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A Novel MILP Model for Human-Robot Collaboration in Assembly Line Balancing under budget constraints^{*}

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Abstract: We develop a novel mathematical approach to enhance the efficiency of assembly line with Human-Robot Collaboration (HRC). The developed model strives to reduce cycle time while considering operational limitations and budgetary factors. It tackles key challenges such as task distribution, setup durations, and the complexities of multi-station, multi-mode assembly processes. Tested across different scenarios, the model demonstrates significant improvements in workflow efficiency and resource allocation. Ultimately, this research presents a structured methodology to foster more flexible and high-performing assembly lines through optimized HRC scheduling.

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Keywords: Manufacturing flexibility, Assembly line balancing, Human-robot collaboration, Mathematical modeling, Optimization

1. INTRODUCTION

The significance of human-robot collaboration (HRC) in modern manufacturing is constantly supported by increasing investments. A recent report projects that the global market for collaborative robots, or “cobots,” is expected to grow from \$710 million in 2018 to more than \$12 billion by 2025, driven by a compound annual growth rate (CAGR) of more than 50% (Matheson et al., 2019). This rapid growth highlights the increasing reliance on HRC, particularly as small and medium-sized enterprises (SMEs) adopt these technologies to improve flexibility and reduce costs.

These collaborative systems combine human dexterity and problem solving ability with the precision and strength of robots, making them indispensable in industries such as automotive and electronics. Traditional automation, while effective in repetitive tasks, often lacks the adaptability required for today’s highly customized and rapidly evolving product cycles. As manufacturing moves towards Industry 4.0, there is a pressing need to explore human-robot collaboration (HRC) as a way to harness the unique strengths of both humans and machines. By working together, humans can leverage robots’ precision and speed for physically demanding tasks, while robots benefit from human cre-

ativity and decision-making in complex or nonstandard situations. Human-Robot Collaboration (HRC) can help to improve the efficiency and agility in manufacturing systems and as a result to enhance the productivity and to reduce the workload of human operators (Mao et al., 2024; Kheirabadi et al., 2023). The use of HRC systems can reduce cycle times and increase production rate by up to 85%, while improving safety and adaptability in complex manufacturing environments (Othman and Yang, 2023).

Several studies have reviewed different aspects of HRC in manufacturing systems. Kheirabadi et al. (2023), Keshvarparast et al. (2024), Fathi et al. (2024) focused on the productivity benefits of HRC without going into details of solution methods for task allocation and scheduling in such environments. Dalle Mura and Dini (2019) focused on fully collaborative operational modes in HRC systems.

Other recent studies have proposed mathematical models for optimizing HRC systems in assembly lines. Sikora and Weckenborg (2023) formulated a Mixed-Integer Programming (MIP) model for the Assembly Line Balancing Problem (ALBP) with HRC, aiming to minimize cycle times by allowing workers and robots to perform tasks either in parallel or jointly. Stecke and Mokhtarzadeh (2022) extended this by developing both a Mixed Integer Linear Programming (MILP) model and a Benders decomposition algorithm, focusing on minimizing cycle time while also reducing ergonomic risks for human workers.

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Mao et al. (2023) and Boschetti et al. (2021) discussed innovative modeling approaches to schedule assembly lines and reduce cycle time through effective HRC. Nourmohammadi et al. (2024) investigated U-shaped and straight assembly lines considering several aspects (e.g., the number of stations, cycle time, and costs of stations, operators, and energy consumption of robots). Inkulu and Bahubalendruri (2023) proposed a task allocation and estimation procedure in an assembly line balancing scheme with HRC. Christensen et al. (2017) and Li et al. (2023) addressed large-scale batch production challenges, integrated Industry 4.0 technologies for the possibility of HRC, and proposed heuristic algorithms that handle task assignments, resource allocation, and scheduling.

In this study, we model both assembly line balancing and scheduling problems for multi-station, multi-robot, multi-human, and multi-mode assembly lines. For the first time in the literature, the proposed model addresses task assignments, mode of operations selection, and the adherence to strict budgetary and operational constraints. By ensuring a balanced distribution of tasks in the different modes of operation, we propose a solution that optimizes line throughput and improves the adaptability and flexibility of the designed assembly line.

This paper is organized as follows. Section 2 presents the mathematical model of the problem considered. Section 3 conducts numerical experiments and analyzes the results. Section 4 provides the conclusion and research perspectives.

2. PROBLEM DEFINITION

The proposed model addresses the challenge of optimizing task assignments and scheduling in a multi-station assembly line, in which tasks can be performed by humans, robots, or through human-robot collaboration. The primary objective is to minimize the total cycle time while adhering to operational constraints, such as budget, resource availability, and precedence relationships between tasks.

There is a set of tasks; $i, j \in \{1, 2, \dots, |I|\}$ to be scheduled to a set of stations; $k \in \{1, 2, \dots, |K|\}$. Each task can be performed in one of operational modes: $m \in \{1, 2, 3\}$ (1 - for human mode, 2 - for robotic mode, and 3 for collaborative mode). There is a set of humans that can be assigned to stations $h \in \{1, 2, \dots, |H|\}$ and a set of robots $r \in \{1, 2, \dots, |R|\}$. Time and cost to perform task i vary depending on operational mode m : they are defined by parameters τ_i^m and C_i^m , respectively. There is also a setup time after execution of task i in operational mode m' to perform task j in operational mode m'' : $s_{ij}^{m'm''}$. The total available budget for the task execution is given by B .

In order to assign robots and human operators to stations as well as schedule the task, we propose a MILP model which uses the following decision and auxiliary variables:

Decision Variables

$x_{ih}^k = 1$ if task i is performed by human h at station k ; 0, otherwise
 $y_{ir}^k = 1$ if task i is performed by robot r at station k ; 0, otherwise
 $z_i^{km} = 1$ if task i is performed at station k in operational

mode m ; 0, otherwise

$p_{ij}^k = 1$ if task i is performed before task j at station k ; 0, otherwise

Auxiliary Variables

$x_i^k = 1$ if task i is performed by a human at station k ; 0, otherwise

$y_i^k = 1$ if task i is performed by a robot at station k ; 0, otherwise

$z_i^k = 1$ if task i is assigned to station k ; 0, otherwise

$z^k = 1$ if at least one task is assigned to station k ; 0, otherwise

t_i^k Actual time to perform task i at station k

$u_h^k = 1$ if human h is active at station k ; 0, otherwise

$v_r^k = 1$ if robot r is active at station k ; 0, otherwise

$w_{ij}^k = 1$ if tasks i and j are performed at station k ; 0, otherwise

$z_i^{km'} = 1$ if task i is performed at station k in operational mode m' ; 0, otherwise

$z_{ij}^{km'm''} = 1$ if task i is performed at station k in operational mode m' and task j is performed at station k in operational mode m'' ; 0, otherwise

s_{ij}^k Setup time between task i and task j at station k

T_i Starting time of task i

T Cycle time

The developed mathematical model is given here below. It allows multiple (more than 2) robots and human operators per station. This model provides a comprehensive framework for balancing and scheduling the assembly line in HRC environment, optimizing both task assignments and operational modes in order to achieve minimal cycle time as defined in (1).

$$\text{Minimize } T \quad (1)$$

The model is subject to the following constraints:

Constraints (2) and (3) define if task i is performed by a human and/or by a robot at station k , respectively.

$$\sum_{h \in H} x_{ih}^k = x_i^k \quad \forall i \in I, \forall k \in K \quad (2)$$

$$\sum_{r \in R} y_{ir}^k = y_i^k \quad \forall i \in I, \forall k \in K \quad (3)$$

Constraint (4) guarantees that the human operational mode at station k is active if the task is performed by a human without collaboration.

$$z_i^{k1} = x_i^k - z_i^{k3} \quad \forall i \in I, \forall k \in K \quad (4)$$

Constraint (5) requires that the robot operational mode at station k is active when the task is performed by a only robot.

$$z_i^{k2} = y_i^k - z_i^{k3} \quad \forall i \in I, \forall k \in K \quad (5)$$

Constraints (6) - (8) enforce that the collaborative mode is selected only when both human and robot are assigned to the task.

$$z_i^{k3} \leq x_i^k \quad \forall i \in I, \forall k \in K \quad (6)$$

$$z_i^{k3} \leq y_i^k \quad \forall i \in I, \forall k \in K \quad (7)$$

$$z_i^{k3} \geq x_i^k + y_i^k - 1 \quad \forall i \in I, \forall k \in K \quad (8)$$

Constraint (9) guarantees that each task i is assigned exactly one operational mode (human, robot, or collaborative) at each station k .

$$\sum_{m \in M} z_i^{km} = z_i^k \quad \forall i \in I, \forall k \in K \quad (9)$$

Constraint (10) ensures that each task is assigned to a single station.

$$\sum_{k \in K} z_i^k = 1 \quad \forall i \in I \quad (10)$$

Constraints (11) and (12) define if station k is active.

$$\sum_{i \in I} z_i^k \geq z^k \quad \forall k \in K \quad (11)$$

$$|I|z^k \geq \sum_{i \in I} z_i^k \quad \forall k \in K \quad (12)$$

Constraint (13) maintains a sequential order of active stations, which means that there are no empty stations between active stations.

$$z^k \geq z^{k+1} \quad k = 1, 2, \dots, |K| - 1 \quad (13)$$

Constraint (14) represents the budget constraint, which ensures that the total cost of performing tasks in all stations and operational modes does not exceed the available budget B .

$$\sum_{i \in I} \sum_{k \in K} \sum_{m \in M} C_i^m z_i^{km} \leq B \quad (14)$$

Constraints (15) - (17) and (18) - (20) guarantee that each human and robot can be active only at one station.

$$u_h^k \leq \sum_{i \in I} x_{ih}^k \quad \forall h \in H, \forall k \in K \quad (15)$$

$$|I|u_h^k \geq \sum_{i \in I} x_{ih}^k \quad \forall h \in H, \forall k \in K \quad (16)$$

$$\sum_{k \in K} u_h^k \leq 1 \quad \forall h \in H \quad (17)$$

$$v_r^k \leq \sum_{i \in I} y_{ir}^k \quad \forall r \in R, \forall k \in K \quad (18)$$

$$|I|v_r^k \geq \sum_{i \in I} y_{ir}^k \quad \forall r \in R, \forall k \in K \quad (19)$$

$$\sum_{k \in K} v_r^k \leq 1 \quad \forall r \in R \quad (20)$$

Constraints (21) - (23) determine if tasks i and j are performed at the same station k .

$$w_{ij}^k \leq z_i^k \quad \forall i, j \in I, i < j, \forall k \in K \quad (21)$$

$$w_{ij}^k \leq z_j^k \quad \forall i, j \in I, i < j, \forall k \in K \quad (22)$$

$$w_{ij}^k \geq z_j^k + z_i^k - 1 \quad \forall i, j \in I, i < j, \forall k \in K \quad (23)$$

Constraint (24) provides that task i can be only performed before or after task j at station k if they both assigned to station k .

$$p_{ij}^k + p_{ji}^k \leq w_{ij}^k \quad \forall i, j \in I, i < j, \forall k \in K \quad (24)$$

Constraint (25) prevents the execution of task j before task i if task i is a predecessor of task j in the transitive closure of E .

$$p_{ji}^k = 0 \quad \forall i, j \in I, (i, j) \in E^c, \forall k \in K \quad (25)$$

Constraint (26) obliges the execution of task i before task j if they both are assigned to station k and task i is a predecessor of task j in the transitive closure of E .

$$p_{ij}^k = w_{ij}^k \quad \forall i, j \in I, (i, j) \in E^c, \forall k \in K \quad (26)$$

Constraints (27) and (28) guarantee if task i and task j are performed in parallel at station k ($p_{ij}^k - p_{ji}^k = 0$), then

they should be assigned to different humans and robots ($x_{ih}^k + x_{jh}^k \leq 1$ and $y_{ir}^k + y_{jr}^k \leq 1$). Otherwise ($p_{ij}^k - p_{ji}^k = 1$) they can be assigned to the same human and/or robot.

$$x_{ih}^k + x_{jh}^k - p_{ij}^k - p_{ji}^k \leq 1, \forall i, j \in I, i \neq j, \forall h \in H, \forall k \in K \quad (27)$$

$$y_{ir}^k + y_{jr}^k - p_{ij}^k - p_{ji}^k \leq 1, \forall i, j \in I, i \neq j, \forall r \in R, \forall k \in K \quad (28)$$

Constraints (29) - (38) calculate time t_i^k necessary to perform task i at station k in the chosen operational mode.

$$\sum_{m \in M} \tau_i^m z_i^{km} = t_i^k \quad \forall i \in I, \forall k \in K \quad (29)$$

$$z_{ij}^{km'm''} \leq z_{ij}^{km'}, \forall i, j \in I, i \neq j, \forall m', m'' \in M, \forall k \in K \quad (30)$$

$$z_{ij}^{km'm''} \leq z_j^{km''}, \forall i, j \in I, i \neq j, \forall m', m'' \in M, \forall k \in K \quad (31)$$

$$z_{ij}^{km'm''} + p_{ij}^k \geq z_{ik}^{m'} + z_{jk}^{m''} - 2, \forall i, j \in I, i \neq j, \forall m', \forall m'' \in M, \forall k \in K \quad (32)$$

$$s_{ij}^k = \sum_{m' \in M} \sum_{m'' \in M} s_{ij}^{m'm''} z_{ij}^{km'm''}, \forall i, j \in I, i \neq j, \forall k \in K \quad (33)$$

$$s_{ij}^k \leq s_{ij}^{\max} p_{ij}^k, \forall i, j \in I, i \neq j, \forall k \in K \quad (34)$$

$$s_{ij}^k \geq s_{ij}^{\max} p_{ij}^k + s_{ij}^{\min} - s_{ij}^{\max}, \forall i, j \in I, i \neq j, \forall k \in K \quad (35)$$

$$\text{where} \quad (36)$$

$$s_{ij}^{\max} = \max\{s_{ij}^{m'm''} \mid m', m'' \in M\} \quad (37)$$

$$s_{ij}^{\min} = \min\{s_{ij}^{m'm''} \mid m', m'' \in M\} \quad (38)$$

Constraint (39) maintains the precedence between tasks at station k .

$$T_i + t_i^k + s_{ij}^k - T_j - UB(1 - p_{ij}^k) \leq 0 \quad \forall i, j \in I, (i, j) \notin E^c \setminus E, \forall k \in K \quad (39)$$

Constraint (40) with the minimization of (1) defines the cycle time T .

$$T_i + t_i^k \leq T \quad \forall i \in I, \forall k \in K \quad (40)$$

Constraint (41) restricts actual time t_i^k to be within the maximum possible task time in all modes.

$$0 \leq t_i^k \leq \max\{\tau_i^m \mid m \in M\} \quad \forall i \in I, \forall k \in K \quad (41)$$

Constraint (42) bounds the starting time T_i of each task within an upper limit UB .

$$0 \leq T_i \leq UB \quad \forall i \in I \quad (42)$$

Finally, Constraint (43) limits the total cycle time T to the calculated upper bound UB , derived in Constraint (44), ensuring that the cycle time does not exceed the worst-case scenario.

$$0 \leq T \leq UB \quad (43)$$

UB can be calculate as follows:

$$UB = \sum_{j \in I} \max\{s_{ij} \mid i \in I\} + \sum_{i \in I} \max\{\tau_i^m \mid m \in M\} \quad (44)$$

3. COMPUTATIONAL RESULTS

The proposed approach was tested on the examples with 20 tasks derived from *SALBP - 2013/small data set.n = 20/instance.n = 20.1.alb*. The modification concerns the generation of execution times of tasks τ_i^m for modes $m = 2$ and $m = 3$, execution costs C_i^m , $m = 1, 2, 3$ as well as setup (transition) times $s_{ij}^{m'm''}$. The costs C_i^1 were taken as $10 \tau_i^1$. The execution times τ_i^m and costs C_i^m for modes $m = 2$

Table 1. Results for the test example.

<i>B</i>	<i>T</i>	<i>C</i>	<i>TS</i>	<i>Gap</i>	<i>Bnd</i>	<i>TS_b</i>	<i>TS_g</i>	<i>NK</i>	<i>NH</i>	<i>NR</i>	<i>NT₁</i>	<i>NT₂</i>	<i>NT₃</i>
$ K = 3, H = 3, R = 3, NV = 14592, NC = 41670$													
29000	934	28991	3600	19.38	753	12	3386	3	3	2	18	2	0
31000	476	30979	3600	23.11	366	1554	3221	3	3	3	9	11	0
35000	440	31554	3600	27.5	319	351	3555	3	3	3	11	9	0
40000	440	31933	3601	40	264	2544	2544	3	3	3	10	9	1
$ K = 3, H = 5, R = 5, NV = 14844, NC = 43978$													
29000	564	28991	3600	0.89	559	1473	3600	3	5	2	18	2	0
31000	325	30984	69	0	325	69	69	3	5	5	12	8	0
35000	287	33460	3600	7.67	265	77	3600	3	5	5	7	9	4
40000	286	33504	3600	14.34	245	2921	3472	3	5	5	8	8	4
$ K = 3, H = 8, R = 8, NV = 15222, NC = 47440$													
29000	428	28820	11	0	428	10	10	3	8	0	20	0	0
31000	323	30983	15	0	323	14	14	3	8	4	13	6	1
35000	230	34358	14	0	230	13	13	3	8	8	6	9	5
40000	230	36091	12	0	230	12	12	3	8	8	5	5	10
$ K = 5, H = 5, R = 5, NV = 24726, NC = 73266$													
29000	564	28974	3600	11.35	500	116	3514	5	5	2	18	2	0
31000	286	30994	3600	19.23	231	1065	1972	5	5	5	8	12	0
35000	277	31571	3600	31.77	189	1215	3437	5	5	5	9	10	1
40000	279	32349	3600	34.77	182	914	3600	5	5	5	9	10	1
$ K = 5, H = 8, R = 8, NV = 25356, NC = 79032$													
29000	363	28974	1405	0	363	1182	1405	5	8	2	18	2	0
31000	221	30984	2000	0	221	157	2000	5	8	7	13	7	0
35000	185	34329	3601	8.65	169	1349	3601	5	8	8	7	7	6
40000	185	34020	3600	5.41	175	342	3600	5	8	8	7	8	5
$ K = 8, H = 8, R = 8, NV = 40557, NC = 126420$													
29000	363	28974	3600	22.31	282	1063	1063	8	8	2	18	2	0
31000	220	30979	3600	2.73	214	190	3600	8	8	7	13	7	0
35000	186	34851	3600	18.82	151	282	282	8	8	8	6	6	8
40000	184	34084	3601	17.93	151	861	861	8	8	8	7	7	6

and $m = 3$ were calculated by multiplying τ_i^1 and C_i^1 by coefficients randomly generated from $[0.75,0.85]$, $[0.5,0.6]$, $[1.1,1.2]$, and $[1.3,1.5]$, respectively. The values $s_{ij}^{m'm''}$, $m', m'' \in M$ were calculated by multiplying $0.5(\tau_i^{m'} + \tau_j^{m''})$ by a factor randomly generated from $[0.03,0.05]$.

In Table 1, we present results of the experiments for different values of numbers of stations, humans, robots and budget. Lower and upper bounds on the cost are 28820 and 40191.4. The time limit for Gurobi 9.5.1 was 60 min. The results comprise the obtained cycle time T , Gurobi solution time TS in sec, the total cost C of the solution, Gurobi optimality gap Gap in %, Gurobi best bound Bnd , the numbers NK , NH , and NR of used stations, humans, and robots in the solution, respectively, the numbers NT_m , $m = 1, 2, 3$ of tasks performed in mode m . Additionally we provide the number of variables NV and constraints NC of MIP model as well as times TS_b and TS_g which correspond to finding the best objective value and the best gap, respectively.

If the operational modes for each task $i \in I$ are fixed to $m(i) \in \{1, 2, 3\}$ then the optimal assignment of tasks to stations can be obtained by solving model (1)-(44) with the following additional constraints

$$x_{ih}^k = 0, i \in I, m(i) = 2, \forall k \in K, \forall h \in H \quad (45)$$

$$y_{ir}^k = 0, i \in I, m(i) = 1, \forall k \in K, \forall r \in R \quad (46)$$

$$z_i^{km} = 0, i \in I, m \in M, m \neq m(i), \forall k \in K \quad (47)$$

$$\begin{aligned} z_{ij}^{km'm''} &= 0, i, j \in I, i \neq j, m', m'' \in M, m' \neq m(i), \\ m'' &\neq m(j), \forall k \in K \end{aligned} \quad (48)$$

$$\sum_{k \in K} x_i^k \geq 1, i \in I, m(i) = 1 \quad (49)$$

$$\sum_{k \in K} y_i^k \geq 1, i \in I, m(i) = 2 \quad (50)$$

$$\sum_{k \in K} z_i^{k3} \geq 1, i \in I, m(i) = 3 \quad (51)$$

We propose two decomposition schemes for solving the initial problem. In the first scheme at the first stage we solve the problem for values of $m(i)$, $i \in I$ which correspond to the solution of the following problem

$$\text{Minimize } \sum_{i \in I} \sum_{m \in M} \tau_i^m z_i^m \quad (52)$$

$$\sum_{i \in I} \sum_{m \in M} C_i^m z_i^m \leq B \quad (53)$$

$$\sum_{m \in M} z_i^m = 1, \forall i \in I \quad (54)$$

where $m(i) = m$ if $z_i^m = 1$.

At the second stage, we address the initial problem by using the obtained solution as an MIP start for the solver.

In the second approach, the first stage involves solving the problem within a predefined time limit. In the second stage, we refine the solution by considering the values of $m(i)$, $i \in I$ derived from the initial solution. This refined solution is then used as an MIP start, while the best MIP bound obtained serves as a lower bound on the cycle time.

In Table 2, We compare the results obtained using different methods to solve the test example. The first approach, A1 employs the MIP model with a time limit of 60 minutes.

The second and third approaches, A2 and A3 correspond to the first and second decomposition schemes, both with a total time limit of 60 minutes. In these approaches, the time limits for the first stage are set to 20 minutes for 20 min and 40 min for A2 and A3, respectively.

Table 2. Comparison of results of different approaches .

$ K = 3, H = 3, R = 3$						
	A1		A2		A3	
<i>B</i>	<i>T</i>	<i>TS</i>	<i>T</i>	<i>TS</i>	<i>T</i>	<i>TS</i>
29000	934	3600	934	3600	934	3601
31000	476	3600	488	3600	488	2402
35000	440	3600	440	3600	443	2401
40000	440	3601	444	3600	444	2401
$ K = 3, H = 5, R = 5$						
29000	564	3600	564	3600	564	2791
31000	325	69	325	110	325	119
35000	287	3600	287	3600	287	2401
40000	286	3600	287	3600	287	2401
$ K = 3, H = 8, R = 8$						
	A1		A2		A3	
<i>B</i>	<i>T</i>	<i>TS</i>	<i>T</i>	<i>TS</i>	<i>T</i>	<i>TS</i>
29000	428	11	428	10	428	11
31000	323	15	323	16	323	14
35000	230	14	230	9	230	13
40000	230	12	230	6	230	12
$ K = 5, H = 5, R = 5$						
	A1		A2		A3	
<i>B</i>	<i>T</i>	<i>TS</i>	<i>T</i>	<i>TS</i>	<i>T</i>	<i>TS</i>
29000	564	3600	564	3600	563	3600
31000	286	3600	311	3600	282	2403
35000	277	3600	279	3600	277	2401
40000	279	3600	279	3600	279	2411
$ K = 5, H = 8, R = 8$						
	A1		A2		A3	
<i>B</i>	<i>T</i>	<i>TS</i>	<i>T</i>	<i>TS</i>	<i>T</i>	<i>TS</i>
29000	363	1405	363	1604	363	1389
31000	221	2000	221	3600	221	1983
35000	185	3601	185	3601	185	2401
40000	185	3600	186	3600	185	2401
$ K = 8, H = 8, R = 8$						
	A1		A2		A3	
<i>B</i>	<i>T</i>	<i>TS</i>	<i>T</i>	<i>TS</i>	<i>T</i>	<i>TS</i>
29000	363	3600	363	3600	363	3600
31000	220	3600	220	3601	220	2416
35000	186	3600	185	3600	186	2404
40000	184	3601	185	3600	184	2505

4. CONCLUSION

This paper introduced an innovative mathematical model designed to optimize Human-Robot Collaboration (HRC) in assembly line balancing while accounting for budget constraints. The Mixed-Integer Linear Programming (MILP) framework effectively handled task allocation, operational modes, and precedence relationships, all while considering the financial limitations of employing both human workers and robots. The model's primary goal was to minimize cycle time, thereby enhancing efficiency and resource utilization. Computational experiments demonstrated substantial performance improvements, with substantial cycle time reductions through optimized resource allocation. These findings highlight the model's effectiveness, particularly in small- and medium-scale manufacturing environments.

Future research could focus on scaling the model for larger and more complex manufacturing systems, as well

as adapting it to fluctuating demand and operational uncertainties. Additionally, incorporating dynamic elements such as real-time decision-making and predictive maintenance could enhance its applicability in Industry 4.0 settings. The integration of heuristic and metaheuristic algorithms, including simulated annealing and genetic algorithms, may help reduce computational time, while hybrid approaches could further improve efficiency for large-scale industrial applications.

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