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An energy-like indicator to assess opinion resilience

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Abstract

Using the bounded-confidence model, with fixed uncertainties and extremists, we investigate how resilient the moderate mean opinion of a population is to the arrival in it of a new group of agents, when the energy of the opinion of this group (extremeness \tilde{A} group size) is varied. We say moderate mean opinion is resilient when, even though it may become temporarily more extreme after the arrival of the new agents, it later recovers its moderate value. We show that such resilience is displayed up to a threshold value of the equivalent energy of the group. We also show that when the agent-based model spontaneously converges to a single extreme, then this energy threshold is nil.

Keywords: BC model, Opinion dynamics, Resilience, Tipping energy

1. Introduction

1 The propagation of extremism is a growing subject of interest for both
2 policy makers and scholars, now that religious extremism leading to terrorist
3 attacks has become a serious concern in Western societies. Here we consider
4 religious extremism as a particular case in a broader definition of extrem-
5 ism as a very strong commitment for or against something, whether social,

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E-mail address: jean-denis.mathias@irstea.fr (J.-D. Mathias).

6 political, ethical, religious, or other. We make no normative assessment of
7 extremism: an extreme opinion is generally considered by the majority as
8 disturbing or dangerous only so long as it remains a minority view. If it
9 gets adopted by a majority, then that majority's assessment changes. Such
10 switches from minority to majority have occurred for various issues, e.g.
11 abortion or divorce in Europe in the last century, or the prohibition of smok-
12 ing in public places a few decades ago. Of course, not all extreme minority
13 opinions get adopted by the majority.

14 Outside the traditional media (newspapers, television, talks, etc.), more
15 and more attention is now paid to the impact of new technologies on the
16 propagation of extremism [1] [2] which are making this issue more and more
17 difficult to address. It is also well known that uncertainty influences the
18 propagation of opinion within a population [3], this uncertainty decreasing
19 [3] or increasing [1] with time according to how and with whom views are
20 discussed [1].

21 We argue that besides experiments and surveys, computer simulations can
22 offer further insights into this issue by testing a set of different assumptions.
23 Among the several models available on extreme opinion propagation, we
24 focus on the bounded-confidence (BC) model [4, 5, 6, 7] close to the theory
25 of social comparison [8] and to the social judgment theory [9]. Specifically, a
26 new stationary state of the BC model has been recently highlighted that is
27 particularly relevant to our topic, in which uncertain agents avoid converging
28 to extreme positions: depending on uncertainty and/or the proportion of

29 extremists, their opinions keep fluctuating with ranging amplitudes instead of
30 becoming extreme [10]. Our study goes further by investigating the behavior
31 of such a population, assumed to be capable of resisting extremism, when
32 faced with the arrival of a group of agents ranging in size with an extreme
33 opinion of ranging intensity, as in [11]. One such group might be workforce
34 immigrants, for whom local social networks support common norms [12].
35 We characterize an arriving group by two attributes: i) its *size* and ii) the
36 *extremeness of its opinion*. We assume that the arriving agents have the
37 same level of uncertainty as the moderates in the initial population. In what
38 follows, we simulate the arrival of different groups in different populations.
39 We use a density distribution model (DDM) derived from the agent-based
40 model (ABM) of the BC model. Such a model avoids the uncertainty due to
41 stochasticity (unlike the agent-based model) and requires less computation
42 time than the agent-based model. The DDM runs much faster than the
43 ABM, and we showed in [10] that it reliably mimicks the behavior of the
44 ABM in most cases. Moreover, when it does not, the discrepancy is very
45 informative about the role of the noise in the ABM.

46 We take the moderate mean opinion to be resilient to the arrival of a new
47 group when the mean opinion of the whole set of agents (initial ones plus the
48 new group) recovers its initial value some time after the arrival of the group.
49 Conversely, if the mean opinion of the whole population becomes extreme,
50 then we consider that the moderate mean opinion is not resilient. We show
51 that an indicator inspired from energy in material science, the size of the

52 group multiplied by the absolute extremeness of its opinion, can be used to
 53 predict whether the moderate mean opinion will be resilient. Simulations
 54 show that beyond a given threshold of this indicator, the moderate mean
 55 opinion is no longer resilient, and switches to extreme.

56 **2. The bounded-confidence model (BC)**

57 *2.1. The agent-based model (ABM)*

58 Following the original model of [13], we consider a population of N agents
 59 composed of moderate and extremist agents. The moderate agents exhibit
 60 the same uncertainty u , whereas the extremists exhibit very low value of
 61 uncertainty u_e compared with u (extremist opinions can be considered as
 62 quasi-static). The initial opinions of the moderate agents are uniformly dis-
 63 tributed along an opinion axis between -1 and +1. An extremist agent ex-
 64 hibits the following two features: (i) it expresses an extreme opinion, and
 65 (ii) it has a high certainty about its opinion. The proportion of extremists in
 66 the population is noted p_e : half of them are initialized with opinion +1 and
 67 the other half with opinion -1. Each agent is paired once in one period: $\frac{N}{2}$
 68 random pairs interact at each time step. When an agent i (with opinion x_i
 69 and uncertainty u_i) meets an agent j (with opinion x_j and uncertainty u_j),
 70 their opinions are modified as follows:

$$\left\{ \begin{array}{l} \text{if } |x_i - x_j| < u_i \text{ then } x_i := x_i + \mu(x_j - x_i) \\ \text{if } |x_i - x_j| < u_j \text{ then } x_j := x_j + \mu(x_i - x_j) \end{array} \right. \quad (1)$$

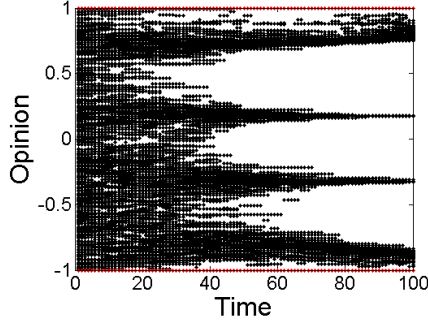
71 where μ models the intensity of the interaction (generally $0 < \mu \leq 0.5$).
72 All individuals' opinions are updated simultaneously at each time step yield-
73 ing a simultaneous updating of the opinions as in the density distribution
74 model described in the next section. From this agent-based model, we iden-
75 tify four main patterns of convergence [10](see Figure 1): central, double
76 extreme, single extreme convergence and no convergence, the latter corre-
77 sponding to the case where opinions keep fluctuating indefinitely.

78 2.2. The density distribution model (DDM)

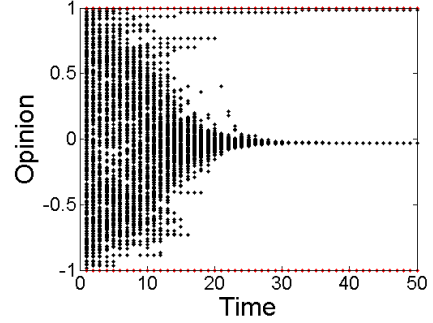
79 We sought to analyse different initial configurations of populations sub-
80 jected to different rises in opinion energy. However, an agent-based model
81 is inherently stochastic, and simulations require high computation time. To
82 avoid stochasticity and shorten computation time, a solution of interest is
83 to study a density distribution model derived from the agent-based model
84 that represents the case of an infinite number of agents. This type of model
85 makes the different phenomena more apparent because the initial uniform
86 distribution can be perfect, and eliminate the random irregularities of the
87 agent-based model. The model includes an array of n values ρ_i representing
88 the probability that agents have an opinion x located in an interval of size
89 $1/n$ and centered on i/n :

$$\rho_i = P\left(\frac{2i-1}{2n} \leq x \leq \frac{2i+1}{2n}\right) \quad (2)$$

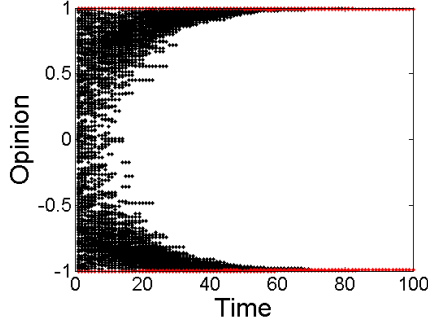
90 The density distribution of the moderate opinions is initiated with a uni-



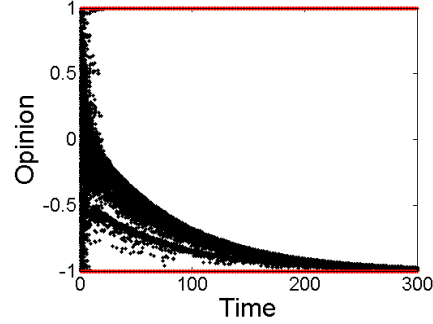
a - $p_e = 0.05$, $u = 0.25$, central clusters, red dots in Fig. 1-f



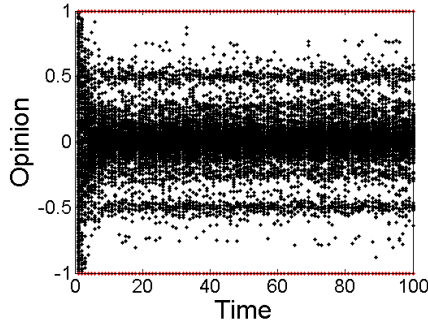
b - $p_e = 0.05$, $u = 0.75$, single central cluster, light blue in Fig. 1-f



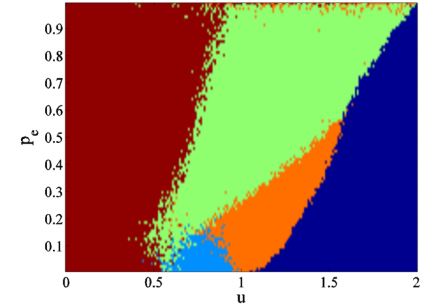
c - $p_e = 0.3$, $u = 0.8$, double extreme, green dots in Fig. 1-f



d - $p_e = 0.05$, $u = 1$, single extreme, orange dots in Fig. 1-f



e - $p_e = 0.05$, $u = 1.8$, opinions fluctuate, dark blue dots in Fig. 1-f



f - Map of convergence patterns

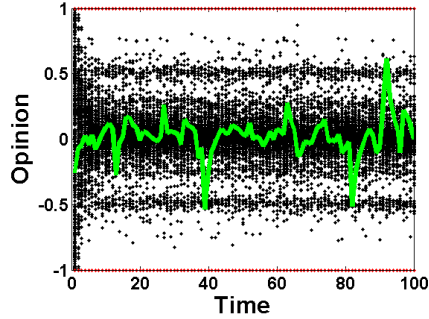
Figure 1: The four convergence patterns observed with the BC model [10]. The opinions of the 400 agents are represented at each time step (black: moderate agents; red: extremist agents). p_e is the proportion of extremists and u is the uncertainty of the moderates for the map of convergence patterns (Fig. 1-f).

91 form distribution (for all i , $\rho_i = \frac{1-p_e}{n}$). The extreme values of the distribution
92 corresponding to extremists are equal to $p_e/2$ at each extremity. We then
93 compute the flows between the cells of the density because of the interac-
94 tions, and update iteratively (see [10] for details). In Figure 2 we compare
95 the density distribution model with the agent-based model (ABM) after the
96 density convergence of the two models. The density distribution calculated
97 with the DDM provides a close approximation to that derived from the ABM
98 with a large population for $u = 1.8$ (see [10] for more details about the lumpy
99 pattern of the density distribution). However, for $u = 1$, the DDM predicts a
100 density convergence whereas the ABM converges to a single extreme cluster.

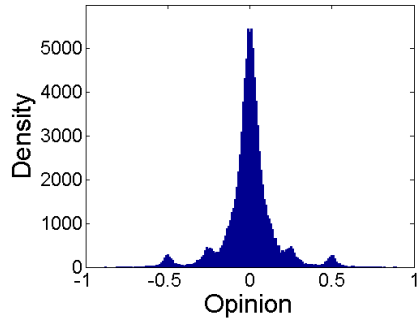
101 *2.3. Assessing opinion resilience*

102 **As explained above, we set out to study opinion resilience of a**
103 **population when a new group arrives. In what follows, we focus**
104 **particularly on two zones:**

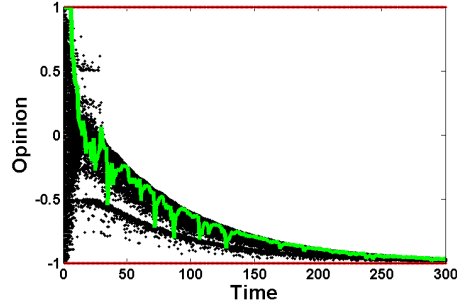
- 105 • **first the zone where opinions keep fluctuating (dark blue area**
106 **in Fig. 1-f). In a large parameter zone (high values of u) the**
107 **opinions do not stabilize: moderate opinions keep fluctuating,**
108 **while the density distribution stabilizes. Interactions between**
109 **moderate agents tend to form a central maximum of density,**
110 **keeping the opinions in a zone where they can be attracted**
111 **by either extremes [10]. This leads to a stable density of opin-**
112 **ions (see [10] for more details about mechanisms yielding these**



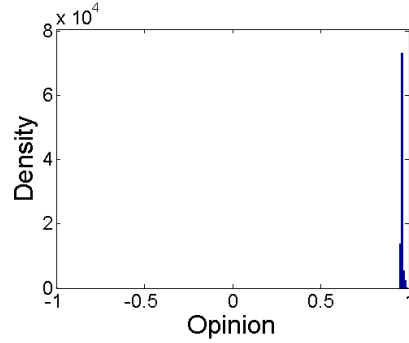
a - $p_e = 0.05$, $u = 1.8$, ABM
opinions fluctuate



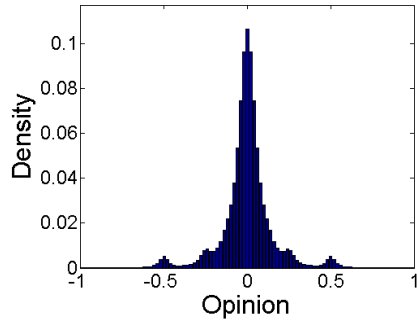
c - $p_e = 0.05$, $u = 1.8$, ABM,
density distribution



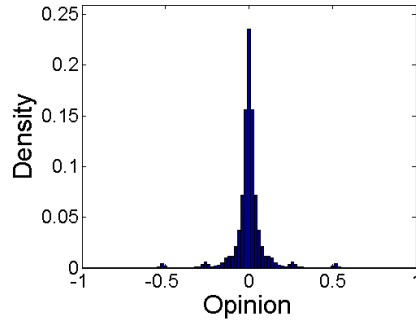
b - $p_e = 0.05$, $u = 1$, ABM,
single extreme



d - $p_e = 0.05$, $u = 1$, ABM,
density distribution



e - $p_e = 0.05$, $u = 1.8$, DDM,
density distribution



f - $p_e = 0.05$, $u = 1$, DDM,
density distribution

Figure 2: Comparison of the density distributions obtained with the ABM and with the DDM (for clarity, densities were plotted with 100 000 agents, whereas other Figures used 400 agents). In Figures 2-a and 2-b, the green line corresponds to the opinion trajectory of one agent. Although the DDM provides close approximations of the ABM in many cases, the DDM cannot reproduce the density distribution in the case of the single extreme (Figure d versus Figure f).

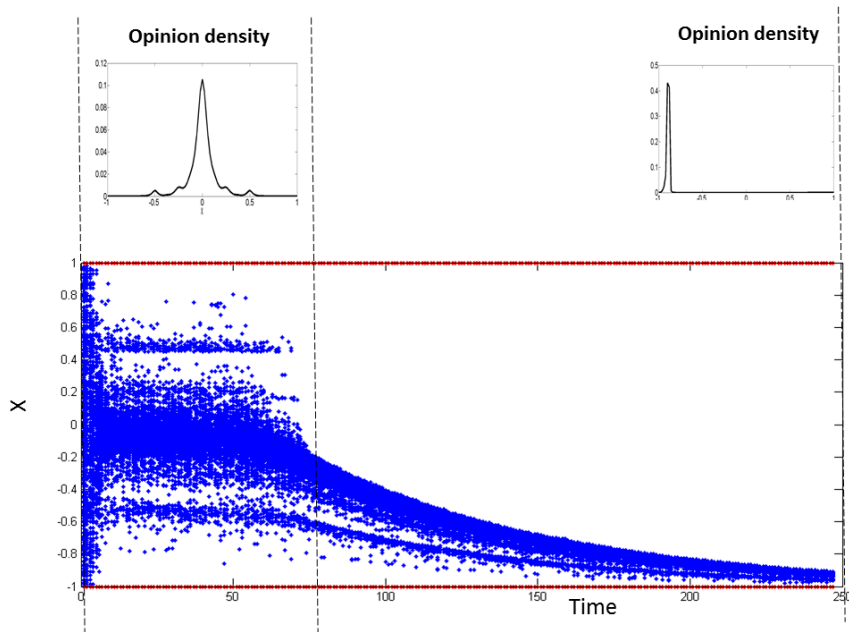


Figure 3: For $p_e=0.05$ and $u = 1.1$, ABM converges to a single extreme, unlike the DDM. On the left, we see that opinions first keep fluctuating and then converges to a single extreme.

113 fluctuations and this density stabilization). The first issue re-
114 lies on showing whether the stationary distribution of opinions
115 bounces back to its initial shape with an average opinion close
116 to 0, or converges to a single extreme with an average opinion
117 close to +1 or to -1. The corresponding results are reported
118 in Section 4.1;

- 119 • second, the zone where the ABM converges to a single ex-
120 treme. For $u = 1$, the DDM predicts a stationary density of
121 0 average opinion, while the ABM converges to a single ex-
122 treme cluster: the DDM, with symmetric initial conditions,
123 cannot produce asymmetric patterns (as already observed for
124 instance in [14]). However, when we further analyse the ABM
125 results (see Figure 3), we see that before converging to a sin-
126 gle extreme, the ABM first converges to a quasi-stationary
127 state of fluctuating opinions. Our second focus thus implies
128 studying with what perturbation we can obtain this single ex-
129 treme with the DDM. The corresponding results are reported
130 in Section 4.2;

131 Both cases amount to perturbing a population with a new group
132 arrival and observing whether the population recovers the same av-
133 erage opinion as beforehand. This therefore corresponds to study-
134 ing the resilience of the average moderate opinion. For this pur-

135 **pose, the concept of resilience has to be clearly defined.**

136 **3. Characterizing the resilience of population opinion**

137 *3.1. Resilience of what to what? keeping a null moderate mean opinion*

138 We use the DDM to characterize how a population behaves when a new
139 group arrives: either the opinion of the population polarizes, or the popu-
140 lation is resilient and recovers its initial moderate mean opinion. However,
141 concepts of resilience (and many associated ones) are used broadly in the
142 literature, under ranging definitions depending on the context of the study.
143 Here, we choose a definition originating from [15, 16, 17] that stresses the
144 need to define the property or function whose resilience is being studied, and
145 the perturbations by which the system may lose this property [18]. Here,
146 the perturbation is defined by the group arrival, noted r . The property of
147 interest that we consider here is based on the moderate mean opinion $X(t)$
148 of the moderate agents' opinions ρ_i^{mod} defined as:

$$X(t) = \frac{\sum_1^n \rho_i^{mod}}{n} \quad (3)$$

149 **In the case of fluctuating opinions**, the stationary state of the mod-
150 erate mean opinion $X(t_0)$ equals 0. This is the property that we want to
151 recover after the arrival of new group within the population: either the
152 moderate mean opinion bounces back to the initial moderate mean opin-
153 ion ($X(t) = 0$, and the system is defined as resilient) or it converges towards

154 a single extreme ($|X(t)| = 1$, and the system is defined as not resilient).

155 3.2. Group arrival r

156 As **explained** above, a small perturbation can cause the collapse of a
157 population from a stationary density to a single extreme cluster. We perturb
158 the DDM (after a density convergence) with a new group arrival $r(\varepsilon, x, \tau)$
159 at time τ ($\tau > t_0$), opinion x of size (percentage of the initial population)
160 $\varepsilon \in [0; 1]$ defined as:

$$r(\varepsilon, x, \tau) = \varepsilon \delta(x) \delta(\tau) \quad (4)$$

161 δ being the Dirac function. We note that the new group is composed
162 of moderate agents (with the same uncertainty u as the moderate agents of
163 the population). With the arrival of the new group, a percentage ε of the
164 moderate agents suddenly exhibits an opinion x at time τ :

$$\rho_j(\tau) = \rho_j(\tau - 1)(1 - \varepsilon), \forall j \quad (5)$$

$$\rho_i(\tau) = \rho_i(\tau - 1) + \varepsilon \sum_j \rho_j(\tau - 1), \quad (6)$$

165 where i corresponds to the discrete index of location x . Of course, a
166 percentage $\varepsilon = 1$ corresponds to a purely theoretical situation where the
167 arriving group is of infinite size.

168 *3.3. An energy-based indicator for characterizing how a population behaves*
169 *after a new group arrival*

170 In material science, the resilience of a material is characterized by the
171 energy that it absorbs during loading and subsequently releases when the
172 load is removed. We therefore consider the group size ε as a force and its
173 opinion x as a displacement and we calculate the following energy associated
174 with the rise r :

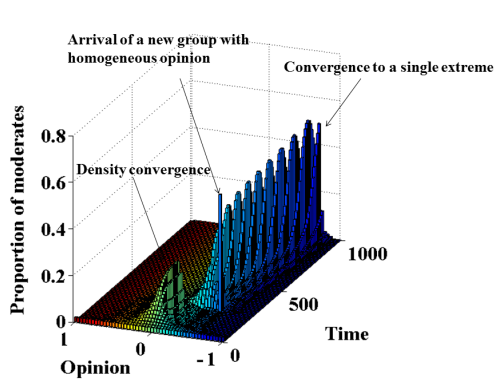
$$E(r(\varepsilon, x, \tau)) = E(\varepsilon, x) = \varepsilon \times \text{abs}(x) \quad (7)$$

175 We note that $E(\varepsilon, x) \in [0; 1]$. After the group arrival, either the moderate
176 mean opinion reverts to 0 (where opinions keep fluctuating with a null mean
177 opinion), or the moderate mean opinion converges to an extreme.

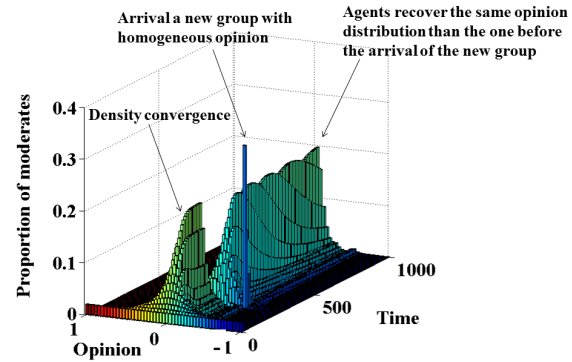
178 **4. Results**

179 *4.1. Assessing the tipping energy E^r*

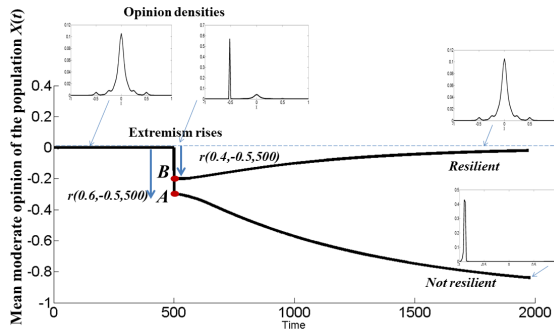
180 For a first study we choose a point in the parameter space located in the
181 fluctuations area presented in Figure 1-f : $u = 1.3$ and $p_e=0.05$. We test
182 different values of the group arrival $r(\varepsilon, x, \tau)$ after the density convergence
183 ($\tau = 500$). In Figure 4-a, we can see that **group arrival A causes a**
184 **population convergence** to a single extreme cluster. On the other hand,
185 the population can absorb a smaller energy group arrival (group arrival B,
186 see Figure 4-b) and reverts to a null moderate mean opinion. Figure 4-c
187 shows the evolution of the moderate mean opinion of moderate agents.



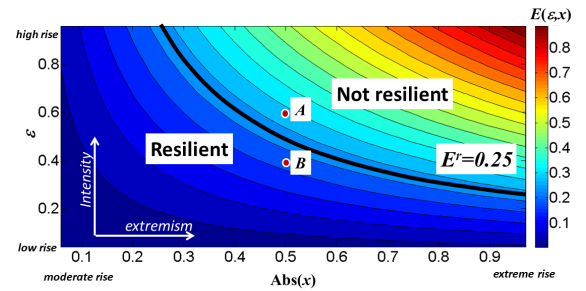
a - Group arrival A, group size $\varepsilon = 0.6$



b - Group arrival B, group size $\varepsilon = 0.4$



c - Evolution of the moderate mean opinion of the population



d - Resilience map according to the rise in energy

Figure 4: Two examples of the response of a population to a group arrival, $u = 1.3$ and $p_e = 0.05$. A population can absorb the group arrival B and reverts to a null moderate mean opinion. By contrast, group arrival A is too powerful, causing a population convergence to a single extreme. The tipping energy separates the resilience zone from the non-resilience zone.

188 Figure 4-d plots the energy of group arrival according to the resilience
 189 of the population. Two zones appear on this energy map: the first is a
 190 "resilience" zone that corresponds to group arrivals to which the moderate
 191 mean opinion is resilient, and the second to the group arrivals to which the
 192 moderate mean opinion is not resilient. The main result is that a constant
 193 energy separates the two zones: the moderate mean opinion can "absorb" a
 194 maximum energy by changing its distribution dramatically, and then recover
 195 it. Whatever the location and intensity of the energy rise, its energy level
 196 alone enables us to predict the resilience (or non-resilience) of the moderate
 197 mean opinion. This energy threshold hence defines tipping points of the
 198 opinion dynamics. Accordingly, we call "tipping energy" E^r , the maximum
 199 energy of the group arrival that can be absorbed by the population. More
 200 specifically, we define the tipping energy E^r associated with the stationary
 201 state $X(t_0)$ (the mean opinion of the population at date t_0) as:

$$E^r = \max[E|\forall r(\varepsilon, x, \tau), t_0 < \tau \ll t, X(t) = X(t_0)] \quad (8)$$

202 In our case ($u = 1.3$ and $p_e=0.05$), the tipping energy E^r equals to 0.25
 203 and characterizes the resilience of the population. We note that the resilience
 204 energy is constant if almost every agent interacts with all the others in the
 205 population (like a particle model in physics) with a high value of uncertainty
 206 u . But, in some cases (u around 1.2), the tipping energy E^r is not exactly
 207 constant: there are very small variations according to group size and opinion

208 because some agents do not see the whole population.

209 4.2. Emergence of a single extreme cluster due to instability

210 We now focus on the zone where the ABM converges to a single extreme
211 whereas the DDM does not (see Figures 1-d and 1-f) in order to explain
212 the emergence of single extreme cluster. For this purpose, we calculate the
213 tipping energy E^r for different values of u . Compared with $u = 1.3$, we
214 expect lower values of uncertainty u to lower the tipping energy, causing the
215 convergence of the population towards extremists (see Figure 5).

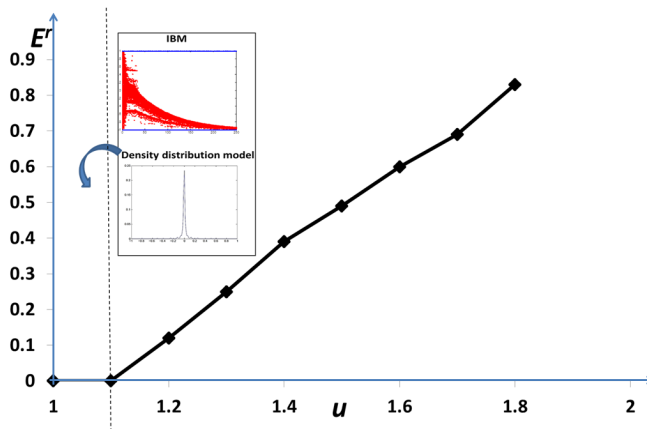


Figure 5: Resilience energy according to u . For $u < 1.1$, the tipping energy is nil. It corresponds to the threshold location of single extreme in the convergence map of the ABM for $p_e = 0.05$ (see Figures 2 and 3).

216 As expected, the tipping energy increases with the uncertainty u : the
217 higher the uncertainty of the moderate agents, the higher the resilience of
218 their moderate mean opinion. On the left of **Figure 5**, the tipping energy is
219 nil: it corresponds to **the areas of the parameter space** of a single extreme

220 in the convergence map of the ABM. This zone appears in the convergence
221 type maps (for u around 1 and $p_e < 0.2$, orange in Figure 1-f) where a
222 single extreme cluster emerges. This cluster is attracted by extremists, and
223 the population switches to one extremist side, leading to a single extreme
224 convergence. However, the single extreme zone cannot be found using the
225 density model without perturbations: the "single extreme" zone in the ABM
226 (orange in Figure 1-f) corresponds to a "opinions keep fluctuating" zone with
227 the density model.

228 For $u \in [1; 1.1]$, the density model yields an "opinions keep fluctuating"
229 zone, whereas the ABM converges to a single extreme cluster (see Figure
230 5). The "single extreme" zone corresponds to an "opinions keep fluctuat-
231 ing" zone where the resilience energy is very low: a very small perturbation
232 causes a convergence towards a single extreme. First, the ABM converges
233 towards an "opinions keep fluctuating" zone, predicted by the DDM. A small
234 perturbation due to the stochasticity then causes the population to converge
235 to a single extreme (see Figure 3). We note that the results are similar
236 when the sizes of opposite extremist groups are different (see Annex A for
237 more details). Also, the tipping energy assessed from ABM simulations fits
238 the tipping energy assessed from DDM simulations (see Annex B for more
239 details).

240 5. Discussion and conclusion

241 Our main finding - based on the BC model with fixed uncertainties - is
242 that the resilience of a moderate mean opinion depends on an equivalent
243 energy of the arriving group, that is the size of the group multiplied by the
244 absolute extremeness of its opinion. Thus either a small group with a very
245 extreme opinion or a large group with a less extreme opinion may push the
246 population towards a single extreme. This is consistent with the results of
247 experiments on the influence of minorities in small groups [19]. In addition,
248 the higher the uncertainty of the moderate agents, the higher the resilience of
249 their moderate mean opinion. Our results also show that even when opposite
250 extremists are not exactly the same in number (see Annex A), the presence
251 of extremists at both ends may prevent extremization of a population of
252 uncertain agents. However, when the extremists are significantly more nu-
253 merous at one end, the energy required for the convergence of the moderate
254 to the stronger extreme is lower, yielding more polarized situations - when
255 the arriving group is on the side of the stronger extreme. Finally, our work
256 can be extended by associating a specific uncertainty to the opinions of the
257 arriving agents (while in this paper, the uncertainty of the new-arrival group
258 is taken to be the same as that of the moderate agents). For instance, we
259 can consider the arrival of an extremist group (agents with extreme opinion
260 and small uncertainty) or we can test strategies to polarize the population
261 [20]: from the tipping energy, we can deduce the size of the group of a given
262 opinion value and a given uncertainty, which is necessary to bring about con-

263 vergence towards a single extreme. Of course, these quantitative results are
264 probably not be directly applicable to real life surveys or experiments. How-
265 ever, the concept of tipping energy may be qualitatively helpful in diverse
266 situations of opinion dynamics, especially with the presence of minorities.

267 **6. Acknowledgments**

268 J.-D. Mathias thanks the French National Research Agency (project VIRGO,
269 ANR-16-CE03-0003-01 grant) for their financial support.

270 **7. References**

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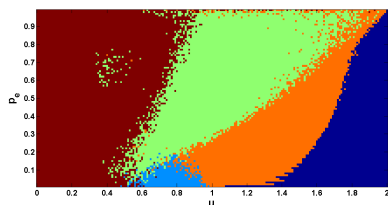
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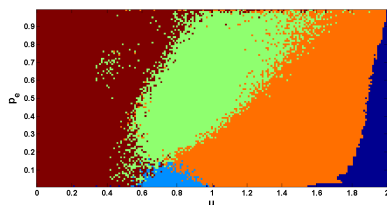
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329 **AppendixA. Unbalanced opposite extremists may also favor the**
 330 **recovery of a moderate opinion**

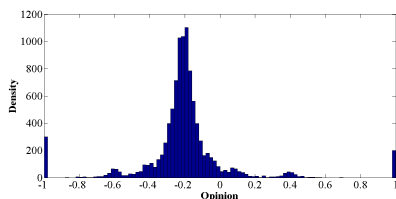
331 In the previous section, we showed that small perturbations may lead to a
 332 single extreme in the case of an equal distribution of extremists at each ends.
 333 Calculations are performed in the case of unbalanced opposite extremists to
 334 see whether the initial distribution of extremists influences our results. We
 335 tested a 60%/40% distribution (60% of extremists located in -1 and 40% of
 336 extremists located in +1) and a 80%/20% distribution (see Figure A.6).



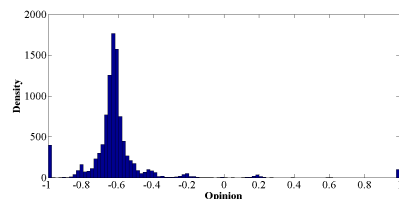
a - Map of convergence, [60%;40%];



b - Map of convergence, [80%;20%]



c - Opinion density [60%;40%],
 $p_e = 0.05, u = 1.8$



d - Opinion density [80%;20%],
 $p_e = 0.05, u = 1.8$

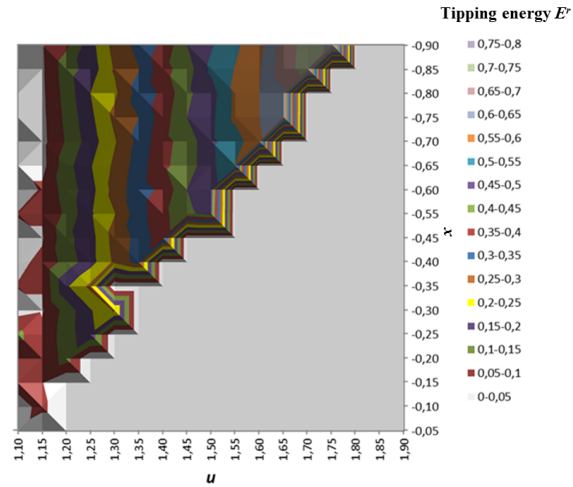
Figure A.6: Influence of unbalanced extremists on the convergence map. Despite the unbalanced extremists, opinions may keep fluctuating. However, the area of single extreme swells with the imbalance.

337 Results in Figure A.6 show that the area where opinions keep fluctuating
 338 shrinks when the extremists distribution is unbalanced: the imbalance (in
 339 terms of extremist location) decreases the resilience energy, and so leads to

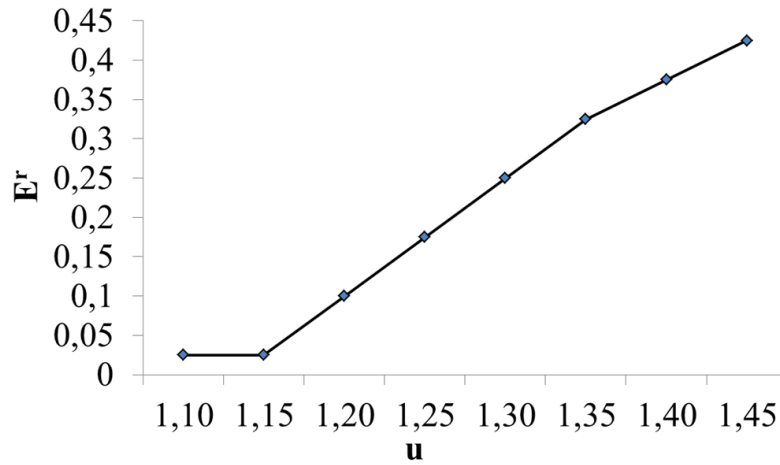
340 more single extreme cases. This is confirmed by the density of the opin-
341 ion which is not centered at zero but is shifted towards the most numerous
342 extremists (see Figures A.6-c and -d).

343 **AppendixB. Assessing the tipping energy with the ABM**

344 We assessed the tipping energy with the ABM to see whether it was the
345 same as the tipping energy with the DDM. We ran 30 replicates with a group
346 arrival located at x . Different values of the groupe size ε were tested. Tipping
347 energy is represented in Figure [B.7](#). Results show a close fit between ABM
348 and DDM results (see Figure 5).



a - Tipping energy according to u , $p_e=0.05$



b - Mean of tipping energy according to u , $p_e=0.05$

Figure B.7: Tipping energy calculated with the ABM, $p_e = 0.05$. For $u < 1.1$, the tipping energy is nil.