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NORWEGIAN MATHEMATICS TEACHERS' USE OF RESOURCES IN REASONING AND PROVING

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Learning reasoning and proving (R&P) is an important part of learning mathematics. In this paper, we present the resources that 161 Norwegian teachers reported using when preparing and teaching lessons on R&P. Follow-up interviews with ten of these teachers provided additional insights into how these resources are used. We applied the curricular noticing framework (Dietiker et al., 2018) to identify how teachers attend to, interpret, and respond to different curricular materials. Findings reveal varied approaches to engaging with resources based on teachers' instructional goals, as well as challenges in interpreting the new curriculum and navigating available resources. These findings underscore the need for clearer curricular guidance and more accessible resources to support the curriculum's focus on R&P.

INTRODUCTION

In mathematics education, reasoning and proving (R&P) are recognized as vital processes that foster deeper understanding and critical thinking among students (NCTM, 2009). However, teaching R&P effectively across grade levels and content areas remains challenging, as it depends on teachers' knowledge (Buchbinder & McCrone, 2022) and involves difficulties such as selecting tasks that encourage reasoning or finding appropriate resources for assessment (Clarke et al., 2012; Sari et al., 2020). Some researchers (e.g., Thompson et al., 2012) also highlight the lack of examples in curricular materials for effectively engaging students in R&P.

Despite the accessibility of various resources, the integration of these resources in lesson planning and enactment, particularly for R&P-focused instruction, varies across countries. The most-used resources are consistent, however, their order of use differs slightly (Çakıroğlu et al., 2023). As part of the European MaTeK project, which includes partners from Slovakia, the Czech Republic, Italy, Norway, and Turkey, this study investigates the resources Norwegian mathematics teachers draw upon and the way they engage with these resources when designing and implementing R&P lessons. Norwegian curriculum changes also provide a timely context for examining R&P instruction. The 2020 reform, known as LK20, introduced a competency-based curriculum, which, compared to prior curricula, places significant weight on reasoning, as reflected in the new core element of Reasoning and Argumentation (Ministry of Education and Research, 2019). However, studies suggest that competence aims in LK20 are not clear, leaving teachers to interpret broad curriculum guidelines with limited detailed exemplification (e.g., Lada et al., 2025). Consequently, Norwegian

teachers may experience challenges in aligning resources with these curricular goals, which underscores the need to understand their resource choices and instructional practices within the evolving educational landscape. Thus, this paper aims to address this gap by exploring the following research question: *What resources do Norwegian mathematics teachers attend to, and how do they interpret and respond to them when planning and enacting lessons on reasoning and proving?*

THEORETICAL FRAMING

Reasoning and proving

In the context of school mathematics, R&P is often conceptualized as a unified construct (e.g., Cai & Cirillo, 2014; Stylianides, 2008). This perspective suggests that activities essential to understanding and establishing mathematical knowledge, such as identifying patterns, generalizing, formulating conjectures, and exploring examples and counterexamples, naturally integrate with processes of justification and proving. Rather than viewing reasoning and proving as discrete, in our study, we consider reasoning activities as foundational steps that lead to proof and validation, supporting the idea of R&P as a unified construct.

Resources and the curricular noticing framework

In the educational context, a "resource" encompasses any asset teachers use to design and implement instruction. Pepin and Gueudet (2018) differentiate between material and nonmaterial resources. Nonmaterial resources include *social resources*, such as interactions with colleagues, and *cognitive resources*, such as mathematical and pedagogical concepts, routines, and frameworks that teachers use or develop. Material resources, specifically *curriculum resources*, are defined as “all the material resources that are developed and used by teachers and students in their interaction with mathematics in/for teaching and learning, both inside and outside the classroom” (p. 1). When teachers design lessons, resources assist them in making decisions regarding mathematical content (what to teach and in what sequence), instructional strategies (how and why to teach it), and lesson enactment (Gustafsson et al., 2024).

To better understand teachers' interactions with curriculum materials, Dietiker et al. (2018) introduced the *curricular noticing* framework, inspired by the concept of teacher noticing. While they define curriculum materials as “written and digital textbooks” (p. 522), in our study, we extend this term to encompass any resource that teachers participate with. This framework divides noticing into three skills: (1) *curricular attending*, skills used when knowing how and where to search in resources when planning a particular lesson; (2) *curricular interpreting*, skills used when making sense of this information and linking it to the knowledge the teacher has about the class where the lesson will be taught and about mathematics teaching in general; and (3) *curricular responding*, where teachers decide on “what they want to do and how” (p. 527), based on their interpretations. This might include selecting or adapting a task, deciding how to present content, or choosing not to use the task. Hence, this framework

aids in identifying what teachers notice and how their noticing shapes resource use. For instance, McDuffie et al. (2018) found that middle school mathematics teachers attended to similar features of resources but interpreted and planned differently depending on their teaching orientations and the type of curriculum.

METHOD

Data for this study were collected through an online questionnaire developed as part of the MaTeK Project (Çakıroğlu et al., 2023). In Norway, the questionnaire was piloted with 20 respondents in November 2021, followed by the main study (with minor adjustments) conducted in March-April 2022 with 141 respondents. The same questionnaire (with language adaptations) was used across all five participating countries. It asked mathematics teachers in grades 1–13 about the resources they use for various teaching purposes, including lesson preparation, assessments, personal knowledge refreshment, and specifically for planning lessons focused on R&P. The resource list, adapted from Kock and Pepin (2019), included examples tailored to the Norwegian context and an open field where teachers could specify additional resources they used beyond the listed options. In Norway, teachers were recruited via emails sent to principals nationwide, requesting that they forward the questionnaire to their mathematics teachers. Additionally, the authors directly invited teachers they knew, resulting in a diverse sample in terms of gender, experience, education, and school level. Respondents willing to participate in a follow-up interview provided their email addresses. These Zoom interviews, lasting 46 to 90 minutes, explored teachers' resource use across teaching purposes and their approaches to R&P-focused lesson planning and instruction. Teachers also reflected on the adequacy of mathematics textbooks for supporting R&P and whether the new curriculum sufficiently emphasizes pupils' R&P skills. Interviews were conducted with all teachers who responded: one from the pilot (December 2021) and nine from the main study (May-August 2022), referred to as T0-T9. Of these, T3, T5, T6, T7, and T9 taught at the primary level (grades 1–7), T0, T2, T4, and T8 at lower secondary (grades 8–10), and T1 at upper secondary (grades 11–13). The analysis focused on the second half of each interview, where teachers were specifically asked about preparing for and teaching R&P-focused lessons. Both authors independently reviewed all ten interview transcripts to identify segments where teachers discussed which resources they use and how they use them. These identified segments were then further analysed using the curricular noticing framework.

FINDINGS

Although this paper focuses mainly on the interview data, we first present questionnaire findings on the resources teachers usually use for planning and teaching R&P, followed by an analysis of how interviewed teachers engage with these resources.

Resources used

Teachers vary in the number of resources they use, with some relying on only one (6.8%), always a textbook or teacher guide. Most use multiple resources, commonly three (20.5%) or four (19.9%), suggesting a preference for combining resources. The graph in Figure 1 shows that textbooks emerged as the most commonly used resource (67%), often alongside a teacher guide (43%). About 46% of respondents utilize self-created materials, and 43% of teachers indicated they use professional books on mathematics or mathematics teaching. In the follow-up interviews, teachers clarified that by “other professional books,” they refer to books from their teacher education, covering both mathematics and mathematics teaching, as well as popularization books on mathematics teaching. About 40% of respondents selected the *social resource* of consultation with colleagues, and 40% indicated using digital *curriculum resource* - online professional platforms. Interviews confirmed that teachers visit educational websites such as matematikk.org (a collaboration between several Norwegian universities) or the Norwegian Centre for Mathematics Education’s website, both of which were also listed as examples in the questionnaire.

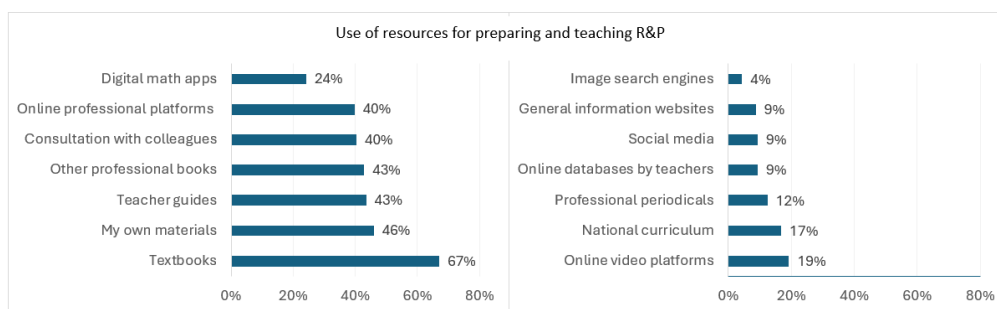


Figure 1: Percentage of respondents in the questionnaire (n=161) using particular resources for preparing and teaching R&P.

Resource use based on search aims

Interview data revealed that teachers’ engagement with resources depends on their needs. Some seek written-out proofs/solutions or “complete” R&P tasks, while others prioritize discussion-based learning or seek pedagogical guidance to improve their teaching of R&P. These differences shape how they interact with resources.

a) Teachers Seeking written-out Proofs/Solutions

Some teachers, particularly those in the higher grades, are *looking for written-out proofs or problem solutions*, either as a way of refreshing their own memory before teaching or because they want their students to engage in verifying the proof or solution step-by-step and convince themselves that each step is logical. When attending to resources with this aim, these teachers typically start with textbooks or, as T6 mentions, searching online, identifying easily a written-out proof or a worked solution.

T6: how was... the binomial formula [...]. You forget the math you haven’t had in 28 years. Then I need to find, yes that’s how it was. [...] Then I simply search different websites.

Interpreting involves verifying whether the material aligns with the topic and is manageable for students. As T1 explains, “seeing if I can find a proof that is manageable within the current topic”. Responding then entails selecting proofs or worked solutions and deciding how to engage students in checking each step.

b) Teachers Seeking Discussion-Driven Tasks

Many interviewed teachers are *looking for tasks to use as a starting point for a discussion*, focusing on R&P through the discussion itself. When attending to curricular materials with this aim, they often search across a range of sources: textbooks, self-created materials, social media (like Facebook teacher groups), and general websites. When interpreting, they assess topic relevance and discussion potential. While most teachers are not explicit about their criteria when interpreting and deciding how to respond, they emphasize relying on their own expertise:

T7: I feel that you need a bit more knowledge yourself to really focus on what’s important, what’s interesting, and what aligns with the lesson goals for a deeper understanding and to highlight it so that more students can also gain the same understanding.

T9: So, a task can really be worked with for a whole lesson if you use it in a creative way.

Some teachers, like T5, follow guidance from teacher guides when deciding how to respond, and T2 is looking for tasks without an answer key available so that students have to compare their answers and discuss discrepancies.

T5: They [teacher guides] provide explanations and even tips on almost exactly what you should say. So, I have to say, if I only used the textbook and not the teacher guide, then it would be a bit... You’d need to bring a bit more of your own knowledge into the lesson, in your head, to ensure reasoning and proof.

c) Teachers Seeking Complete R&P Tasks and Lesson Plans

When preparing a lesson on R&P, some teachers are *looking for a “complete” R&P-task*, meaning a task where the prompt for students to start reasoning and/or proving is part of the task. Here the most common strategy when attending to curricular material, looking in the textbook (digital or paper-based), fails. When T2 is asked if textbooks are good enough at providing help with planning R&P-lessons, he replies: “No! No, textbooks that I know, haven’t really given any .. any input on or help to practice tasks like that, where they [pupils] should discuss this.”

One teacher (T7) is looking for *suggestions for a lesson plan* focusing on R&P or a sequence of lesson plans. When attending to this, the teacher is using ‘other professional books’ and, in interpreting these materials, primarily assesses whether the content level is appropriate for his class; if so, he uses the lesson plan as it is.

d) Teachers Seeking Pedagogical Guidance

Some teachers in our study express uncertainty about how best to foster their students’ R&P skills and thus they sometimes look for *teaching advice on teaching R&P*. However, most of these teachers report limited options for finding such resources.

T1: I don't know. [...] How to get them to understand the proof. One thing is to go through the proof and work yourself through it, but.. To get the formal proof writing in a way that you could explain ... that I would want some more resources on.

One teacher, T4, has adopted Peter Liljedahl's 'Thinking Classroom' method as a way to develop R&P skills with his students:

T4: I would say every time we have «thinking classroom». Then we do it [work with students' R&P-skills], and we try to have it two times per month.

T4 mentioned first learning about this method at a conference organized by the Norwegian Centre for Mathematics Education, which also provides information about it on its website.

Challenges in resource use

Teachers face several challenges when they engage with resources to teach R&P, including difficulties interpreting the curriculum, limited textbook support, and challenges navigating and accessing resources.

The shift to the new LK20 curriculum disrupted established attending strategies, like using old exam tasks. With no completed LK20 exam at the time of the interviews, teachers relied on sample exams (with examples of low, medium, and high-achieving responses) provided by the Norwegian Directorate for Education and Training (Udir). These examples, however, were often difficult to interpret. T2 describes the confusion:

T2: [...] some students write three pages and some write three lines. And this student who has written three lines is high attaining and the one who has written three pages is medium. [...] I think this is hard to explain to the students what we are expecting.

When T2 struggles with the interpretation, he also finds it challenging to know how to respond. Interpreting the suitability of these tasks requires careful alignment with the new curriculum, which many teachers find challenging. As T6 notes, "a lot of work before we get that [curriculum] under our skin [...] I think the curriculum, reading and interpreting it, I think it is really hard".

Many interviewed teachers noted that textbooks provide limited support for R&P, often focusing on procedural tasks rather than exploring underlying reasoning. T9 commented: "Too few 'why' questions are asked. The newest editions might be a bit better at that, but I think they move too quickly." Additionally, teachers mentioned challenges in navigating and accessing available resources. For example, T5 explained: "It certainly does [exist] if you search for it, but I can't say it's something that pops up in my face." Similarly, T8 echoed this frustration: "there probably are, but how easy it is to find them, I don't know. But you have to be a bit used to it, searching both online and in books to find good ways to do this."

DISCUSSION AND CONCLUSIONS

Our analysis reveals that teachers use a variety of resources to plan and teach R&P-focused lessons, with distinct patterns emerging among interviewed teachers based on

their specific aims. Consistent with McDuffie et al. (2018), we observed that in the curricular attending phase, these teachers either look for tasks or whole lesson plans. However, our study extends these two categories by identifying an additional type of engagement: some teachers also seek *meta-resources*—guidance on conducting R&P lessons and improving their teaching of R&P—highlighting the need for resources that provide not only content but also pedagogical strategies, supporting mathematics teachers’ planning of teaching and learning activities (Gueudet et al., 2023). Furthermore, our findings highlight that many interviewed teachers face difficulties navigating a diverse set of available resources. In our analysis, we incorporated this navigation skill as part of the curricular attending process, but it could be recognized as a distinct preliminary step within Dietiker et al.’s (2018) framework, particularly if the framework is to encompass all resources beyond traditional textbooks.

A significant finding from our study is the challenge some teachers encounter in interpreting the updated curriculum, particularly the new emphasis on reasoning and argumentation. These teachers would benefit from targeted support, which aligns with what McDuffie et al. (2018) concluded. Stakeholders such as National Centre, Udir, and teacher educator institutions, could collaboratively operationalize what it means to enact this curriculum as intended, for example by developing exemplary curricular materials or professional development programs that model R&P-focused instruction across various grade levels. For further studies, analysing the entire interview to study each teacher’s noticing skills more in depth and in relation to their conceptualization of R&P in school mathematics could provide valuable insights. Additionally, now that some time has passed since the introduction of LK20, conducting new interviews could shed light on the development of teachers’ noticing skills and the resources they need.

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