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Robust Filtering Design for Periodic Piecewise Polytopic Systems over a Low-Frequency Range

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Abstract

This paper presents a novel approach based on linear matrix inequalities (LMIs) for the design of filters for continuous-time periodic piecewise systems with polytopic uncertainties, focusing on the low-frequency range. The main contribution is the extension of the generalized Kalman–Yakubovich–Popov (gKYP) lemma to periodic piecewise linear polytopic systems. By exploiting the structural properties of these systems, new sufficient conditions are derived to guarantee the asymptotic stability of the estimation error dynamics while ensuring H_∞ performance over a specified frequency range of the exogenous inputs, despite the presence of admissible uncertainties. Simulation results illustrate the effectiveness and practical applicability of the proposed method.

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Keywords: Periodic Piecewise Systems; Filtering; low-frequency Analysis; Polytopic Uncertainties; Linear Matrix Inequalities (LMIs).

1. Introduction

In recent years, periodic systems have attracted significant research interest, as many engineering applications exhibit inherently periodic behavior, such as rotor-blade systems [25], satellites [26], and vibratory systems [19]. Consequently, numerous studies have focused on analyzing and controlling such systems. For example, using structured Lyapunov functions, the authors in [17] analyzed the stability of periodic systems under parametric uncertainties. The filtering problem under polytopic uncertainties was investigated in [24], stabilization was addressed in [5], and filtering was also studied in [1].

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With technological advancements, periodic models are increasingly regarded as realistic and effective representations of many practical phenomena. A notable subclass of these systems is piecewise periodic systems, where the overall period is divided into several sub-periods, each exhibiting time-invariant behavior. Such models can be derived using techniques like the averaging method [23, 13], which approximates the system by its averaged dynamics.

Over the past decades, significant efforts have been devoted to state estimation and filtering. Numerous techniques aiming to minimize the filtering error have been developed. The seminal work of [16] remains foundational, providing optimal filtering results. Subsequent studies addressed robust filtering for uncertain systems [14], robust filtering for singular switched systems [4], and the design of filters ensuring minimal energy-to-peak gain between noise and output [3, 27]. Additionally, H_∞ filtering techniques were studied in [28].

Parameter uncertainty is common in system modeling. For polytopic uncertainties, some works employ Lyapunov functions independent of the polytopic parameter or within a quadratic framework [2, 6], while others use parameter-dependent Lyapunov functions [20]. Research on periodic piecewise systems, however, remains limited. For instance, [12] investigated peak-to-peak filtering, [22] addressed H_∞ filtering with missing output measurements, and [21] designed filters with dissipative specifications.

It is important to note that most of the aforementioned filtering approaches for both LTI and periodic systems are developed over the entire frequency (EF) domain of disturbances or noise. However, in many practical applications, the frequency range of disturbances is known a priori. In such cases, designing filters over a finite-frequency (FF) domain is more appropriate. Several studies have addressed this problem [7, 18, 8], including filtering of Takagi–Sugeno systems [10, 9].

Despite these developments, efficient filtering techniques for continuous-time periodic piecewise linear polytopic systems specifically within a predefined frequency band remain largely unexplored. This gap motivates the present work.

In this paper, we propose a novel method based on linear matrix inequalities (LMIs) for the design of filters for continuous-time periodic piecewise linear polytopic systems. The main contribution is the extension of the generalized Kalman–Yakubovich–Popov (gKYP) lemma to such systems, together with the development of new LMI-based conditions to ensure both asymptotic convergence of the estimation error dynamics and H_∞ performance against external disturbances in a low-frequency domain for all admissible uncertainties. Simulation results are provided to illustrate the effectiveness of the proposed approach.

Notation: We denote by \mathbb{R}^n the n -dimensional Euclidean space. The notation $P > 0$ (respectively, $P \geq 0$) indicates that P is a symmetric positive definite (respectively, positive semi-definite) matrix. The symbol $*$ denotes symmetric terms in block matrices. The superscript \top represents the matrix transpose. The operator $\text{sym}(A)$ is defined as $\text{sym}(A) = A + A^\top$.

2. Problem Formulation and Preliminaries

Consider a class of uncertain continuous-time periodic piecewise linear systems described by

$$\begin{cases} \dot{x}(t) &= A(t; \rho)x(t) + B_\omega(t; \rho)\omega(t), \\ y(t) &= C(t; \rho)x(t) + D_\omega(t; \rho)\omega(t), \\ z(t) &= L(t; \rho)x(t), \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ and $y(t) \in \mathbb{R}^p$ denote the state and output vectors, respectively, $\omega(t) \in \mathbb{R}^m$ is the disturbance input vector, and $z(t) \in \mathbb{R}^l$ is the signal to be estimated. The parameter vector ρ is assumed to be constant over time.

The system’s periodicity is reflected in the matrices $A(t; \rho)$, $B_\omega(t; \rho)$, $C(t; \rho)$, $D_\omega(t; \rho)$, and $L(t; \rho)$, which satisfy

$$\begin{aligned} A(t + \alpha\mathcal{T}_p; \rho) &= A(t; \rho), & B_\omega(t + \alpha\mathcal{T}_p; \rho) &= B_\omega(t; \rho), \\ C(t + \alpha\mathcal{T}_p; \rho) &= C(t; \rho), & D_\omega(t + \alpha\mathcal{T}_p; \rho) &= D_\omega(t; \rho), \\ L(t + \alpha\mathcal{T}_p; \rho) &= L(t; \rho), \end{aligned}$$

where \mathcal{T}_p is the fundamental period and $\alpha \in \{0, 1, 2, \dots\}$ for all $t \geq 0$.

We assume that each interval $[\alpha\mathcal{T}_p, (\alpha+1)\mathcal{T}_p)$ can be partitioned into \mathcal{N} sub-intervals $[\alpha\mathcal{T}_p + t_{i-1}, \alpha\mathcal{T}_p + t_i)$, $i \in \mathbb{N} = \{1, 2, \dots, \mathcal{N}\}$, with $t_0 = 0$ and $t_{\mathcal{N}} = \mathcal{T}_p$. Within each sub-interval, the system matrices are constant and denoted by $(A_i, B_{\omega_i}, C_i, D_{\omega_i}, L_i)$. The dwell time is $\mathcal{T}_i = t_i - t_{i-1}$, satisfying $\sum_{i=1}^{\mathcal{N}} \mathcal{T}_i = \mathcal{T}_p$.

Accordingly, equation (1) can be equivalently represented as

$$\begin{cases} \dot{x}(t) &= A_i(\rho)x(t) + B_{\omega_i}(\rho)\omega(t), \\ y(t) &= C_i(\rho)x(t) + D_{\omega_i}(\rho)\omega(t), \\ z(t) &= L_i(\rho)x(t), \quad t \in [\alpha\mathcal{T}_p + t_{i-1}, \alpha\mathcal{T}_p + t_i). \end{cases} \tag{2}$$

The system matrices of each subsystem are assumed to be partially unknown but belong to a convex bounded polytopic set \mathcal{X}_i with r vertices:

$$\mathcal{X}_i \triangleq \left\{ (A_i(\rho), B_{\omega_i}(\rho), C_i(\rho), D_{\omega_i}(\rho), L_i(\rho)) \mid (A_i, B_{\omega_i}, C_i, D_{\omega_i}, L_i) = \sum_{j=1}^r \rho_j (A_{i,j}, B_{\omega_{i,j}}, C_{i,j}, D_{\omega_{i,j}}, L_{i,j}), \rho \in \mathbb{W} \right\}, \tag{3}$$

where

$$\mathbb{W} = \left\{ (\rho_1, \dots, \rho_r) \mid \sum_{j=1}^r \rho_j = 1, \rho_j \geq 0 \right\}. \tag{4}$$

Here, $(A_{i,j}, B_{\omega_{i,j}}, C_{i,j}, D_{\omega_{i,j}}, L_{i,j})$, $j = 1, \dots, r$, represent the vertices of the polytope \mathcal{X}_i for the i -th subsystem. Such polytopic uncertainties provide a convenient framework for modeling parametric uncertainties.

The objective of this work is to design a full-order filter, which is assumed to be periodic with the same fundamental period \mathcal{T}_p as system (1), in the following form:

$$\begin{cases} \dot{x}_f(t) &= A_{f_i}x_f(t) + B_{\omega_{f_i}}y(t), \\ z_f(t) &= L_{f_i}x_f(t) + D_{\omega_{f_i}}y(t), \quad t \in [\alpha\mathcal{T}_p + t_{i-1}, \alpha\mathcal{T}_p + t_i), \end{cases} \tag{5}$$

where $x_f(t) \in \mathbb{R}^{n_r}$ denotes the filter state vector and $z_f(t) \in \mathbb{R}^l$ is the estimation of the signal $z(t)$. The periodic filter gains to be determined are A_{f_i} , $B_{\omega_{f_i}}$, L_{f_i} , and $D_{\omega_{f_i}}$.

Based on (2) and (5), the corresponding error system can be written as:

$$\begin{cases} \dot{x}_e(t) &= A_{e_i}(\rho)x_e(t) + B_{e_i}(\rho)\omega(t), \\ z_e(t) &= L_{e_i}(\rho)x_e(t) + D_{e_i}(\rho)\omega(t), \end{cases} \tag{6}$$

where

$$\begin{aligned} x_e(t) &= \text{error state vector}, & z_e(t) &= z(t) - z_f(t), \\ A_{e_i}(\rho) &= \begin{bmatrix} A_i(\rho) & 0 \\ B_{\omega_{f_i}} & C_i(\rho) \end{bmatrix}, & B_{e_i}(\rho) &= \begin{bmatrix} B_{\omega_i}(\rho) \\ B_{\omega_{f_i}} & D_{\omega_i}(\rho) \end{bmatrix}, \\ L_{e_i}(\rho) &= [L_i(\rho) - D_{\omega_{f_i}}C_i(\rho) - L_{f_i}], & D_{e_i}(\rho) &= -D_{\omega_{f_i}}D_{\omega_i}(\rho), \end{aligned}$$

for all $t \in [\alpha\mathcal{T}_p + t_{i-1}, \alpha\mathcal{T}_p + t_i)$.

Definition 1 The error system (6) is said to satisfy the H_∞ performance with attenuation level $\gamma > 0$ if:

1. The error system (6) is asymptotically stable when $\omega(t) = 0$.
2. For a given $\gamma > 0$ and zero initial condition, the following inequality holds:

$$\int_{|\bar{\omega}| \leq \omega_t} \hat{z}_e(\bar{\omega})^\top \hat{z}_e(\bar{\omega}) d\bar{\omega} \leq \gamma^2 \int_{|\bar{\omega}| \leq \omega_t} \hat{\omega}(\bar{\omega})^\top \hat{\omega}(\bar{\omega}) d\bar{\omega}, \tag{7}$$

for any $\hat{\omega}(\bar{\omega}) \in L_2[0, \infty)$ [15].

The following lemmas are used in the development of our results.

Lemma 1 [9] Let matrices $Z \in \mathbb{R}^{m \times n}$, $X \in \mathbb{R}^{p \times n}$, and a symmetric matrix $\Sigma = \Sigma^\top \in \mathbb{R}^{n \times n}$ be given, with $\text{rank}(Z) < n$ and $\text{rank}(X) < n$. Then the following statements are equivalent:

1. There exists a matrix $Y \in \mathbb{R}^{p \times m}$ such that

$$\Sigma + Z^\top Y^\top X + X^\top Y Z < 0. \tag{8}$$

2. The following conditions hold:

$$Z^\perp{}^\top \Sigma Z^\perp < 0, \quad X^\perp{}^\top \Sigma X^\perp < 0,$$

where Z^\perp and X^\perp denote the orthogonal projections of Z and X , respectively.

Lemma 2 [11] Assume that the error dynamics (6) is asymptotically stable and $\gamma > 0$ is given. The system (6) has a low-frequency (LF) L_2 -gain bounded by γ if there exist parameter-dependent periodic matrices $P_i(\rho) > 0$, $i \in \{1, \dots, \mathcal{N}\}$, and a symmetric matrix $M > 0$ such that

$$\psi = \begin{bmatrix} A_{e_i}(\rho) & B_{e_i}(\rho) \\ I & 0 \end{bmatrix}^\top \Pi_i \begin{bmatrix} A_{e_i}(\rho) & B_{e_i}(\rho) \\ I & 0 \end{bmatrix} + \begin{bmatrix} L_{e_i}(\rho) & D_{e_i}(\rho) \\ 0 & I \end{bmatrix}^\top \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} L_{e_i}(\rho) & D_{e_i}(\rho) \\ 0 & I \end{bmatrix} < 0, \tag{9}$$

$$\phi = \begin{bmatrix} A_{e_i}(\rho) & B_{e_i}(\rho) \\ I & 0 \end{bmatrix}^\top \Pi_{i+1} \begin{bmatrix} A_{e_i}(\rho) & B_{e_i}(\rho) \\ I & 0 \end{bmatrix} + \begin{bmatrix} L_{e_i}(\rho) & D_{e_i}(\rho) \\ 0 & I \end{bmatrix}^\top \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} L_{e_i}(\rho) & D_{e_i}(\rho) \\ 0 & I \end{bmatrix} < 0, \tag{10}$$

with

$$P_i = P_{\mathcal{N}+i}, \quad \Pi_i = \begin{bmatrix} -M & P_i(\rho) \\ P_i(\rho) & \omega_l^2 M + \frac{P_i(\rho)}{\tau_i} \end{bmatrix}, \quad \Pi_{i+1} = \begin{bmatrix} -M & P_{i+1}(\rho) \\ P_{i+1}(\rho) & \omega_l^2 M + \frac{P_{i+1}(\rho) - P_i(\rho)}{\tau_i} \end{bmatrix},$$

for all admissible uncertainties.

3. Main Results

In the following, one sufficient condition is presented for error system (6) having a low-frequency (LF) with H_∞ performance index γ , for all admissible uncertainties.

Theorem 1 Given scalars \mathcal{T}_i and a performance index $\gamma > 0$ and for a frequency interval $|\bar{\omega}| \leq \omega_l$, system (6) is asymptotically stable and satisfying the (LF) specification if there exist parameter-dependent positive definite \mathcal{T}_p -periodic matrix $P_i(\rho)$, constant positive definite matrix M , \mathcal{T}_p -periodic matrices $\mathcal{G}_i, \mathcal{V}_i, \mathcal{A}_{f_i}, \mathcal{B}_{f_i}, L_{f_i}, D_{\omega_{f_i}}, \mathcal{H}_i$ and \mathcal{R}_i of appropriate dimensions such that

$$\begin{bmatrix} -I & 0 & 0 & \psi_{14} & -L_{f_i} & -D_{\omega_{f_i}} D_{\omega_i}(\rho) \\ * & -\mathcal{M}_1 & -\mathcal{M}_3 & \psi_{24} & \psi_{25} & -\mathcal{H}_i^\top \\ * & * & -\mathcal{M}_2 & \psi_{34} & \psi_{35} & 0 \\ * & * & * & \psi_{44} & \psi_{45} & \psi_{46} \\ * & * & * & * & \psi_{55} & \psi_{56} \\ * & * & * & * & * & \psi_{66} \end{bmatrix} < 0 \tag{11}$$

$$\begin{bmatrix} -I & 0 & 0 & \phi_{14} & -L_{f_i} & -D_{\omega_{f_i}} D_{\omega_i}(\rho) \\ * & -\mathcal{M}_1 & -\mathcal{M}_3 & \phi_{24} & \phi_{25} & -\mathcal{H}_i^\top \\ * & * & -\mathcal{M}_2 & \phi_{34} & \phi_{35} & 0 \\ * & * & * & \phi_{44} & \phi_{45} & \phi_{46} \\ * & * & * & * & \phi_{55} & \phi_{56} \\ * & * & * & * & * & \phi_{66} \end{bmatrix} < 0 \tag{12}$$

$$P_i = P_{N+i} \tag{13}$$

where

$$\begin{aligned} \psi_{14} &= L_i(\rho) - D_{\omega_{f_i}} C_i(\rho) & \psi_{24} &= \mathcal{P}_{1_i}(\rho) - \mathcal{G}_i^\top, & \psi_{25} &= \mathcal{P}_{3_i}(\rho) - \mathcal{R}_i^\top, & \psi_{34} &= \mathcal{P}_{3_i}^\top(\rho) - \mathcal{V}_i^\top, \\ \psi_{35} &= \mathcal{P}_{2_i}(\rho) - \mathcal{V}_i^\top, & \psi_{44} &= \frac{\mathcal{P}_{1_{i+1}}(\rho) - \mathcal{P}_{1_i}(\rho)}{\mathcal{T}_i} + \omega_i^2 \mathcal{M}_1 + \text{sym}(\mathcal{G}_i A_i(\rho) B_{\omega_{f_i}} C_i(\rho)), \\ \psi_{45} &= \frac{\mathcal{P}_{3_{i+1}}(\rho) - \mathcal{P}_{3_i}(\rho)}{\mathcal{T}_i} + \omega_i^2 \mathcal{M}_3 + A_i^\top(\rho) \mathcal{R}_i^\top + \mathcal{A}_{f_i} + C_i(\rho)^\top B_{\omega_{f_i}}^\top, \\ \psi_{46} &= \mathcal{G}_i B_{\omega_i}(\rho) + \mathcal{B}_{f_i} D_{\omega_i}(\rho) + A_i^\top(\rho) \mathcal{H}_i^\top, & \psi_{55} &= \frac{\mathcal{P}_{2_{i+1}}(\rho) - \mathcal{P}_{2_i}(\rho)}{\mathcal{T}_i} + \omega_i^2 \mathcal{M}_2 + \mathcal{A}_{f_i} + \mathcal{A}_{f_i}^\top, \\ \psi_{56} &= \mathcal{R}_i B_{\omega_i}(\rho) + \mathcal{B}_{f_i} D_{\omega_i}(\rho), & \psi_{66} &= -\gamma^2 I + \text{sym}(\mathcal{H}_i B_{\omega_i}(\rho)); \\ \phi_{14} &= L_i(\rho) - D_{\omega_{f_i}} C_i(\rho) & \phi_{24} &= \mathcal{P}_{1_{i+1}}(\rho) - \mathcal{G}_i^\top, & \phi_{25} &= \mathcal{P}_{3_{i+1}}(\rho) - \mathcal{R}_i^\top, & \phi_{34} &= \mathcal{P}_{3_{i+1}}^\top(\rho) - \mathcal{V}_i^\top, \\ \phi_{35} &= \mathcal{P}_{2_{i+1}}(\rho) - \mathcal{V}_i^\top, & \phi_{44} &= \frac{\mathcal{P}_{1_{i+1}}(\rho) - \mathcal{P}_{1_i}(\rho)}{\mathcal{T}_i} + \omega_i^2 \mathcal{M}_1 + \text{sym}(\mathcal{G}_i A_i(\rho) B_{\omega_{f_i}} C_i(\rho)), \\ \phi_{45} &= \frac{\mathcal{P}_{3_{i+1}}(\rho) - \mathcal{P}_{3_i}(\rho)}{\mathcal{T}_i} + \omega_i^2 \mathcal{M}_3 + A_i^\top(\rho) \mathcal{R}_i^\top + \mathcal{A}_{f_i} + C_i(\rho)^\top B_{\omega_{f_i}}^\top, \\ \phi_{46} &= \mathcal{G}_i B_{\omega_i}(\rho) + \mathcal{B}_{f_i} D_{\omega_i}(\rho) + A_i^\top(\rho) \mathcal{H}_i^\top, & \phi_{55} &= \frac{\mathcal{P}_{2_{i+1}}(\rho) - \mathcal{P}_{2_i}(\rho)}{\mathcal{T}_i} + \omega_i^2 \mathcal{M}_2 + \mathcal{A}_{f_i} + \mathcal{A}_{f_i}^\top, \\ \phi_{56} &= \mathcal{R}_i B_{\omega_i}(\rho) + \mathcal{B}_{f_i} D_{\omega_i}(\rho), & \phi_{66} &= -\gamma^2 I + \text{sym}(\mathcal{H}_i B_{\omega_i}(\rho)). \end{aligned}$$

for all admissible uncertainties and $i \in \{0, 1, \dots, N\}$ moreover, if the above conditions are satisfied, the robust filter matrices in (5) are given by

$$A_{f_i} = \mathcal{V}_i^{-1} \mathcal{A}_{f_i}, \quad B_{\omega_{f_i}} = \mathcal{V}_i^{-1} \mathcal{B}_{f_i},$$

Proof 2. Matrices ϕ and ψ can be written as follows

$$\psi = \begin{bmatrix} A_{e_i}(\rho) & B_{e_i}(\rho) \\ I & 0 \\ 0 & I \end{bmatrix}^\top \Xi_i \begin{bmatrix} A_{e_i}(\rho) & B_{e_i}(\rho) \\ I & 0 \\ 0 & I \end{bmatrix} < 0; \quad \phi = \begin{bmatrix} A_{e_i}(\rho) & B_{e_i}(\rho) \\ I & 0 \\ 0 & I \end{bmatrix}^\top \Xi_{i+1} \begin{bmatrix} A_{e_i}(\rho) & B_{e_i}(\rho) \\ I & 0 \\ 0 & I \end{bmatrix} < 0 \tag{14}$$

where

$$\Xi_i = \begin{bmatrix} -M & P_i(\rho) & 0 \\ * & \frac{(P_{i+1}(\rho)-P_i(\rho))}{\mathcal{T}_i} + \omega_l^2 M + L_{e_i}^\top L_{e_i}(\rho) & L_{e_i}^\top(\rho)D_{e_i}(\rho) \\ * & * & -\gamma^2 I + D_{e_i}^\top(\rho)D_{e_i}(\rho) \end{bmatrix}.$$

and

$$\Xi_{i+1} = \begin{bmatrix} -M & P_{i+1}(\rho) & 0 \\ * & \frac{(P_{i+1}(\rho)-P_i(\rho))}{\mathcal{T}_i} + \omega_l^2 M + L_{e_i}^\top L_{e_i}(\rho) & L_{e_i}^\top(\rho)D_{e_i}(\rho) \\ * & * & -\gamma^2 I + D_{e_i}^\top(\rho)D_{e_i}(\rho) \end{bmatrix}$$

Thus, according to Lemma 1, we have the following inequalities, which are equivalent to equation (8)

$$\begin{bmatrix} -M & P_i(\rho) & 0 \\ * & \frac{(P_{i+1}(\rho)-P_i(\rho))}{\mathcal{T}_i} + \omega_l^2 M + L_{e_i}^\top L_{e_i}(\rho) & L_{e_i}^\top(\rho)D_{e_i}(\rho) \\ * & * & -\gamma^2 I + D_{e_i}^\top(\rho)D_{e_i}(\rho) \end{bmatrix} + \text{sym} \left\{ \begin{bmatrix} 0 \\ \mathcal{K}_i \\ \mathcal{F}_i \end{bmatrix} \begin{bmatrix} -I & A_{e_i} & B_{e_i} \end{bmatrix} \right\} < 0 \quad (15)$$

$$\begin{bmatrix} -M & P_{i+1}(\rho) & 0 \\ * & \frac{(P_{i+1}(\rho)-P_i(\rho))}{\mathcal{T}_i} + \omega_l^2 M + L_{e_i}^\top L_{e_i}(\rho) & L_{e_i}^\top(\rho)D_{e_i}(\rho) \\ * & * & -\gamma^2 I + D_{e_i}^\top(\rho)D_{e_i}(\rho) \end{bmatrix} + \text{sym} \left\{ \begin{bmatrix} 0 \\ \mathcal{K}_i \\ \mathcal{F}_i \end{bmatrix} \begin{bmatrix} -I & A_{e_i} & B_{e_i} \end{bmatrix} \right\} < 0 \quad (16)$$

By applying Schur complement to the inequalities (16) and (15), and assuming that the following variables are in the following form

$$\mathcal{K}_i = \begin{bmatrix} \mathcal{G}_i & \mathcal{V}_i \\ \mathcal{R}_i & \mathcal{V}_i \end{bmatrix}, \quad \mathcal{F}_i = [\mathcal{H}_i \ 0], \quad P_i = \begin{bmatrix} \mathcal{P}_{1_i}(\rho) & \mathcal{P}_{3_i}(\rho) \\ * & \mathcal{P}_{2_i}(\rho) \end{bmatrix}, \quad M = \begin{bmatrix} \mathcal{M}_1 & \mathcal{M}_3 \\ * & \mathcal{M}_2 \end{bmatrix}$$

it is easy to obtain the results from Theorem 1, which completes the proof.

Remark 1 It should be noted that we are limiting ourselves here to the study of piecewise periodic systems that can be represented by equation (2). Issues of finite frequency for other types of periodic systems, particularly those with different periodicities or without a common fundamental period \mathcal{T}_p , are not addressed in this article. This restriction constitutes a limitation of our work, which could be overcome in future studies.

4. Simulation Results

To highlight the advantages of the proposed technique, we apply our approach to two numerical examples, and the simulation results illustrate the effectiveness of the proposed method. The first example, without uncertainties, demonstrates effective filtering with good disturbance attenuation. The second example, with uncertainties, provides a performance comparison.

4.1. Example 1

The dynamics of a two-degree-of-freedom vibration system are represented by [12]:

$$M\ddot{\mathcal{Q}}(t) + C\dot{\mathcal{Q}}(t) + K\mathcal{Q}(t) = H\omega(t), \tag{17}$$

where $\mathcal{Q}(t) = [d_1(t) \ d_2(t)]^T$ denotes the displacement of the masses. The system is excited by a disturbance input $\omega(t)$ applied to mass m_2 .

M is the mass matrix, C is the damping matrix, and K is the stiffness matrix. These matrices are defined as follows:

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \quad C = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}, \quad K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}.$$

Assuming the state vector $x(t) = [Q(t) \dot{Q}(t)]^T$, the state-space representation of the system is:

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B_\omega = \begin{bmatrix} 0 \\ -M^{-1}H \end{bmatrix}.$$

The subsystem matrices are given as follows:

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -37.50 & 12.50 & -2.50 & 1.25 \\ 27.78 & -27.78 & 2.78 & -2.78 \end{bmatrix}, \quad B_{\omega_1} = \begin{bmatrix} 0 \\ 0 \\ 0.25 \\ 0.56 \end{bmatrix}, \quad L_1 = [1 \ 1 \ 0 \ 0], \quad C_1 = [0 \ 1 \ 0 \ 0], \quad D_{\omega_1} = 1.$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -37.50 & 12.50 & -2.50 & 1.25 \\ 25.00 & -25.00 & 2.50 & -2.50 \end{bmatrix}, \quad B_{\omega_2} = \begin{bmatrix} 0 \\ 0 \\ 0.25 \\ 0.56 \end{bmatrix}, \quad L_2 = [1 \ 1 \ 0 \ 0], \quad C_2 = [0 \ 1 \ 0 \ 0], \quad D_{\omega_2} = 1.$$

$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -37.50 & 12.50 & -2.50 & 1.25 \\ 22.73 & -22.73 & 2.27 & -2.27 \end{bmatrix}, \quad B_{\omega_3} = \begin{bmatrix} 0 \\ 0 \\ 0.25 \\ 0.56 \end{bmatrix}, \quad L_3 = [1 \ 1 \ 0 \ 0], \quad C_3 = [0 \ 1 \ 0 \ 0], \quad D_{\omega_3} = 1.$$

All system matrices share the same fundamental period $\mathcal{T}_p = 15$ s, with $\mathcal{T}_1 = \mathcal{T}_2 = \mathcal{T}_3 = 5$ s. The low-frequency limit is selected as $\omega_l = 10$ rad/s.

The periodic filter parameters for system (5) with performance index $\gamma = 0.59$ are given as follows:

$$A_{f_1} = \begin{bmatrix} -0.1744 & 0.0917 & 1.0376 & -0.0177 \\ 0.1808 & -0.0948 & -0.0285 & 1.0128 \\ -38.6712 & 12.8194 & -3.3303 & 1.7123 \\ 28.8788 & -28.8690 & 3.6612 & -3.2682 \end{bmatrix}, \quad B_{\omega_{f_1}} = \begin{bmatrix} 0.0002 \\ -0.0002 \\ -0.2467 \\ -0.5629 \end{bmatrix}, \quad D_{\omega_{f_1}} = -0.0014,$$

$$A_{f_2} = \begin{bmatrix} -0.1856 & 0.1014 & 1.0381 & -0.0175 \\ 0.1845 & -0.1004 & -0.0317 & 1.0142 \\ -38.4252 & 12.6128 & -3.2933 & 1.6837 \\ 25.8023 & -25.8585 & 3.2996 & -2.9360 \end{bmatrix}, \quad B_{\omega_{f_2}} = \begin{bmatrix} 0.0002 \\ -0.0002 \\ -0.2467 \\ -0.5634 \end{bmatrix}, \quad D_{\omega_{f_2}} = -0.0014,$$

$$A_{f_3} = \begin{bmatrix} -0.1866 & 0.1047 & 1.0371 & -0.0168 \\ 0.1796 & -0.1008 & -0.0342 & 1.0154 \\ -38.3007 & 12.4834 & -3.2716 & 1.6658 \\ 23.3657 & -23.4490 & 2.9980 & -2.6623 \end{bmatrix}, \quad B_{\omega_{f_3}} = \begin{bmatrix} 0.0003 \\ -0.0002 \\ -0.2465 \\ -0.5640 \end{bmatrix}, \quad D_{\omega_{f_3}} = -0.0016$$

$$L_{f_1} = [-0.6912 \ -1.1613 \ 0.0477 \ -0.0314], \quad L_{f_2} = [-0.6806 \ -1.1571 \ 0.0482 \ -0.0313], \\ L_{f_3} = [-0.6649 \ -1.1559 \ 0.0602 \ -0.0377];$$

The disturbance input signal is $\omega(t) = e^{-0.1t} 1.25 \sin(0.314t)$ for $t \in [0, 100]$. The resulting original output $z(t)$, estimated output $z_f(t)$, and filtering error $z_e(t)$ are presented in Fig. 1a and Fig. 1b.

It can be observed that the estimated state $z_f(t)$ closely follows the signal $z(t)$, resulting in the filtering error $z_e(t)$ approaching zero.

4.2. Example 2

Consider an uncertain continuous-time periodic piecewise system with $\mathcal{T}_p = 4$ s and $\mathcal{T}_1 = 1.5$ s, $\mathcal{T}_2 = 2.5$ s and we select $\omega_l = 10$ rad/s. There are two groups of vertex matrices in each subsystem and the parameters are given as follows:

Uncertain subsystem 1:

$$A_1(\rho) = \rho A_{1,1} + (1 - \rho)A_{1,2}, \quad B_1(\rho) = \begin{bmatrix} 1.2 \\ -1.0 \end{bmatrix}, \quad C_1(\rho) = [0.8 \ 1.2], \quad D_1(\rho) = 1.2, \quad L_1(\rho) = [1.2 \ 1.0].$$

where

$$A_{1,1} = \begin{bmatrix} -2.5 & 2.1 \\ 1.6 & -3.4 \end{bmatrix}, \quad A_{1,2} = \begin{bmatrix} -2.5 & 2.1 \\ 1.6 & -2.9 \end{bmatrix}$$

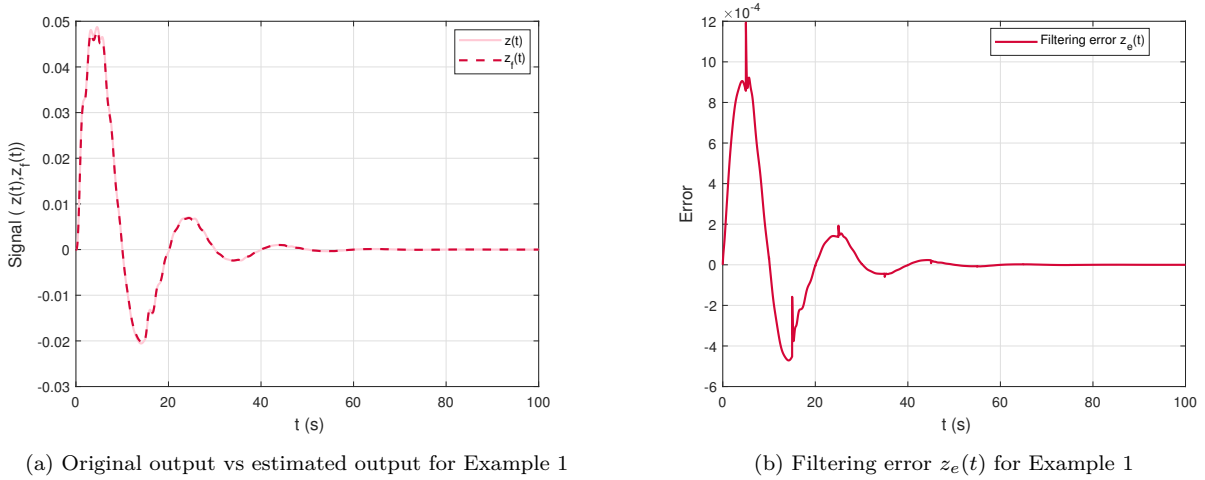


Fig. 1: Simulation results for Example 1.

Uncertain subsystem 2:

$$A_2(\rho) = \rho A_{2,1} + (1 - \rho)A_{2,2}, \quad B_2(\rho) = \begin{bmatrix} 1.0 \\ -0.8 \end{bmatrix}, \quad C_2(\rho) = [0.8 \ 1.0], \quad D_2(\rho) = 1.0, L_2(\rho) = [1.2 \ 0.9]$$

where

$$A_{2,1} = \begin{bmatrix} -2.8 & 2.2 \\ 1.8 & -4.2 \end{bmatrix}, \quad A_{2,2} = \begin{bmatrix} -2.8 & 2.5 \\ 1.8 & -4.2 \end{bmatrix}$$

The periodic filter parameters of the system (5) with the performance index $\gamma = 0.0800$ can be determined and given as follows

$$A_{f_1} = \begin{bmatrix} -3.2971 & 0.9361 \\ 2.3198 & -2.2913 \end{bmatrix}, \quad B_{\omega_{f_1}} = \begin{bmatrix} -1.0040 \\ 0.8592 \end{bmatrix}, \quad L_{f_1} = [-1.0862 \ -0.8529], \quad D_{\omega_{f_1}} = 0.0034,$$

$$A_{f_2} = \begin{bmatrix} -3.5707 & 1.4505 \\ 2.5378 & -3.3150 \end{bmatrix}, \quad B_{\omega_{f_2}} = \begin{bmatrix} -1.0002 \\ 0.8229 \end{bmatrix}, \quad L_{f_2} = [-1.0548 \ -0.6221], \quad D_{\omega_{f_2}} = 0.0125.$$

Considering the polytopes with two vertices of uncertain subsystems 1 and 2, we take three groups of uncertainty parameters as $\rho = 0$, $\rho = 0.4$ and $\rho = 1$. Under zeros initial conditions. The disturbance input signal is chosen as $\omega(t) = e^{-0.1t}1.25 \sin(0.314t)$, with $t \in [0, 100]$. The disturbance frequency is $\bar{\omega} = 0.314$ rad/s and must be less than ω_l . The resulting outputs and filtering errors for Example 2 are shown in Fig. 2a and Fig. 2b.

We define the ratio

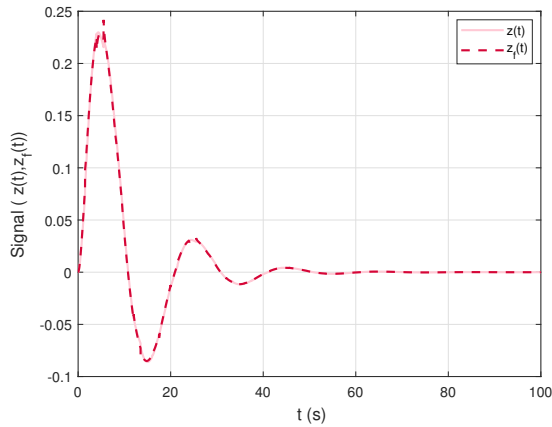
$$\Gamma(t) = \frac{\int_0^\infty z_e(t)^\top z_e(t) dt}{\int_0^\infty \omega^\top(t)\omega(t) dt}. \tag{18}$$

Fig. 3 presents the values of $\Gamma(t)$ for both examples. In each case, the maximum values remain below the prescribed performance levels ($\gamma = 0.59$ for Example 1 and $\gamma = 0.080$ for Example 2) in the frequency range $|\bar{\omega}| \leq \omega_l$, confirming that the H_∞ performance objectives are satisfied.

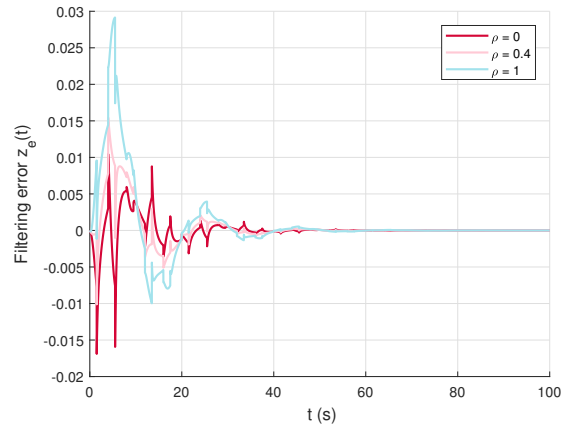
The simulation results confirm the validity of the proposed approach under low-frequency disturbances, demonstrating robust H_∞ filtering and asymptotic stability of the periodic piecewise filtering error system for all admissible uncertainties.

5. Conclusion

This work addressed the design of filters for continuous-time periodic piecewise polytopic linear systems in the low-frequency range. By exploiting the generalized KYP (gKYP) lemma, the properties of piecewise periodic systems, the

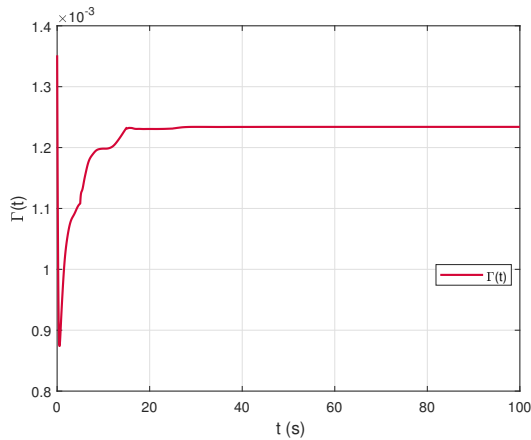


(a) Original output vs estimated output for Example 2

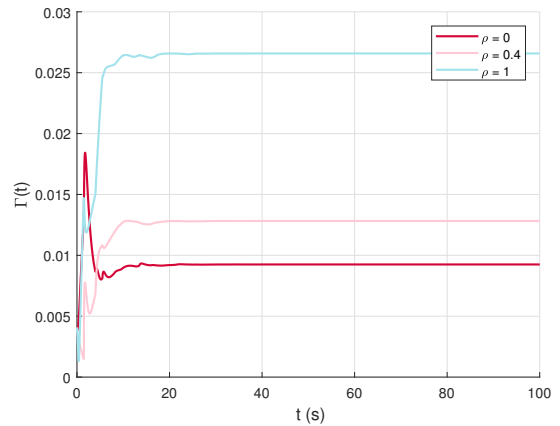


(b) Filtering error $z_e(t)$ for Example 2

Fig. 2: Simulation results for Example 2.



(a) Example 1



(b) Example 2

Fig. 3: Values of $\Gamma(t)$ for frequencies below ω_l for both examples.

projection lemma, and slack matrices, the proposed approach reduces estimation errors and yields less conservative results. Numerical examples confirm the effectiveness of the method across different frequency ranges. Future work may focus on filter and reduced-order model design for periodic fractional-order systems, including the case with time-varying delays, extending the applicability of the proposed approach.

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