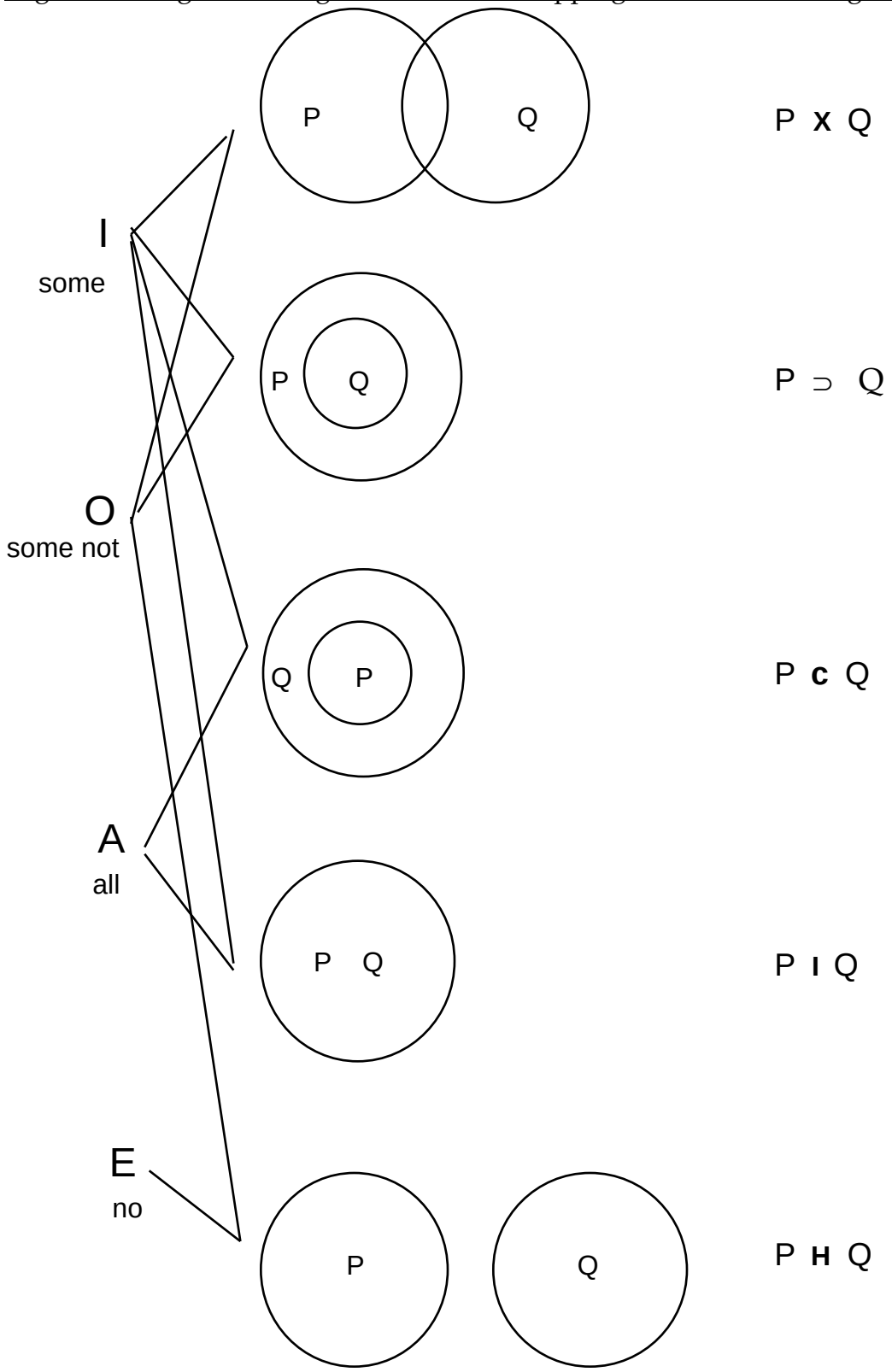
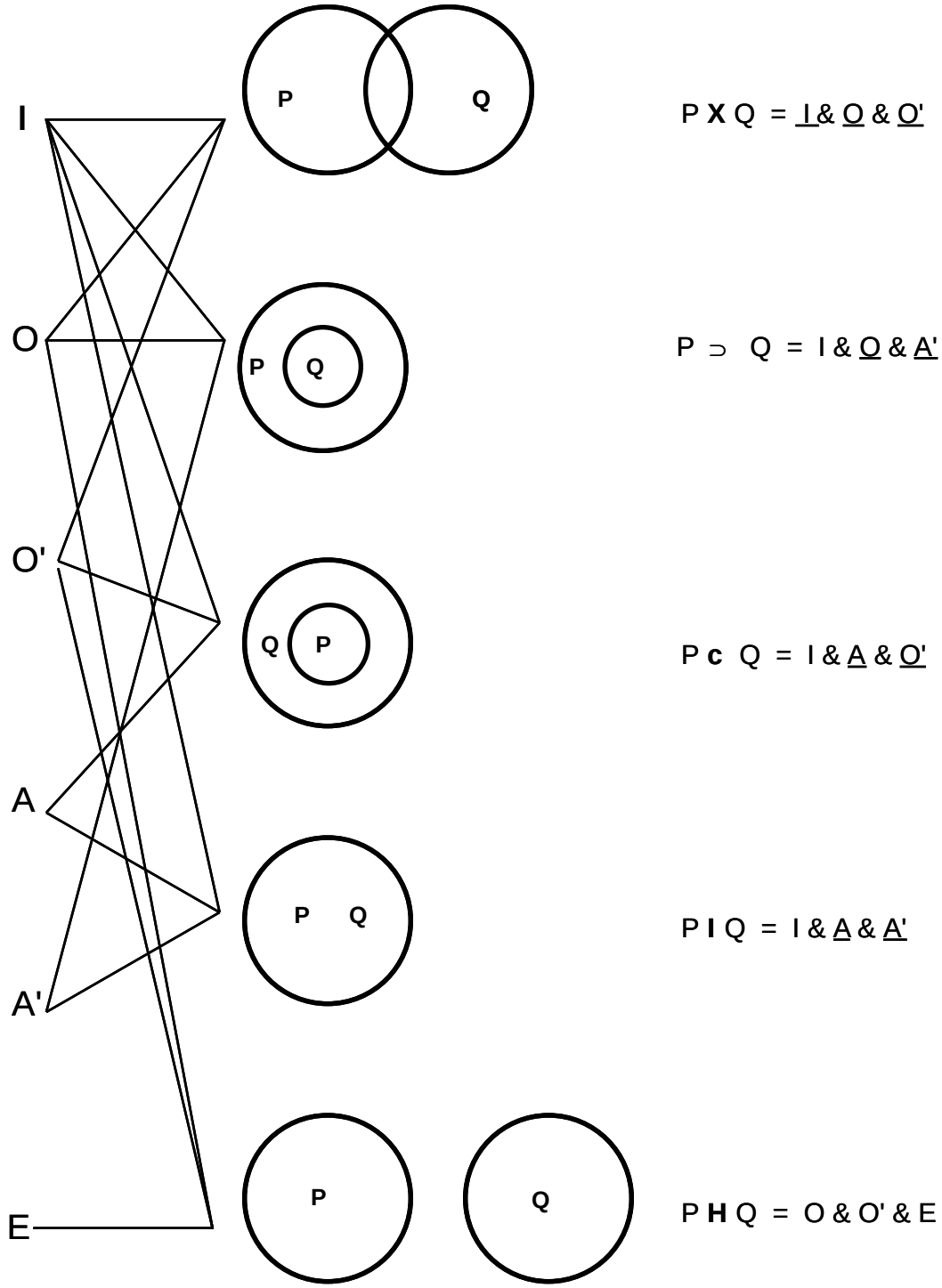



Figure 7. Gergonne's diagrams complemented with converse propositions and their formulas.