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► **To cite this version:**

Mohamad Janbein, Céline Roudet, Christian Gentil, Clément Poull, Lucie Druoton. A Survey of Roughness-Based Applications Intended for Characterising, Controlling or Generating Real or Synthetic Non-Smooth Surfaces. 21st International Conference on Computer Graphics, Interaction and Visualization Theory and Applications, Mar 2026, Marbella, Spain. pp.476-487, <10.5220/0014633700004728>. <hal-05552290>

HAL Id: hal-05552290

<https://hal.science/hal-05552290v1>

Submitted on 13 Mar 2026

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A Survey of Roughness-Based Applications Intended for Characterising, Controlling or Generating Real or Synthetic Non-Smooth Surfaces

Mohamad Janbein¹^a, Céline Roudet²^b, Christian Gentil²^c, Clément Poull²^d and Lucie Druoton²^e

¹LORIA, Université de Lorraine, Nancy, France

²LIB, Université Bourgogne Europe, Dijon, France

mohamad.janbein@univ-lorraine.fr, {celine.roudet, christian.gentil, clement.poull, lucie.druoton}@ube.fr

Keywords: Roughness, Surface Texture, 3D Mesh, Terrain, Statistical Characterisations, Quality Control, Simulation, Surface Metrology, Manufactured Object, Machining Process, Standards.

Abstract: Surface roughness is an inherently complex notion for which no unique definition exists, as its meaning depends on the considered application domain. This difficulty has led different fields to develop their own interpretations, characterisations, generations (and even tools) to address roughness. This is an essential concept in Computer Graphics (CG), when it comes to modelling 3D content for realistic scene creation, subject to an evaluation of their visual quality after undergoing geometric operations (e.g., simplification, compression, remeshing, or smoothing). It is also central in other fields, such as industry or engineering, where its characterisation and control are essential for design, manufacturing, and quality control. This survey examines the most significant works on roughness, exploring the various ways it is defined, utilized, modeled, and generated across the considered fields. It highlights how these initiatives differ and complement each other, with the hope of inspiring the development of new trends in CG or geometric modelling.

1 INTRODUCTION


Roughness is a complex concept, and it is difficult to give a general definition. It is a notion that depends on the application context and encompasses a large number of possible geometric configurations. In Computer Graphics (CG), the appearance of object surfaces plays a key role in the realism of scenes. Rendering algorithms take material properties into account to calculate interactions with light. Roughness is one of the important factors influencing these interactions. Associated with the concept of microgeometry, it is both complex to describe and control. Even if we could describe these geometries, it would still be difficult or very costly to calculate interactions with light, due to the complexity of the geometries. Other fields, such as metrology, tribology, mechanics, physics, chemistry, and biology, are also interested in surface roughness in order to characterise, model,


control, study, and understand its impact on surface properties, whether in research or industry.


We propose a review of the most significant works that have been developed in recent years. We chose to mainly emphasise how they considered surface roughness in their respective field of application.


In general, the control and objective measurement of roughness encounter two essential problems. The first, which concerns only real surfaces (in industry or medical field), is data acquisition. It requires sophisticated and precise digitisation procedures (Leach, 2011; Barbero and Ureta, 2011), due to the observation scale to be considered. This latter is on the order of the micron, for example, in surface metrology of mechanical parts, and of the kilometer in bathymetry of the world ocean (Fox and Hayes, 1985).


The second concerns how to quantify or characterise roughness. In CG, various perceptual metrics, based on curvature and roughness computations, have been proposed since 2007. Taking into account geometric aspects first, they nowadays also consider the texture and light-matter interactions. We review them in section 3.3. For quality control in industry (detailed in section 4.4), where validation processes ensure the conformity of machined parts, stan-

^a <https://orcid.org/0000-0003-3271-0712>

^b <https://orcid.org/0000-0002-0704-081X>

^c <https://orcid.org/0000-0002-0343-3456>

^d <https://orcid.org/0000-0002-4402-2928>

^e <https://orcid.org/0000-0002-6409-8516>

standard quantifiers have been proposed (De Chiffre et al., 1994; De Chiffre et al., 2000), of which about fifteen are commonly used (and detailed in section 4.1). But the analysed data can come from a wide variety of instruments (including profilometers, spectroscopic techniques, optical or scanning probe/electron microscopes). That’s why nearly 80 different types of measurements for processing these data are provided, for example, by Mountains®: the surface and microscopic image analysis software platform developed by Digital Surf (Mountains®, 2025).

This paper presents a non-exhaustive list of the main application areas in which substantial work related to roughness has been proposed. To fully appreciate the complex notions roughness encompasses, section 2 explores the mathematical concepts of irregularity and non-differentiability. In section 3, we emphasise that roughness plays an important role in CG applications, seeking to reproduce real-world visual appearances as accurately as possible. Section 4 focuses on industrial applications, which are very diverse but all intended to analyse and/or control roughness during the machining or manufacturing processes. We end up by analysing the similarities and differences between the methods proposed in these two distinct domains (CG and engineering), aiming that it could motivate the evolution of new research approaches in CG.

2 MATHEMATICAL CONCEPTS OF IRREGULARITY RELATED TO ROUGHNESS

It is customary to oppose roughness to what is smooth. In mathematics, smooth surfaces have long been studied through the notions of regularity, continuity, and differentiability, but this represents only a very small part of the set of continuous surfaces (Hunt, 1994). Early examples of continuous nowhere-differentiable functions were introduced in the 19th and early 20th centuries (e.g., Bolzano, Weierstrass, Takagi), relying on iterative and multiscale constructions related to fractal geometry (Mandelbrot, 1977; Tricot, 1995; Allaart and Kawamura, 2012). Hence, roughness is rather referred to as “irregularity” in the non-differentiable sense. The Hölder exponent or the fractal dimension (explained below) are often used for characterising these irregularities. Recent works have introduced a new framework to extend the definition of differentiability to a set of fractal curves (Janbein et al., 2024) and surfaces (Poull et al., 2025).

The regularity class C^n of a function $f(t)$ indicates

that, up to order n , its derivative exists and is continuous, regardless of the shape or behavior of the function. The Hölder (or Lipschitz) regularity of f provides a much more precise characterisation, thanks to the pointwise *Hölder exponent*, that is not necessarily an integer. The *Hölder exponent* characterises a singular behavior when f is locally non-differentiable at a point v and is linked with the *fractal dimension*. The latter encompasses several definitions, the most commonly used being the Minkowski-Bouligand or box-counting dimension (Kolmogorov, 1958), the generalized Renyi dimensions, also called correlation dimensions (Grassberger and Procaccia, 1983), and the *Hausdorff dimension*, based on measure theory (Tricot, 1995). For planar curves, the fractal dimension ranges between 1 for smooth curves and 2 for highly irregular ones. As a global frequency analysis tool, the Fourier transform provides neither spatial localization nor characterisation of irregular points. The wavelet transform (WT) is the preferred tool for evaluating the pointwise *Hölder exponent* and therefore for characterising singularities in 1D (Mallat and Hwang, 1992) or 2D (Wang and Yang, 2012) signals.

A function that exhibits the same pointwise regularity everywhere (same Hölder exponent) is called monofractal. Multifractal functions exhibit more complex behaviour. Their fractal dimension is no longer sufficient to describe the complexity of the irregularities, so it is necessary to study the distribution of their local irregularities via the singularity spectrum (Muzy et al., 1993; Jaffard et al., 2007). This spectrum is a very compact global representation of the pointwise regularity properties along the function: it describes the importance of the pointwise values of the Hölder exponent over the entire function. When a function is monofractal, the spectrum reduces to a single point. Unlike the wavelet transform, which is widely adopted in industry, this spectrum remains largely confined to specialized research contexts due to its higher analytical complexity.

3 ROUGH 3D MODELS FOR CG

Nowadays, several industrial or research domains, such as digital entertainment, Virtual or Augmented Reality (VR/AR), scientific visualization, Computer-Aided Design (CAD), architecture, or medicine, exploit 3D content for the production of synthetic images. Current research aims to obtain the most realistic 3D scenes possible (integrating a maximum of details, for example, roughness, when justified), while minimising processing time (for real-time applications) and data transmission. These virtual 3D

scenes are generally composed of polygonal meshes or height maps (for terrain representation), onto which a texture is mapped via a parameterisation from the texture plane. We review below the different models and principles involved in this process, without dwelling on the generative AI techniques for 3D scene generation, recently surveyed in (Silva and de Carvalho, 2025).

3.1 Texture Mapping

For high-quality rendering, having proper material appearance is crucial. Most modern rendering engines are based on Physically-Based Rendering (PBR), be they offline renderers (Cycles, Arnold, RenderMan, ...), or real-time ones (Unreal Engine, Unity, Godot, ...). With PBR, materials are described using a set of properties encoded in several maps, with the following three having a direct impact on small scale roughness (illustrated in Figure 1):

- **Normal Map:** alters the orientation of the surface normals, simulating the direction of light reflection.
- **Roughness Map:** controls the micro-surface details, affecting the scattering of light. The higher the value on the roughness map, the less specular reflections the material exhibits.
- **Displacement Map:** modifies the geometry of the surface, creating real rough geometries.

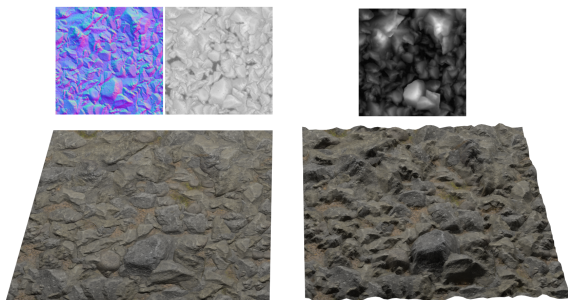


Figure 1: Left: a rock texture (bottom) rendered with normal and roughness maps (top). Right: an additional displacement map (top) modifies the geometry of the mesh (bottom).

There are many ways to create these maps: artist authoring, digital acquisition, (semi-)procedural generation, ... Artist authoring involves manually painting or sculpting these maps with specialized software. It is a very time-consuming and labor-intensive process, but it allows for the creation of highly detailed materials. Digital acquisition involves capturing real-world materials using specialized equipment. It is of-

ten used to create materials that are difficult to replicate with automated methods, but lacks control. In the following paragraphs, we focus on (semi-)procedural methods, as they are among the most used.

Procedural texture synthesis, pioneered by Perlin (Perlin, 1985), involves generating these maps directly, using algorithms or mathematical functions. Among them, sparse convolution noises are used to statistically reproduce many stochastic patterns/structures (like cells, cracks, grains, scratches, spots, stains, and waves), but of limited types and ranges. The control over the visual characteristics requires expert knowledge along trial and error authoring, with limitations when both spatial and spectral controls are required. There have been plenty of mathematical extensions proposed over the years, offering more versatility, by using more advanced convolution kernel functions. Interested readers can refer to these surveys (Lagae et al., 2010; Smelik et al., 2014; Dong et al., 2020) for more details. These methods can be used to construct the textures directly on a surface, avoiding tiling issues.

As for semi-procedural methods, they use exemplars as guides for procedural generation. These exemplars can be realisations of stochastic processes (Heitz and Neyret, 2018) or small bilinear textures composed of microstructures, called textons (Galerie et al., 2017). For aperiodic tilings, a compromise must be found to obtain a result that best respects the input sample, without completely imitating it nor introducing too much randomness. Interested readers can refer to these surveys (Wei et al., 2009; Raad et al., 2017) for more details. Finally, noise by example (Galerie et al., 2012; Guehl et al., 2020) was introduced to avoid the drawbacks of procedural approaches and leverage the advantages of example-based textures. All these works still fail to preserve some structures and are hard to fine-tune for specific looks.

3.2 Modelling, Rendering, and Editing of Synthetic Terrains

Terrains exhibit various levels of roughness (mountain ranges, coastal cliffs, eroded hills, river valleys, gorges, ...) and are fundamental elements of outdoor 3D synthetic scenes (also called virtual worlds). To meet the requirements of the video game industry, cinema, and training simulators, extremely detailed terrains over areas of several thousand kilometers are currently observed (Cordonnier et al., 2016; Schott et al., 2023). Three broad, non-exclusive families of approaches can be distinguished: procedural generation (which attempts to imitate nature by phenomemo-

logical models), physical simulation (which attempts to reproduce it by simulation), and the intermediate approach, example-based synthesis.

The first terrain generation algorithms (Fournier et al., 1982; Musgrave et al., 1989; Voss, 1991) were based on iterative subdivisions and the use of procedural noise. The elevations of the new points (constructed iteratively) depend on those of their neighbors at the lower level (to maintain surface coherence) and are randomly perturbed with decreasing amplitude at each iteration level. The rough appearance is thus introduced, where the noise variance follows a power law, with an exponent linked to the *Hausdorff dimension* of the surface. By their self-affine nature, natural terrains can be accurately generated by locally fractal stochastic processes. Several models (Echelard et al., 2010; Warszawski et al., 2019) are thus based on fractional Brownian (Mandelbrot and Van Ness, 1968) or multifractional Brownian motions (Stoev and Taqqu, 2006) and noises. The very uniform and homogeneous style induced by exploiting self-similarity/affinity leads to a lack of realism, but these methods have the advantage of producing infinite extents with potentially unlimited precision.

To give a more natural aspect to the relief, one can rely on a combination of noise-based functions (Ebert et al., 2002) or on complex stochastic processes (Echelard et al., 2010; Pérez-Ràfols and Almqvist, 2019), producing, for example, greater roughness at the top of mountains than on the plains. Geomorphological erosion simulation algorithms, directly taking physical causes into consideration, are notably capable of reflecting rock fragmentation or sculpting (Paris et al., 2020; Tzathas et al., 2024), the transport of material/particles and its redistribution (by hydraulic or aeolian erosion) (Musgrave et al., 1989), or tectonic uplift (Cordonnier et al., 2016; Cortial et al., 2019); each incorporating various degrees of roughness.

Similar to semi-procedural textures, semi-procedural terrains often use a corpus of real exemplars as a starting point for the synthesis. Several assemblies of these real height maps of regions of the Earth are proposed (Zhou et al., 2007; Gain et al., 2015; Guérin et al., 2016; Guérin et al., 2017). User control is possible by sketching the desired placement of these exemplars with great geological richness, or through even more interactive tools (Guérin et al., 2022; Grenier et al., 2023). Also worth noting are the gains in processing time and memory as techniques advance (Galín et al., 2019; Argudo et al., 2025). But researchers prefer to rely on feedback from graphic designers, artists and experts in the field rather than on metrics, intended to automatically quantify the

perceived realism of the terrains (Rajasekaran et al., 2022).

3.3 Roughness in the Evaluation of the Visual Quality of 3D Meshes

To describe the non-smooth characteristics of a 3D mesh, one mainly refers to sharp/prominent edges, geometric textures, or geometric artifacts, but the term roughness only appeared from 2005 onwards (Corsini et al., 2005; Gelasca et al., 2005). These non-smooth characteristics are mentioned when discussing operations performed on 3D meshes, such as lossy compression, simplification, smoothing, or watermarking, which "damage" the appearance of the initial data. Numerous smoothing or denoising techniques (also called "surface fairing"), seeking to preserve visual appearance, have existed for a long time. To this end, algorithms incorporate precise and objective metrics for evaluating the visual quality of the simplified 3D mesh relative to the original. The aim is to preserve visually salient characteristics (regions that strongly attract the eye), based on an eye-tracking system (Howlett et al., 2004) or in an automatic manner (Lindstrom, 2000; Lee et al., 2005). These studies also exploit the fact that the human vision is sensitive to changes in curvature (Kim et al., 2002). Roughness is not involved when considering these applications, unlike when assessing the visual or quantization artifacts produced by mesh watermarking or lossy compression of these meshes (Corsini et al., 2005; Gelasca et al., 2005; Corsini et al., 2007; Lavoué, 2009; Kim et al., 2010; Lavoué, 2011; Torkhani et al., 2014; Feng et al., 2018). It is indeed necessary to consider the so-called visual "masking effect" (Silverstein, 1996), where one visual signal can mask or reduce the visibility of another if it has a similar frequency or orientation.

The first proposed metrics did not make it possible to distinguish between rough regions and sharp edges. In the context of watermarking, compression, or remeshing, this distinction is essential, as a rough area exhibits a high degree of masking, whereas a geometric change on edges or smooth regions is much more visible (Lavoué, 2009). Interested readers can refer to reviews and analyses of these perceptual/geometric metrics, designed for static or dynamic meshes (Corsini et al., 2013; Moreau et al., 2014; Torkhani et al., 2015). Finally, knowing that the perceptual contribution of texture is generally greater than that of the model geometry, new metrics considering mesh textures (Guo et al., 2017; Zhang et al., 2022; Nehmé et al., 2023), their perception as a function of light-matter interaction (Vanhoey et al., 2017),

or their visualization through an immersive virtual reality environment (Nehmé et al., 2021) have recently emerged.

4 ROUGHNESS CHARACTERISATION IN INDUSTRY

By necessity, the industrial domain was certainly the first to consider analysing and controlling roughness for design, manufacturing, simulation, and quality control. The diagram in Figure 2 illustrates how these latter stages, involved in the machining processes (i.e., turning, grinding, blasting, ...) and detailed in the following subsections, are linked with each other.

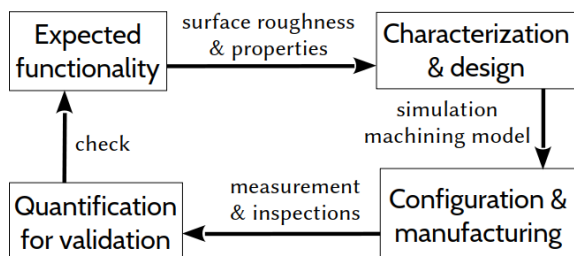


Figure 2: In industry, expected functionalities are translated into "surface texture" specifications at the design stage, which impose machining parameters. Measurement allows the manufactured surface to be characterised and its compliance with the expected functionality to be validated.

Characterising or quantifying roughness is first very useful for providing indications or guidelines regarding the expected machining result (Benardos and Vosniakos, 2003), which must comply with the provided specifications. The study of the relationships between roughness characteristics and surface properties is part of Surface Integrity (SI): a broader field that takes into account both the state and attributes of a machined surface and its relationship to the desired functional performance (Davim, 2010). SI is also described as a multi-physics (biological, mechanical, optical) and multiscale (micro- or nano-structure) residual signature left by the manufacturing processes (or other surface-generation operation) on the surface (M'Saoubi et al., 2008; Jawahir et al., 2011; Lebon and Tapie, 2022).

Quality control or validation is needed to ensure a particular functionality or simply to achieve the desired aesthetic appearance (Rosenboom et al., 2011; Pitard et al., 2015), when controlling defects in manufactured products. In this context, roughness is re-

ferred to as "surface texture", associated with the micro-geometrical alterations of the theoretical (often smooth) geometric surface. These physical and chemical modifications that occur in the fine structure of the material cause short-wavelength irregularities (also called micro-asperities) with variable amplitudes and spacing. Although limited to a thin surface layer, surface texture variations can significantly affect the quality of the component, potentially rendering its surface unacceptable. This is the domain of inspection and measurement: linked to surface metrology (Leach, 2011; Feng et al., 2019). Rough surfaces are also crucial in natural and industrial processes involving heat transfer, given heat can be dissipated more efficiently thanks to a larger exchange surface. A recent review can be browsed (Kadivar and Garg, 2025) for more information. Latest advances in additive manufacturing have enabled the production of rough biomaterial implants. These implants may facilitate the replacement of destroyed or damaged bone, followed by regeneration, thanks to cellular colonization of the implants (i.e., osseointegration). Interested readers can refer to these recent surveys (Romero-Serrano et al., 2024; Lubna et al., 2025) for more details on this topic.

4.1 Statistical Characterisations

As mentioned previously, roughness consists of localized variation on several scale levels. One difficulty is determining the scale level from which this roughness is observed. For industrial applications, this question is rather straightforward, since this roughness generally appears at scales much smaller than that of the shape. Thus the adopted standards (De Chiffre et al., 1994; De Chiffre et al., 2000; Gadelmawla et al., 2002; Blunt and Jiang, 2003) propose segmenting the geometry of surfaces according to five main levels or orders of scale (illustrated in Figure 3): orientation (with respect to a horizontal line: order 0), shape (order 1), waviness (macro-geometrical deformations of large wavelength: order 2), periodic roughness (order 3) and aperiodic roughness (order 4) (De Chiffre et al., 1994). In rarer cases, aperiodic crystalline structure (order 5) and formation (order 6) are also considered (but not shown in Figure 3).

From measurements (most often topographic surveys), statistics characterising the properties of the rough geometry are then extracted (Gadelmawla et al., 2002) (mean amplitude: R_a , standard deviation: R_q , mean slope: Δ_a , ...). They must be as simple as possible and linked to the functionalities that the designer wishes to control. But these quantifiers are also linked to the measuring instruments,

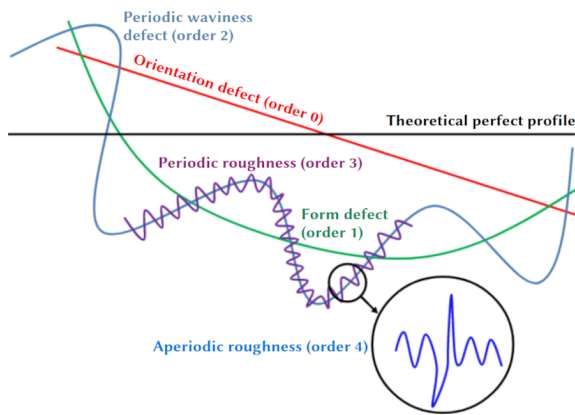


Figure 3: Different scale orders of "surface texture" defects along a 1D profile.

the first of which only allowed the measurement of surface profiles. The resulting characterisations (i.e., profile parameters defined by the ISO 4287 and 21920 standards) could not account for the properties of the overall surface. Then the measuring devices evolved and now make it possible to acquire the topography of the entire surface, allowing for new quantifiers (defined in the ISO 25178 standard).

Four main classes or sets of rough parameters are hence considered by these standards:

- height/amplitude parameters, including the different statistical moments (mean, standard deviation, skewness, and kurtosis coefficients),
- spatial parameters (autocorrelation length and dominant spatial wavelength),
- hybrid parameters (mean quadratic slope, gradients, developed length or area, texture aspect ratio or direction, ...),
- and functional parameters (surface bearing index, surface core/valley fluid retention index, ...).

Each of these parameters highlights certain geometric or functional characteristics of the surface, but most often, a given value can correspond to very diverse geometries and even to non-rough surfaces.

4.2 Characterisation and Design with Respect to an Expected Functionality

The designer who specifies the guidelines by imposing characteristic values must know the relationships between the functionalities he expects (Myers, 1962) or the machining parameters (tool type, cutting speed, depth of cut, ...) and these values. This is the domain of simulation and modelling of machining processes.

Multiscale analysis (Van Gorp et al., 2007; Brown et al., 2018) facilitates the understanding of the relationships between topographies and expected functionalities (Myers, 1962; Le Goïc et al., 2016; Das and Linke, 2017; Brown et al., 2018) or surface interactions in metrology, physics, chemistry, or biology. There are numerous state-of-the-art surveys, each focusing on a specific domain (Khorasani et al., 2012; Gurau and Irle, 2017; Lee and Babadagli, 2021; Song et al., 2022; Kumar et al., 2025): to cite a few.

Regardless of the application domain, these studies differ in their methodological approach and in their way of characterising and generating roughness. From the methodological standpoint, there are first purely experimental approaches that exploit manufacturing processes (for example, sanding a surface with sandpaper of different grits). Roughness is then assessed and quantified (thanks to the computation of the R_a parameter, which is mainly used to characterise the surface topography). Finally, a statistical analysis determines the existence of dependencies between roughness quantifiers and physical properties, like wettability control of polymers, for example (Encinas et al., 2010).

Other studies incorporate more sophisticated knowledge of materials and manufacturing processes. They also require adapting roughness measurement methods. For example, the perceived quality of wood products is strongly impacted by surface roughness, with significant consequences for finishing costs. Since each species has a specific anatomy and wood is a porous material, the recommendations of standards do not apply. Good practices adapted to this context are presented in (Gurau and Irle, 2017) (measuring instrument, resolution, elimination of form defects, ...). It is even necessary to study and formalize subjective notions such as the visual or tactile perception of materials in order to study the impact of roughness on these functionalities (Ramanantoandro et al., 2014).

The choice of roughness parameters is also decisive. In tribology (the study of contact mechanics), the notion of roughness is fundamental for explaining or controlling the observed phenomena. Simple quantifiers (like R_a) are insufficient. Contact phenomena are particularly complex to study and analyse on real surfaces. Firstly, it is difficult to manufacture reference surfaces in accordance with the study being conducted. Furthermore, accurately measuring the surface characteristics is also a complex task (Pérez-Ràfols and Almqvist, 2019). Models for representing or generating surfaces, accounting for their statistical properties, have been proposed (Nayak, 1971; Greenwood and Wu, 2001) and are now used systematically

(Pérez-Ràfols and Almqvist, 2019). The first ones (Nayak, 1971) were based on the theory of random processes. A control of the probability distribution of peaks is therefore accessible (Paggi and He, 2015) from the power spectrum. Developments were also proposed by Greenwood et al. (Greenwood and Wu, 2001) to distinguish between peaks and asperities.

4.3 Configuration and Manufacturing

Another challenge is to control roughness during the manufacturing process. Here again, the studies are specific and depend on the considered process and on the functional roughness parameters taken into consideration (Zhang et al., 2015; Krishnan, 2020). In mechanical engineering, Benardos et al. (Benardos and Vosniakos, 2003) identified four families of factors affecting roughness, during the machining of metallic materials: object characteristics (material hardness and dimensions), cutting tool characteristics (geometry and material), machining parameters (feed and cutting speeds, tool inclination, ...) and the cutting phenomenon (forces, vibration, ...). Among these factors, those related to the machining process, particularly the tool, appear predominant (Lebon and Tapie, 2022). Some approaches consist of modelling the cutting tool geometry, depth of cut, and cutting speed. The mathematical expression of the geometry resulting from the machining is then deduced. From this geometry, the roughness parameters can be calculated, and the dependence on the machining parameters can be established.

Numerical simulation is the standard tool for analysing desired functionalities or properties on surfaces or profiles. For a given engineering study, tolerances are imposed on one or more standardized conventional quantifiers (De Chiffre et al., 1994; De Chiffre et al., 2000). Their selection is crucial and must be linked to the functionalities the designer wishes to control or improve (related to the physical properties of the objects/surfaces concerned). However, a given value of one of these characteristics can correspond to roughness values exhibiting very varied geometries and physical properties. This is mainly due to the fact that conventional roughness measurements are mostly based on global statistical quantification. It hence complicates the precise evaluation and characterisation of roughness, but can be reduced when the industrial process is well defined/known.

4.4 Quality Control and Validation Processes

In the field of quality control and surface metrology, roughness check is part of the validation processes and ensures the conformity of machined parts. The considered roughness measurements are generally simple and often those found in the standards: R_a (arithmetic mean deviation), R_q (standard deviation), etc. But an understanding of the correlation between the quantifier(s) of these surfaces (and/or the appropriate scale(s) to consider) and expected functionality is still lacking (Ramanantoandro et al., 2014; Le Goïc et al., 2016; Das and Linke, 2017; Brown et al., 2018; Liu and Chang, 2019). That's why many studies are turning instead towards a multiscale analysis of the input data, to take into account the irregularities present across a range of scales. It hence facilitates the detection and evaluation of appearance anomalies or scratches on an industrial surface (Rosenboom et al., 2011; Pitard et al., 2015). In this context, these input data are quite often images (height maps or textures) from cameras, or sometimes profile measurements from optical lasers or profilometers.

Initially used to characterise the form defect of a geometry, Discrete Modal Decomposition (DMD) has more recently been generalized to geometric variations of waviness and roughness, to characterise appearance defects (Pitard et al., 2015) (often aperiodic and localized). This transform considers the vibration modes of the reference geometric element as basis functions. A discrete WT can also be considered for the inspection of various manufactured surfaces (Wang et al., 2003a; Wei and Fwa, 2004; Wei et al., 2005; Morala-Argüello et al., 2011; Rosenboom et al., 2011), including fractal representation of engineering surfaces (Xiong et al., 2001). This analysis can be implemented via the "lifting scheme" algorithm (Blunt, 2001; Abdul-Rahman et al., 2013), by a Frequency Normalized WT (FNWT) (Josso et al., 2002), a Continuous WT (Grzesik and Brol, 2009) or wavelet packets (Zawada-Tomkiewicz, 2010; Wang et al., 2017). It finally gives the possibility of estimating the fractal dimension of rough profiles or surfaces (Nayak et al., 2019). Nucleate boiling (giving rise to a heat transfer phenomenon) (Wang et al., 2003b), tribological contact analysis (Bigerelle et al., 2013) or assessment of implant roughness (Perrotti et al., 2011) are some of the involved applications. Many manufactured objects, however, have this fractal appearance only when they are observed at sufficiently fine scales (microstructures with chaotic geometries). By applying Mandelbrot's work to engineering sur-

faces, Brown (Brown, 2013) introduced several multiscale geometric analysis techniques (currently standardized and sometimes designated by the acronym SSFA: Scale-Sensitive Fractal Analysis). The goal is to establish a functional correlation between topography and the expected behavior of the surface, by determining the scale of analysis, most likely to reveal the details and complexity of the relief. This facilitates the design of products having optimal behaviors during interactions of different types (friction, adhesion, contact, wear, ...) or serves to discriminate surfaces and distinguish the good product from the bad in quality control systems (Morala-Argüello et al., 2011; Ma et al., 2021).

5 CONCLUSION

This paper presents a state-of-the-art review of the concept of roughness. After recalling key mathematical notions, we review the role of roughness in Computer Graphics (CG), from generation to rendering, including processing, and then examine how it is perceived, characterised, and exploited in other domains, with a particular focus on industrial applications. The first observation is the difficulty in defining roughness. Although there is a general consensus on the notion of roughness, the geometric complexity of rough surfaces is difficult to define formally and universally. To address this issue, field-specific solutions have been proposed, ranging from perceptual metrics in geometry processing to dedicated tools in CG that allow designers to directly control visual effects. Currently, human intervention or expertise remains necessary to control or assess these complex shapes. In industry, expert knowledge of manufacturing processes enables the characterisation of roughness using simple statistical measures (e.g., R_a/S_a), which are sufficient to ensure a given surface functionality. However, such simple measures are clearly insufficient to reflect the geometric complexity of rough surfaces. The most sophisticated characterisations use wavelet analysis, fractal dimension and/or singularity spectrum (based on multifractal analysis), which handle two intrinsic concepts of roughness: irregularity and self-similarity. Roughness synthesis methods widely exploit those two properties by combining random variables, often relying on statistical shape controls defined in the spectral domain to achieve natural-looking surfaces. However, these parameters remain difficult to interpret, especially for non-expert users. Finally, existing analyses and characterisation tools, roughness measurements, and generation algorithms do not constitute a geometric model of roughness.

We would need one, though, to modify, adjust, or combine rough geometries. The random aspect, although very interesting for giving surfaces a "natural" appearance, is an obstacle to accurately characterising and controlling geometry. Currently, developing a generic model to represent rough surfaces remains a challenge.

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