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Exploring Non-Euclidean Geometry Through Virtual and Real Experiences to Unlock Non-Experts' Intuition

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Figure 1: Two viewpoints on *visual echoes*. Left: a participant experiencing light trajectories in $S \times E$ during the introduction phase. Right: VR exploration scene for understanding *visual echoes*. Participants are equipped with HMD and the associated controllers. They are immersed in $S^2 \times E$, in a scene with seven objects colored in seven different colors that participants could differentiate. For instance, the yellow sphere can be seen twice, with different appearances thanks to two different light paths traveling from the eye to this object. Detailed explanations can be found in Section 3.

ABSTRACT

Three-dimensional non-Euclidean geometries are highly abstract mathematical constructs, typically introduced only at advanced levels of mathematical education due to their conceptual and technical complexity. This paper presents a study exploring whether intuitive understanding of one such geometry – $S^2 \times E$, the product of a 2-sphere and a line – can be facilitated through an interactive virtual reality (VR) experience without relying on formal mathematical instruction. We designed an immersive VR environment that allows users to explore $S^2 \times E$ as inhabitants of this space. Participants, who were neither mathematics experts nor students specializing in mathematics, engaged with the system after a short introduction using a learn-by-doing approach. Results show that participants were able to infer and articulate key properties of $S^2 \times E$ without knowledge prior to the experiment, and that their performance was independent of their educational background or attitude towards mathematics. Our findings demonstrate the potential of VR as a medium for conveying abstract geometric concepts through embodied learning, making them accessible to non-expert audiences.

Index Terms: Virtual Reality, Non-Euclidean Geometry, Pedagogy, Geometric Intuition, Embodied Learning.

1 INTRODUCTION

Understanding complex mathematical structures often requires years of formal education and abstract reasoning skills. Among these, non-Euclidean geometries – geometrical systems that deviate

from the familiar rules of Euclidean space – pose particular challenges due to their counter-intuitive properties and limited visualizability in everyday experience. Three-dimensional non-Euclidean geometries are curved or distorted spaces where, for example, straight lines may converge or diverge, and traditional notions of distance and shape are fundamentally altered. As a consequence, conventional rendering software do not support non-Euclidean geometries, since light rays in these engines are constrained to follow straight lines in Euclidean space. These geometries, especially in three dimensions, are typically introduced only at advanced stages of mathematical education. Yet non-Euclidean geometries are not only of interest to mathematicians. They play a fundamental role in many fields of science, including physics and computer graphics [26, 5]. A concrete example of non-Euclidean geometry in computer graphics is the conformal parameterization of high-genus surfaces using hyperbolic space [17]. By mapping surfaces to their universal covering in the hyperbolic plane, this method reduces distortion in texture mapping and remeshing tasks. Furthermore, engaging with these geometries can foster a valuable shift in perspective, encouraging learners to question deeply held intuitions about space, logic, and truth. For these reasons, it is both intellectually and practically worthwhile to develop alternative ways of accessing and understanding such geometries – especially for non-expert audiences who may benefit from them in interdisciplinary contexts. At the same time, VR has emerged as a promising tool for education, offering immersive and interactive environments that can support experiential learning. Prior research has shown that VR can enhance learners' spatial reasoning, engagement, and motivation, particularly in mathematical domains [32, 11]. While several studies have explored the use of VR in teaching mathematical concepts [15], and others have addressed pedagogical approaches to non-Euclidean geometry [28, 10], relatively little work has been done at the intersection of these two fields.

This study investigates whether an intuitive understanding of a specific three-dimensional non-Euclidean space – $S^2 \times E$, the prod-

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uct of a 2-sphere and a Euclidean line – can be developed through immersive VR, together with a short introduction to the geometry, without requiring formal mathematical instruction. Our objective is not to teach advanced mathematics per se, but to explore whether embodied interaction within a simulated version of $S^2 \times E$ can enable non-experts to grasp some of its key spatial properties through direct experience. To this end, we developed a custom VR environment that allows users to inhabit and navigate $S^2 \times E$ from a first-person perspective. Participants engaged in exploratory tasks designed to reveal geometric features of the space, such as the behavior of *visual echoes* and the relationship between the angles and area of triangles. Our study assesses whether participants were able to internalize and articulate these concepts, and examines how this learning process is influenced by their mathematical background and attitude. Results show that participants were able to infer and articulate key properties of $S^2 \times E$ without knowledge prior to the experiment, and that their performance was independent of their educational background or attitude towards mathematics.

By focusing on $S^2 \times E$ as a case study, this work aims to contribute to a broader understanding of how virtual reality can serve as a bridge between abstract mathematical theory and intuitive human perception. We hope to open new pathways for making complex mathematical ideas more accessible, not only for educational enrichment but also for interdisciplinary scientific understanding.

2 RELATED WORKS

Understanding and teaching non-Euclidean geometry poses significant challenges due to their abstract nature and lack of intuitive physical models. Recent advances in VR offer promising avenues to enhance spatial and mathematical learning by providing immersive, embodied experiences. This section reviews classical educational approaches, explores VR’s potential in mathematical learning, surveys existing VR applications in geometry, and discusses cognitive and affective factors influencing learners’ engagement and understanding.

2.1 Non-Euclidean Geometry in Education and Visualization

Non-Euclidean geometries have long been introduced in mathematics education through formal instruction and symbolic reasoning. Classical approaches typically rely on two-dimensional analogies, such as the Poincaré disk model for hyperbolic geometry [8] or the Riemann sphere for spherical geometry. These models help convey curvature and metric properties within a familiar framework, yet they require significant background knowledge in geometry, analysis, or topology.

Physical models (e.g., crochet hyperbolic planes [29], paper models of spherical triangles) have also been used to provide tangible representations. While these tools offer valuable insights into curvature and angle relationships, they are inherently limited in their ability to capture the full dynamic properties of three-dimensional non-Euclidean spaces.

Despite the utility of these methods, visualizing and internalizing higher-dimensional or non-trivial product spaces – such as $S^2 \times E$ – remains particularly difficult, especially for learners without advanced mathematical training. Three-dimensional non-Euclidean geometries cannot be fully embedded in three-dimensional Euclidean space without distortion, making intuitive representations challenging. As a result, they are rarely taught outside of specialized academic contexts, limiting broader engagement with their conceptual richness.

2.2 VR for Mathematical and Spatial Learning

VR has emerged as a powerful tool for education, particularly in domains that benefit from spatial reasoning and interactive visualization. A growing body of research suggests that immersive en-

vironments can facilitate deeper engagement and improved comprehension of abstract mathematical ideas. For example, VR has been used effectively to teach Euclidean geometry, linear algebra, and multivariable calculus, with studies reporting increased learner motivation, retention, and spatial awareness [19, 11]. By enabling learners to directly interact with mathematical objects and navigate within spatial contexts, VR can bypass some of the cognitive barriers associated with purely conceptual instruction [9]. This is especially valuable when dealing with geometric structures that lack intuitive real-world counterparts.

Embodied cognition theories argue that abstract thinking, including mathematical reasoning, is grounded in sensorimotor experience [3, 31]. Rather than viewing learning as the manipulation of symbols alone, this perspective emphasizes the role of the body in shaping conceptual understanding. Virtual reality provides a natural platform for embodied learning by allowing users to interact physically with spatial structures. This is particularly valuable for mathematical concepts – such as those in non-Euclidean geometry – that are difficult to grasp through diagrams or formulas alone.

Studies show that such interaction supports learning. Abrahamson and Bakker [1] describe how learners form attentional anchors – perceptual cues that coordinate movement and guide mathematical insight – through exploratory interaction. Similarly, Nemirovsky et al. [23] demonstrate how hands-on engagement in interactive exhibits fosters perceptuomotor integration, blending action and perception into meaningful mathematical understanding. These findings suggest that embodied interaction in VR can transform abstract concepts into intuitive experiences, providing a powerful pathway for learning through action and perception.

2.3 Prior VR Applications in Geometry

Several VR projects have enabled users to explore non-Euclidean spaces: Curved Spaces [30] facilitates navigation in multi-connected topologies; 3-dimensional.space [7] immerses users in the eight Thurston geometries; and ZenoRogue’s work [14] offers interactive experiences in exotic geometries. A study was recently conducted comparing learning non-Euclidean geometries properties through VR, a screen simulation and a slides presentation [21]. These environments allow users to explore curved or topologically exotic spaces, revealing properties such as distorted parallelism, closed geodesics, or nontrivial holonomy. While primarily developed for exploratory purposes, these projects demonstrate VR’s capability to render mathematically accurate experiences of alternative geometries, including spherical, hyperbolic, and other curved spaces.

Despite these advances, certain challenges remain in leveraging these applications – whether in VR or on screen – for educational purposes. Many are designed primarily for exploration and aesthetic immersion. As a result, they do not include guided tasks, explanatory scaffolding, or assessment strategies, which are necessary for fostering and measuring conceptual understanding [18, 25]. This does not diminish the value of experiential and discovery-based approaches, which are essential to fostering deep understanding – especially in abstract domains – but highlights the importance of aligning such experiences with appropriate pedagogical frameworks. Second, most existing VR implementations focus on hyperbolic or spherical spaces, with little attention given to less familiar spaces like $S^2 \times E$. Moreover, few studies have empirically evaluated whether non-expert users can meaningfully understand the geometry they are exploring, especially in the absence of prior mathematical training [27].

2.4 Cognitive and Affective Factors in Learning Mathematics and Geometry

Learners’ prior knowledge and affective factors such as mathematics anxiety and self-efficacy significantly influence their en-

agement with and understanding of spatial concepts. Research shows that high levels of math anxiety can impair performance on spatial reasoning tasks [2, 20], while strong self-efficacy is positively correlated with better learning outcomes in mathematics [24]. In immersive learning environments, these factors remain important. Studies indicate that learners with limited mathematical background can still benefit from VR experiences if the design emphasizes exploration and reduces the cognitive load associated with conceptual manipulation [19]. Moreover, embodied interaction in VR may help mitigate affective barriers by shifting the focus from abstract reasoning to concrete sensorimotor experience [1].

Assessing intuitive understanding of non-Euclidean geometry presents methodological challenges. Standard quantitative tests often fail to capture qualitative, embodied insights gained through immersive interaction. Consequently, researchers employ mixed methods such as think-aloud protocols, self-explanation, interactive problem-solving, and semi-structured interviews [12, 22]. These approaches enable the observation of spontaneous reasoning and conceptual articulation without requiring formal mathematical vocabulary, which is crucial when working with diverse learners who vary widely in mathematical expertise. Such qualitative measures complement performance-based assessments and offer richer evidence of embodied learning and conceptual change.

2.5 Summary and Research Gaps

While classical methods and physical models remain foundational, VR’s immersive and embodied capabilities provide novel opportunities to engage learners with complex geometric concepts. Existing VR applications demonstrate the feasibility of exploring non-Euclidean spaces, but more structured pedagogical designs and rigorous evaluations are needed – especially for lesser-studied geometries and non-expert audiences. Understanding cognitive and affective factors will be crucial to realizing VR’s full educational potential in advanced geometry.

3 UNDERLYING CONCEPTS

3.1 Mathematical Concepts

The space explored in this study is denoted $S^2 \times E$ and cannot be accurately represented in our physical world. Intuitively, it is a space in which at each point along a straight infinite line, there is attached a copy of a spherical surface. Alternatively, one can think of $S^2 \times E$ as an infinite stack of identical spheres, each corresponding to a different position along the Euclidean direction. Formally, $S^2 \times E$ is a three-dimensional non-Euclidean geometry formed by taking the Cartesian product of a 2-sphere (S^2) and a one-dimensional Euclidean line (E) (see Figure 2).

This geometry is anisotropic: its properties differ depending on the direction considered. Along the E direction, space roughly behaves like ordinary Euclidean space – straight lines extend infinitely, and familiar geometric relations hold. However, in the directions tangent to S^2 , the geometry is curved like the surface of a sphere. This combination leads to surprising visual and spatial phenomena.

A particularly striking feature is the occurrence of *visual echoes*. Due to the positive curvature¹ of spherical dimensions, geodesics² of S^2 reconverge. As a result, light that travels around the curved component S^2 returns to the observer, leading to multiple images of the same object from different directions. Similarly, multiple rays following composite directions in the product space $S \times E$ (cylinder) can also allow an observer to see the same object from various paths of various lengths, resulting in *visual echoes* of the

¹For a two dimensional space, a positive (resp. negative) curvature roughly means that the space everywhere locally looks like a chunk of sphere (resp. horse saddle).

²The equivalent of straight lines on a curved surface.

object in question. This behavior contrasts with Euclidean space, where parallel lines never meet and *visual echoes* do not naturally occur.

Another key geometric property concerns triangles in the S^2 component. In contrast to Euclidean geometry, where the sum of the interior angles of a triangle is always exactly 180° , triangles on a sphere have angle sums greater than or equal to 180° . The excess is directly proportional to the area of the triangle on the spherical surface. In the context of $S^2 \times E$, this relation still holds for triangles constrained to a fixed spherical slice. Thus, by observing the angles of a triangle drawn in the curved direction, one can infer information about its area – a geometric connection that is deeply non-Euclidean in nature.

These foundational concepts form the basis for the interactive experience developed in our study. By navigating and observing within $S^2 \times E$, participants are exposed to its directional curvature, *visual echoes*, and non-Euclidean triangle geometry – without requiring formal mathematical training.

3.2 Simulating Non-Euclidean Spaces

Standard rendering pipelines are generally limited to Euclidean settings, as they model light propagation using straight-line rays. Inspired by the method introduced by Coulon et al. [7], we developed a ray marching rendering framework in which rays follow the geodesics of the underlying geometry. An open-source, web-based implementation provided by the team of the cited paper [7] is available [6].

4 EXPERIMENT

The purpose of this experiment was to investigate whether non-expert participants could develop an intuitive understanding of the non-Euclidean space $S^2 \times E$ through immersive VR coupled with a short out-of-VR introduction without relying on formal mathematical instruction. Participants explored a custom-designed virtual environment and engaged in a series of tasks, each targeting a specific property of the geometry. The experiment followed a learn-by-doing approach and was divided into structured blocks alternating between exploration, evaluation, and reflection. In each scene, they were asked to concentrate on understanding a specific property of the geometry in which they were immersed. Participants were free to spend as much time per property as they wished (see Figure 8).

4.1 Participants and Apparatus

32 participants took part in the experiment (8 females, 24 males), aged from 20 to 50 ($\bar{X} = 28, SD = 7.2$). All participants were employees from the lab from scientific or administrative teams. Participants were not experts on the subject of the experiment: none were mathematicians, and only two participants declared they had heard of $S^2 \times E$ before, but they could not elaborate or were confused with something else. They did not receive any economical compensation for their participation. They all had normal or corrected vision. They were immersed in the virtual environment using an HTC head-mounted display, and the associated controllers. The VR experiment was performed using a desktop computer using Unity 2022.3. Participant’s working area was a rectangle of 4×5 meters. The experiment was approved by the local ethics committee (COERLE n° 2025-07).

4.2 Hypotheses

Based on our review of the literature, we formulated the following hypotheses:

[H1]: Virtual reality, coupled with a short out-of-VR introduction, can enable non-mathematicians to conceptualize and understand complex geometries such as $S^2 \times E$ without receiving formal explanations. Specifically, we hypothesize that our virtual environment will help participants grasp how *visual echoes* behave in

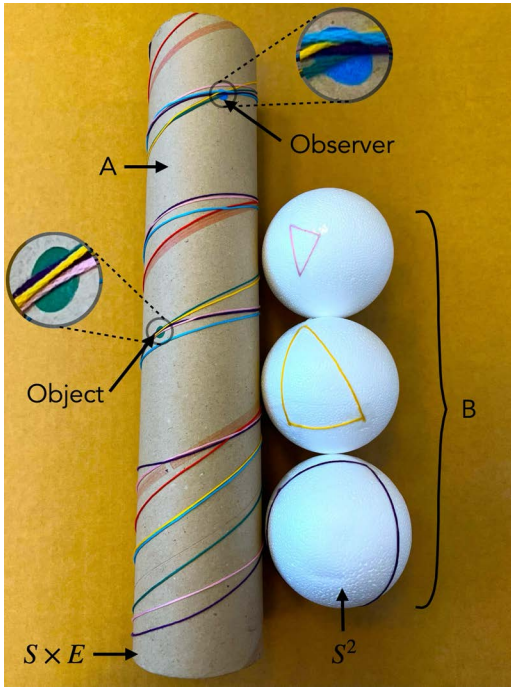


Figure 2: Physical counterparts used for the evaluation tasks outside of VR concerning $S^2 \times E$. The cylinder $S \times E$ and the sphere S^2 represent the different sections of $S^2 \times E$. A: strings represent rays of light. The blue dot represents an observer while the green dot represents an object. This cylinder was used for assessing participants conceptual understanding of *visual echoes*. They were asked which strings allowed the observer to see the object. B: participants had to estimate the value of the sum of the angles of these three triangles. This task was the real counterpart of the virtual task.

$S^2 \times E$ (**H1.a**), and that it will support their understanding of the relationship between the area of a triangle and the sum of its angles (**H1.b**). In addition, we hypothesize that participants will be able to develop and formulate strategies accounting for the specifics of the environment to succeed in the tasks (**H1.c**).

[H2]: Participants' attitude towards mathematics and their prior level of mathematical education will have an impact on their understanding. Specifically, we hypothesize that participants with less mathematical background or a negative disposition towards mathematics will have more difficulty grasping the concepts explored in the virtual environment.

[H3]: Participants will be subjected to a significant raise of their cognitive load. Because the geometry being visited is complex and very different from what the participants are used to, understanding it will be very demanding, as assessed in the corresponding debriefing question.

[H4]: Participants will be bothered by *visual echoes*. As these echoes are a new phenomenon for participants, and as they can change shape and can occupy a large portion of the field of view, participants will have to make an effort to function in spite of them.

4.3 Experimental Protocol and Design

The experiment was divided into four main phases (see Figure 3). The experimenter was present throughout the entire experiment and was responsible for the conceptual briefing (phase 2) and the interview (phase 4). Participants were invited to express any difficulties they encountered, but the experimenter did not answer any questions about geometry so as not to provide more information to one participant than another.

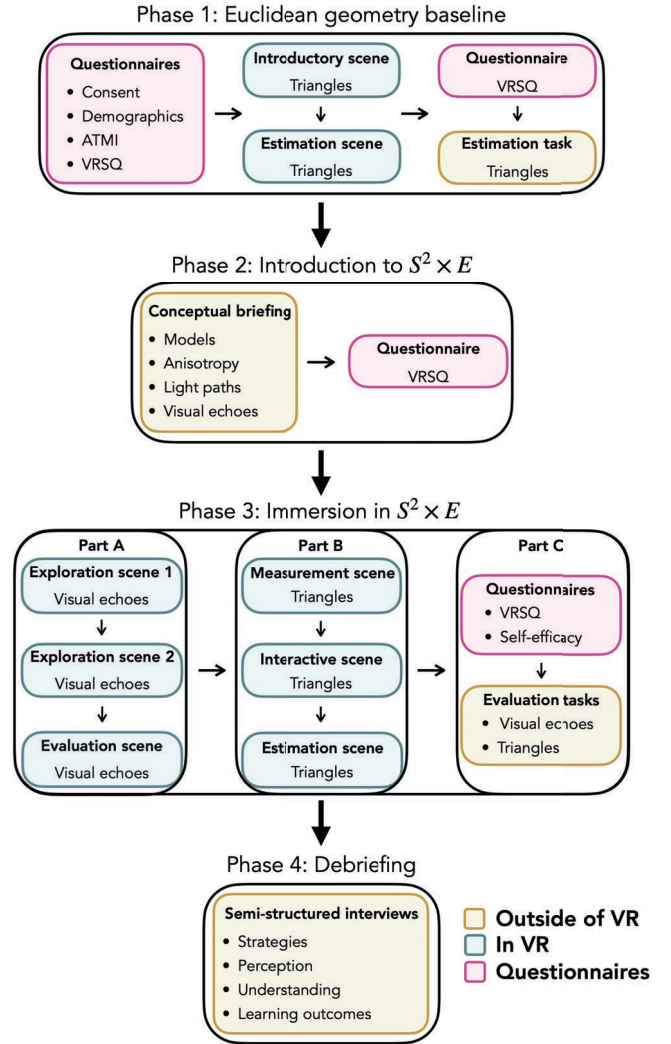


Figure 3: Experimental protocol. In blue, parts of the experiment held in VR, in yellow, parts held outside of VR.

4.3.1 Phase 1: Euclidean Geometry Baseline

Participants first completed a pre-experiment questionnaire, including consent forms, demographic data including level of mathematics education and knowledge about non-Euclidean geometries, the Attitude Towards Mathematics Inventory (ATMI) [16], and the Virtual Reality Sickness Questionnaire (VRSQ) [13]. It should be noted that the ATMI evaluates participants' attitude towards mathematics, not their level of mathematics. Hence the expertise of participants and their score at the ATMI are not to be confused with one another. Expertise was evaluated as part of the demographic questionnaire.

Then, participants were immersed in an introductory VR scene demonstrating triangle geometry in Euclidean space. They manipulated triangle vertices and observed that the sum of the interior angles was always 180° . After that, they completed a triangle estimation task, in which they estimated angle sums of three triangles in a Euclidean scene. The objective of this part of the experiment is threefold: to introduce participants to VR for the first time in a simple and familiar context, to remind them that the sum of the angles of triangles as we know them always equals 180° , and to train participants in the tasks that will be presented to them next.

The VRSQ was administered again to assess any changes in sickness, and a non-VR paper-based Euclidean triangle estimation task

was conducted as a complementary measure.

4.3.2 Phase 2: Introduction to $S^2 \times E$

Before the second immersion, participants received a short guided introduction. Using analogies (such as rolling a sheet of paper into a cylinder), the experimenter explained the anisotropic nature of $S^2 \times E$ and the presence of spherical directions. Participants then learned about light paths in this geometry and identified a path between an object and an observer (see Figure 1), followed by the concept of *visual echoes* – multiple appearances of the same object due to light paths looping around curved space (see Figure 2). This introduction provided only the minimal information necessary for the experiment, without any mathematical formalism or specialized vocabulary. Concepts such as *visual echoes* were introduced explicitly to ensure that questions were unambiguous. Participants began the VR immersion with a limited understanding, enough to explore the environment freely but not detailed enough to guide or bias their exploration.

After this briefing, participants filled out the VRSQ again to track any pre-immersion symptoms.

4.3.3 Phase 3: Immersion in $S^2 \times E$.

The second VR block was split into two themes: *visual echoes* and triangle geometry.

Part A: *visual echoes*

1. Exploration scene 1 – Participants interacted with nine colored objects. They could toggle object visibility (make an object disappear or reappear) and observe how *echoes* behaved (see Figure 1).
2. Exploration scene 2 – A similar scene with seven objects, some sharing the same color, adding complexity.
3. Evaluation scene – Seven gray objects were presented, and participants had to estimate how many were truly present, relying on their understanding of *visual echoes* (see Figure 4). The objects were all gray in order to prevent participants from using color as a visual cue, and to ensure that their judgments were based solely on their interpretation of the *visual echoes*. They had unlimited time (see Figure 8) and up to six attempts (three attempts in the gray scene, and three attempts in the same scene but with colors on the objects). One attempt consisted in the participant guessing how many objects were in the scene. In case of failure, participants continued their exploration without any change in the scene (except for the colors between attempts 3 and 4). This was a deliberate design choice intended to support iterative reasoning and allow participants to refine their judgments and gain confidence in their thought process, rather than to optimize resistance to guessing through scene changes.

Part B: Triangle Geometry in $S^2 \times E$

1. Measurement task – Participants used a virtual protractor to measure the angles of a triangle with a total sum of 310° (see Figure 5).
2. Interactive scene – Participants manipulated triangle vertices and observed real-time updates of angle sums to explore how geometry behaves in this space (see Figure 6).
3. Estimation task – Four triangles were presented sequentially, twice. Participants had to estimate the sum of their interior angles without the protractor.

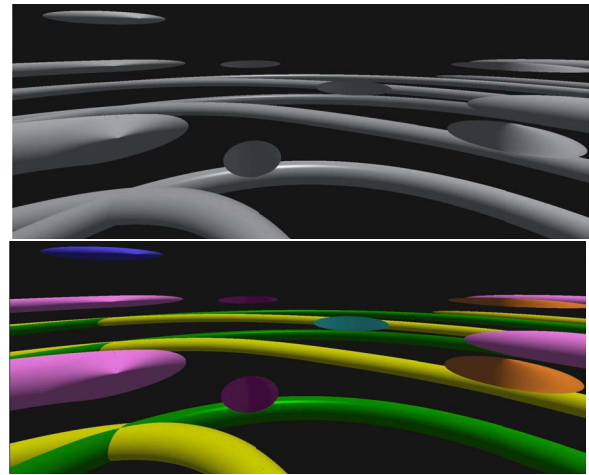


Figure 4: Up: evaluation scene concerning *visual echoes* in $S^2 \times E$. Participants had to count the number of objects in the scene. Down: evaluation scene with colors. Each object has its own color. At the front of the image, we see a purple sphere, a *visual echo* of which can be seen further away in the same direction. There is also a dark blue sphere in the upper left, a turquoise sphere in the middle, and an orange sphere on the right, with a *visual echo* in the background. In pink, we see echoes of a capped cylinder in the scene. Yellow and green show infinite cylinders and their *visual echoes*. The shape and perceived position of the *visual echoes* change when the participant moves.

Part C:

1. The VRSQ was administered again to assess any changes in sickness, as well as a self-efficacy questionnaire, assessing participant's self-efficacy before and after the experiment.
2. A non-VR task concerning *visual echoes* was conducted (see Figure 2).
3. A non-VR spherical triangle estimation task was conducted (see Figure 2).

4.3.4 Phase 4: Debriefing

Participants reviewed video recordings of their evaluation tasks inside the virtual environment alongside the experimenter. A semi-structured interview followed, addressing the following topics:

- Strategies used in Euclidean and non-Euclidean tasks.
- Perception and interpretation of *visual echoes*.
- Understanding of the relation between area and sum of the angles in triangles.
- Experience of cognitive load and visual distraction.
- Insights and learning outcomes.
- Opinions on the pedagogical value of VR for teaching mathematics.

4.3.5 Rationale for the Study Design

Due to known methodological constraints in designing appropriate control conditions in this context – particularly regarding comparability and cognitive accessibility – only an interactive VR condition was implemented. These constraints have been documented in previous work comparing slides, desktop-based simulations and VR for similar geometric content [21]. A more detailed discussion is provided in Section 5.4.7.

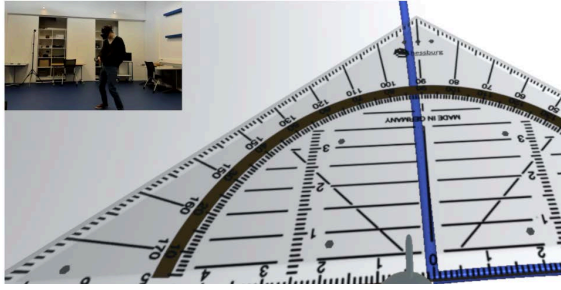


Figure 5: The first step participants took in understanding spherical triangles was to measure each angle of a triangle using a protractor and calculate their sum, thereby realizing that it does not necessarily equal 180° .

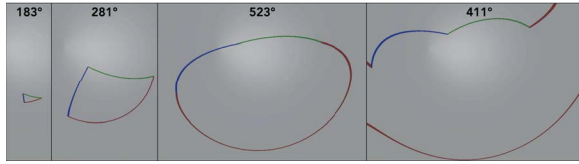


Figure 6: On the exploration scene, participants had to understand the relationship between area and sum of the angles in $S^2 \times E$. Participants could move the vertices of the triangle and read the resulting sum of its angles. Left, the initial triangle with a sum of 183° , second, third, bigger triangles with sums of 281° and 523° . Right, the vertices passed the diameter of the sphere, the triangle reduces in area and sum of its angles with a sum of 411° .

4.4 Experimental Data

4.4.1 Subjective Measures

We used the VRSQ questionnaire [13] to assess participant's VR sickness level. Before starting the experiment, participants were asked to fill in the ATMI [16]. After the second immersion, participants were asked to answer a self-efficacy questionnaire. At the end of the experiment, participants took part in a debriefing that was recorded so that it could be transcribed later. The aim of the debriefing was to gather data on the participants' experiences, the knowledge they had acquired and their intuition, without influencing their answers by using a multiple-choice questionnaire, and allowing them to express their comments more freely than they would have been able to in a written format. The debriefings were not transcribed verbatim. Instead, key elements of participants' responses to each question were manually noted. Once responses from all participants had been collected, these elements were grouped and organized into categories of similar answers.

4.4.2 Objective Measures

We measured the time spent on each task, the answers to the evaluation tasks, as well as the number of time participants pressed the triggers in the scenes where they could have the objects appearing and disappearing. The goal of these measures was to allow correlation tests between participants' behaviours and their performances.

5 RESULTS

Due to the distribution of the data, analysis was conducted using non-parametric statistical methods. Owing to the exploratory nature of this study, correlation analyses were conducted, and no causal relationships are implied by the reported correlations. Given that multiple correlations were tested, the presented p-values should

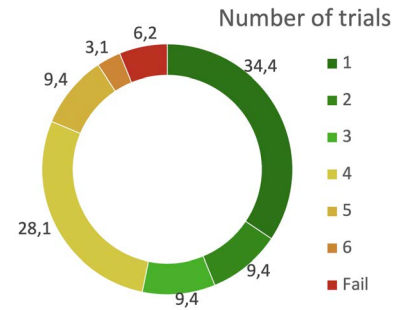


Figure 7: The first three trials (green) were conducted without color cues, whereas the final three trials (yellow) took place within the colored scene. 93.8% of participants successfully completed the counting task. 53.2% succeeded without the use of color cues (green), while 40.6% succeeded using color cues (yellow).

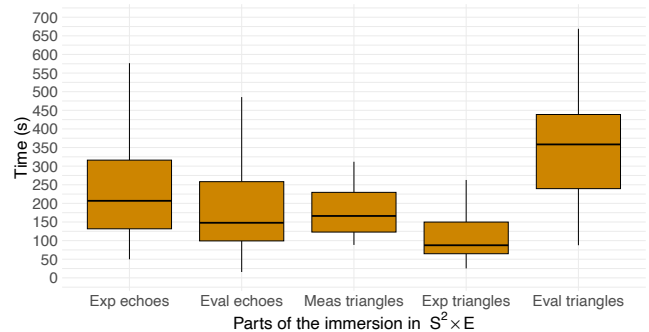


Figure 8: Time spent in each part of the immersion in $S^2 \times E$: exploration scenes concerning the *visual echoes*, counting task for evaluating the understanding of *visual echoes* (time spent per trial), triangle measurement scene, exploration scene concerning triangles, triangles angles estimation scene for evaluating participants' understanding (total time spent for the list of triangles).

be interpreted with caution due to the risk of false positives (multiple comparisons).

5.1 Tasks

To contextualise the results presented in this section, Figure 8 presents the time participants spent to complete the tasks.

5.1.1 Visual echoes

Results concerning the number of trials needed to succeed in the counting task are presented in Figure 7.

We found a moderate negative correlation between the time spent on the exploration scenes and the number of trials needed to succeed at the counting task using a Spearman test $S = 6413$, $\rho = -0.4$, $p\text{-value} < 0.02$, meaning that greater time spent in the exploration scenes was associated with fewer attempts in the evaluation scene.

We also found a moderate positive correlation between the number of time participants had objects appearing and disappearing and the time they spent in the exploratory phase using a Spearman test $S = 3067$, $\rho = 0.4$, $p\text{-value} < 0.01$, meaning that greater time spent in the exploration scenes was associated with a higher number of object appearances and disappearances.

We didn't find any other significant correlation with any other measure, including the score to the ATMI, or the level of mathematics knowledge.

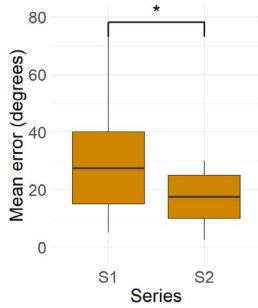


Figure 9: Comparison of mean errors in estimating the sum of the four triangles' angles between the first and second series.

Regarding the out-of-VR task, 72% of participants correctly identified all the strings connecting the observer to the object, without naming any incorrect ones.

5.1.2 Estimation of a triangle's angles

On the estimation of the sum of the angles of the triangles, we found a small negative correlation between the time spent on the task and the error made on the estimation using a Spearman test $S = 3406945$, $\rho = -0.3$, $p\text{-value} < 7.8 \times 10^{-6}$, meaning that greater time spent on the task was associated with better estimations.

We found a significant difference with a moderate effect size ($r = 0.42$) on participant's accuracy on the estimation between first and second time they saw the triangles, using a wilcoxon test $V = 301$, $p\text{-value} = 0.03$. We observe that participants make smaller errors the second time they see the triangles (see Figure 9).

We found a moderate positive correlation between the number of succeeded estimations during the VR part and during the outside-of-VR part using a Spearman test $S = 3352$, $\rho = 0.4$, $p\text{-value} = 0.03$, meaning that participants who were better at estimating the angles of triangles in VR were also better at estimating the angles outside of VR.

Participants accuracy in the estimation tasks was also saved as a complementary measure (see Figure 10). We are unable to analyse these data in depth though, as psychophysics literature on angles estimation does not, to the best of our knowledge, provide a rule for systematic errors in our kind of setting [4]. Figure 10 shows the value of the sum of the angles. However, most of the participants estimated the angles one by one before adding up their estimates. In addition, the psychophysics literature highlights inequalities in the accuracy of angle measurements depending on the values (implicit references, underestimation of small angles, overestimation of large angles, etc.). Here is a list of the value of each angle of each triangle:

- Virtual triangles (four triangles presented sequentially, twice): Triangle 1: $90^\circ, 90^\circ, 60^\circ$ / Triangle 2: $90^\circ, 90^\circ, 160^\circ$ / Triangle 3: $50^\circ, 65^\circ, 100^\circ$ / Triangle 4: $90^\circ, 90^\circ, 125^\circ$;
- Real triangles: Yellow triangle: $55^\circ, 80^\circ, 95^\circ$ / Pink triangle: $40^\circ, 55^\circ, 90^\circ$ / Purple triangle: $105^\circ, 115^\circ, 170^\circ$.

Interestingly, among the four pairs of virtual triangles, the third pair seems to be the easiest one for our participants, when one could hypothesize that it is the hardest one since none of its angles are references (90 degrees or 180 degrees for instance). Concerning the real triangles, it seems that the bigger the triangle the bigger the absolute error, which seems coherent: a 30 degrees error represents 7.7% of the 390 degrees triangle, but would represent 15.8% of the 190 degrees triangle.

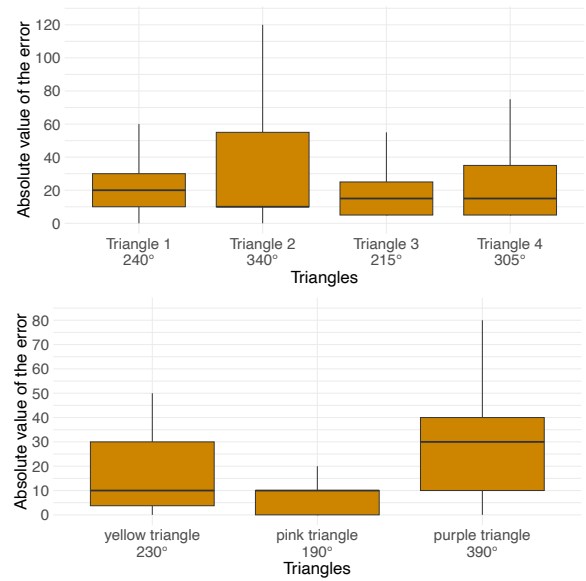


Figure 10: Mean absolute value of the error in triangles angles estimation tasks. Up: VR task. Down: non-VR task.

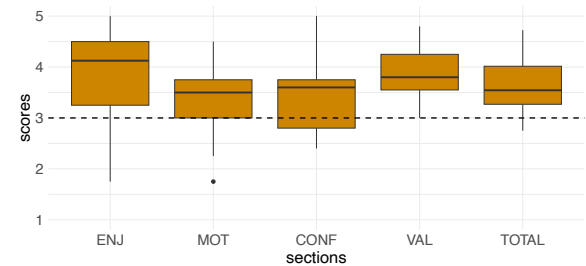


Figure 11: Scores to the ATMI per section (enjoyment, motivation, self-confidence, perceived value) and mean (total).

5.2 Questionnaires

VRSQ scores: we found a statistically significant difference with a large effect size ($r = 0.77$) when running a Wilcoxon test on the VRSQ scores between before the second block and after the second block, even though rates remained low. The rates (on a scale from 0 to 100) were $\bar{X} = 9$, $SD = 9$ before the experiment and $\bar{X} = 13$, $SD = 10$ after, indicating that the level of cybersickness felt is not preventing from using the virtual environment.

ATMI scores: participants had a total score answering to the ATMI of $\bar{X} = 3.6$, $SD = 0.5$ with scores going from 2.8 to 4.7 on a scale from 1 to 5 (see Figure 11).

Self efficacy scores: we found a statistically significant difference with a large effect size ($r = 0.83$) when running a Wilcoxon test on the self efficacy scores between before the experiment and after the second block $V = 14796$, $p\text{-value} < 2.2 \times 10^{-16}$, with a mean total score rising from $\bar{X} = 2.1$, $SD = 2.4$ before the experiment to $\bar{X} = 5.9$, $SD = 2.7$ after the second block on a scale from 1 to 10 .

5.3 Debriefing

5.3.1 Visual echoes

78% of participants developed a good strategy to succeed in the task, with 50% who decided they would group each object with its echoes based on their relative position and shape, 16% who

chose to identify local objects and ignore echoes, and 13% who based their strategy on identifying echoes from their distortions and movements when the participant was moving. We designate a strategy as good when it is founded on a sound reasoning based on the geometry.

Interestingly, 59% of participants declared they were not bothered by the *visual echoes* on this part of the experiment, on top of who 9% said they ignored them, and 6% said that the echoes were helpful to complete the counting task as they allowed participants to group objects with their echoes.

Finally, 69% of participants declared that the cognitive load experienced during this task was ok, while 22% said that they were overloaded, and 9% raised the question of the difficulty they had retaining and summing the objects they already counted.

5.3.2 Relation between area and sum of a triangle's angles

94% of participants were able to articulate relevant factors influencing the sum of the angles in a triangle. Among all participants, 41% used accurate concepts (e.g., area, spatial extent), 38% used moderately accurate terms (e.g., size, curvature, deformation), and 16% used less accurate or confused terminology (e.g., perimeter, side lengths). It is to be noted that the most accurate terms that could be used were terms like area or spatial extent, which are accessible to all the participants because they all went to middle school. In addition, mathematical (area) and non mathematical (spatial extent) terms are valued the same, as our focus is on the developed idea rather than the words used.

5.3.3 Locally Euclidean property

During the debriefing, the experimenter asked participants to describe their strategy for estimating the sum of the angles in the triangles. In this context, 88% explained that they positioned themselves vertically above the angles, having noticed that distortions were locally absent.

Notably, this property of the geometry was neither introduced nor emphasized at any point in the experiment – no introductory explanation or specific task was designed to highlight it. It is therefore particularly encouraging to see that participants were able to identify and leverage this concept independently.

5.3.4 Estimation of a triangle's angles

Most participants, 88%, developed their strategy based on the locally Euclidean property of the geometry, as they decided to go close to each angle to reduce distortions and be able to estimate them properly.

Interestingly, 38% of participants declared they were not bothered by the *visual echoes* on this part of the experiment, on top of who 25% said they ignored them, and 13% said that the echoes were helpful to complete the triangle angle's estimation task, as they were able to get close to angles that were outside of the working area.

In addition, 72% of participants declared that the cognitive load experienced during this task was ok, while 6% said that they were overloaded, and 22% raised the question of the difficulty they had retaining the angles estimated and mental calculation, whatever the geometry.

5.4 Discussion

The results of our experiment support the primary hypothesis [H1] that immersive virtual reality, coupled with a short out-of-VR introduction, can facilitate intuitive understanding of complex non-Euclidean geometries such as $S^2 \times E$, even in participants without formal mathematical training. Both objective and subjective measures indicate that participants were able to engage meaningfully with abstract geometrical concepts through exploratory tasks in VR without any more prior knowledge than the short introduction.

5.4.1 Understanding of Visual Echoes

The high success rate (94%) on the visual echo counting task indicates that most participants were able to adapt to this unfamiliar environment and successfully distinguish between actual objects and their visual echoes. More than half of them succeeded even before color cues were introduced, suggesting that their understanding extended beyond surface-level perceptual cues, therefore supporting our hypothesis [H1.a]. The significant negative correlation between time spent exploring and the number of attempts indicates an association between exploration and performance. This pattern aligns with constructivist principles, which emphasize that active engagement can foster learning; however, any causal interpretation remains tentative, as it cannot be established from correlational evidence alone.

Interestingly, 75% of participants declared that they were not bothered by the *visual echoes* on the counting task as well as on the angles estimation task. 6% even declared that echoes were useful to complete the counting task, and 13% to complete the angles estimation task, therefore not supporting our hypothesis [H4]. It is remarkable that participants adapted to their new environment so quickly. Indeed, it is obvious that counting objects would have been objectively simpler without the echoes. The participants who stated that they had been helped by the presence of the echoes had integrated the environment to the point of not considering any alternative, and they therefore used the characteristics of the environment to their advantage.

5.4.2 Understanding Triangle Geometry in $S^2 \times E$

The vast majority of participants (94%) were able to articulate a meaningful relationship between triangle area and angle sum. This result confirms hypothesis [H1.b] and suggests that intuitive exploration in immersive environments can lead to non-trivial insights about non-Euclidean geometry. Interestingly, participants who spent more time on angle estimation tasks tended to perform better on these estimations. This association is consistent with the idea that success in the task was not automatic but may have developed through engagement and reflection, although a causal relationship cannot be inferred from this correlational pattern. Additionally, participants improved between the first and second angle estimation series, even though no additional instruction was given. This learning effect reinforces the potential of learn-by-doing strategies in VR-based educational tools.

5.4.3 Unprompted Recognition of Local Euclidean Behavior

A particularly compelling result is that 88% of participants spontaneously identified the locally Euclidean property of $S^2 \times E$, despite this not being introduced during the experiment. This suggests that immersive VR can enable participants to observe and interpret subtle properties of the geometry through direct experience, supporting the claim that embodied exploration can be a powerful driver of spatial reasoning in abstract domains.

5.4.4 Developing strategies

81% of participants developed effective strategies to navigate and interpret a highly non-intuitive environment, demonstrating flexibility and perceptual learning. Rather than constructing an explicit mental model of light propagation in $S^2 \times E$, participants appear to have formed task-relevant intuitions that allowed them to solve problems within this space. This distinction is important: their success does not imply a scientific understanding of the geometry's underlying optical behavior, but rather an ability to recognize and respond appropriately to its visual consequences, therefore supporting our hypothesis [H1.c].

5.4.5 Influence of Attitude Towards Mathematics and Participant Demographics

Our results do not support hypothesis [H2]. We found no significant correlation between participants' performance and their scores on the ATMI, nor with their level of prior mathematical education. In other words, participants' attitude towards mathematics and their academic background did not appear to influence their ability to understand and interact with the virtual environment or succeed in the evaluation tasks. This absence of correlation suggests that the immersive and interactive nature of the virtual environment may help reduce the gap usually observed between individuals with different levels of mathematical familiarity. Such environments may thus offer accessible and inclusive ways of engaging with abstract concepts, even for participants with limited formal training in mathematics.

5.4.6 VR Sickness, Self-Efficacy and Cognitive Load

While we observed a slight increase in VRSQ scores after the second block, the overall level of reported sickness remained low. Moreover, mild symptoms did not appear to interfere with task performance. Self-efficacy scores increased significantly after the second immersion, indicating that participants left the experiment feeling more confident in their ability to reason about unfamiliar geometrical concepts. This is a critical indicator for educational effectiveness. In addition, 71% of participants reported no increase of their cognitive load, and 16% more attributed the increasing cognitive load to information retention and mental calculation rather than the geometry in itself, indicating that adapting to $S^2 \times E$ was not as demanding as we thought, therefore not supporting our hypothesis [H3].

5.4.7 Limitations

This study has several limitations. First, although the sample included participants with diverse professional backgrounds – including human resources, communications, finance, doctoral students, interns, and engineers – they were all employees from the lab, and we did not have volunteers with very low scores to the ATMI. These neutral to positive attitude towards mathematics are not to be linked with mathematical training or expertise, as they simply reflect neutral to positive enjoyment, motivation, self-confidence when doing mathematics. It would have been interesting to observe participants with negative attitude, but the participation to the experiment being based on volunteering, people with negative attitude did not wish to participate.

Second, while the inclusion of a control group is typically considered essential for establishing benchmarks in experimental studies, we deliberately chose not to implement one in our protocol. This decision was informed by a prior study [21] conducted in a similar context – non-Euclidean geometries in VR – which highlighted significant challenges in designing an effective control condition. In that study, the authors compared traditional slides, desktop-based simulations, and immersive VR. They encountered critical difficulties in ensuring both the comparability across conditions and the delivery of a fully interactive, immersive VR experience. Moreover, for participants with no prior knowledge of the subject matter, learning these abstract geometrical concepts through traditional or screen-based modalities proved to be disproportionately difficult, making comparisons problematic and potentially misleading. In light of these findings, our study focused exclusively on an immersive VR condition designed with a learn-by-doing approach. Despite the absence of a control group, our results demonstrate a clear and meaningful understanding of the core concepts – concepts that were unfamiliar to participants prior to the experiment. While this study alone does not allow us to make definitive claims about the superiority of VR over other modalities, it nonetheless provides valuable insight into the potential of immersive VR for

making abstract mathematical ideas more accessible and engaging.

Third, the experiment was exploratory in nature, and the VR experience was guided and scaffolded. It remains to be seen how well participants retain or transfer their insights to other contexts or problem types. Fourth, self-efficacy was only measured once after the experiment, limiting our ability to assess changes over time.

5.4.8 Implications and Future Work

These results underscore the potential of immersive VR for introducing abstract mathematical concepts in a tangible and engaging way. In particular, geometries that resist static visualization, like $S^2 \times E$, benefit greatly from experiential immersion. Future work should explore how such tools can be integrated into educational settings, assess long-term retention, and investigate how different design elements (e.g., feedback, interaction types, pacing) influence learning outcomes. Moreover, further studies could investigate how to best scaffold the experience for learners with minimal scientific background, and how to balance visual richness with cognitive load. There is ongoing work on developing VR interactions compatible with non-Euclidean geometries to maximize the educational benefits of this tool. Comparative studies with non-VR versions would also be valuable to determine which aspects of participants' understanding can be specifically attributed to the VR environment. Finally, more extensive research using standardized cognitive load questionnaires could provide deeper insights into how the tool affects learners' mental effort and engagement.

6 CONCLUSION

In this experiment, we had non-mathematicians who were unfamiliar to non-Euclidean geometries and in particular to $S^2 \times E$ explore this geometry in a learning-by-doing approach. The purpose of the experiment was to investigate whether non-expert participants could develop an intuitive understanding of the non-Euclidean space $S^2 \times E$ through immersive VR without relying on formal mathematical instruction. Participants explored a custom-designed virtual environment and engaged in a series of tasks, each targeting a specific property of the geometry.

Results show that most participants were able to infer and articulate the two key properties showcased in the experiment (*visual echoes* and relation area/sum of a triangle's angles), as well as the locally Euclidean property that was not explicitly introduced during the experiment. Additionally, their performance was independent of their educational background or attitude towards mathematics. We also observed that participants accepted the specifics of the geometry quickly, adapted their reasoning, indicating a remarkable and unexpected flexibility in questioning their acquired knowledge about geometry.

Beyond the work that remains to be done to enable the use of virtual reality in teaching non-Euclidean geometries to non-experts (see Section 5.4.8), it would also be worthwhile to explore its potential for expert mathematicians. Although this group already possesses an in-depth understanding of such geometries and may be more accustomed to abstract reasoning based on non-Euclidean principles, they also have years – if not decades – of experience with a formal, symbolic approach to mathematics. It is therefore worth investigating whether they can benefit from this new modality, which quite literally offers a shift in perspective, or whether the paradigm change is too abrupt to be effective.

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