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Mayeul Arminjon

Abstract

In the Lorentz-Poincaré version of special relativity (SR), the “relativistic” effects, beginning with the Lorentz contraction, are seen as absolute, as being due to a motion relative to a privileged frame of reference or “ether”. We present a scalar theory of gravitation that extends Lorentz-Poincaré SR. We provide motivations for such a theory, including critical remarks about general relativity. That theory starts from an interpretation of gravity as Archimedes’ thrust in a fluid “micro-ether”, of which Lorentz’s inertial ether is seen as the mean rest frame. Newton’s theory propagates instantaneously, hence corresponds with the limiting case of an incompressible fluid. When combined with the Lorentz-Poincaré version of SR, the assumption of a heterogeneous gravitational ether leads naturally to assume effects of gravity on clocks and rods, hence to a curved metric. Motion is governed by an extension of Newton’s second law to a curved spacetime, that implies a dynamics for a continuous medium. The main observational tests are discussed. The Maxwell equations in the presence of gravity need to postulate an “interaction energy”, which might contribute to dark matter. The present state of calculating that energy in a galaxy is shown.

MSC: 83C05, 83C25, 83D05

Keywords: Alternative theory of gravitation, mechanism for gravity, metric effects of gravity, Newton’s second law, geodesic motion, dark matter

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1 Introduction

After the realization of the wave nature of light in the first halfth of the 19th century, and even more after the discovery of the electromagnetic waves in the second halfth of that same century, it was just natural to admit the existence of a medium for the propagation of light and other electromagnetic waves: the ether. As is well known, all experiments that attempted at detecting the motion of the Earth through the ether failed, which led to the raise of the (special) Theory of Relativity and to Einstein's statement that "the introduction of a light ether will appear superfluous". However, it may be argued [15] that the Theory of Relativity is the result of a historical genesis which led first to a version that keeps the concept of an ether — more precisely, Lorentz's ether, that is simply an inertial frame in which Maxwell's equations are valid. This version may be called "Lorentz-Poincaré Relativity", because also Poincaré played a prominent role in its development, e.g. through the introduction of the "Principle of Relativity" in 1904 and the first exact writing, in 1905, of what we call the Lorentz transformation, and notwithstanding the fact that other talented scientists brought very important contributions, e.g. FitzGerald, Larmor, Michelson. In Lorentz-Poincaré Relativity, there is a preferred time: the inertial time in the

ether reference frame. Hence, our intuitive notions of separated time and space, and of the existence of an observer-independent present, are saved. Now Lorentz-Poincaré Relativity, because it is really a relativistic theory, makes the ether and hence also this preferred time undetectable. However, Special Relativity does not contain gravitation. It might be the case that a satisfying theory of gravity be a preferred-frame theory with a detectable ether. The purpose of the present paper is to present the motivation and to summarize the development of a simple theory of that kind, which is based on a merely scalar gravitational field, and to discuss its main experimental tests and predictions.

2 Motivations

As indicated in the Introduction, a basic motivation of the investigated theory was to extend Lorentz-Poincaré theory to include gravity: the undetectability of Lorentz's ether makes it somehow "metaphysical". In that extension, thus in the investigated theory, the ether is detectable, i.e., it has physical effects. However, there are several other motivations.

2.1 The Search for a Physical Explanation of Gravity

There is no "physical explanation" of gravity in the known theories, be these Newton's theory, or Einstein's general relativity (GR) and its numerous extensions or modifications. First, Newton's instantaneous action at a distance turns out to be very efficient, but not only does it obviously not contain a mechanism for gravity: it also can hardly be regarded as physically plausible — and this was already Newton himself's opinion, which he stated quite clearly at the beginning of the *Principia*. Indeed, how to imagine that arbitrarily distant astronomical bodies might influence instantaneously the motion of a given body?

As to Einstein's theory and its variants, they assert that gravitation is essentially the *curvature* of *spacetime*. This is even much more abstract than is Newton's distant action. In fact, spacetime can a priori be considered as merely an artificial construct (even though this construct turns out to be very useful in theoretical physics), and the notion of its curvature is remote from any contact with our physical perception. Nevertheless, these theories have over Newton's theory the important advantage that, in them, gravitation propagates with a

finite velocity (exactly that of light, according to GR).

In the investigated theory, there is a physical mechanism for gravity: Archimedes' thrust, see subsect. 3.1.

2.2 The Problems Bound with General Relativity

1) A first, well-known problem, is that general relativity is hardly compatible with quantum theory, that has an impressive success in describing the microscopic world. The difficulties begin already with the simple and undoubtedly necessary task of writing quantum mechanics in a gravitational field. In quantum mechanics, time is a well-identified variable, whereas GR has plethora of equally good times: in GR, the time coordinate x^0 can be transformed as an arbitrary function of the spacetime coordinates x^μ ($\mu = 0, \dots, 3$), thus $x'^0 = \phi((x^\mu))$. In contrast, in quantum mechanics, only changes of the time origin and the time unit, $x'^0 = ax^0 + b$, are allowed — at least in a given reference frame. Thus, in a Minkowski spacetime, each inertial frame has its inertial time. The Lorentz-invariant quantum wave equations have to use that inertial time, because they are not form-invariant even under a general space-independent transformation of time, $x'^0 = \phi(x^0)$. Now some theories of gravitation have a flat “background metric”, say γ^0 . Such is the case for the “relativistic theory of gravitation” [24], and for the investigated theory. For such theories, in a reference frame that is an inertial frame for γ^0 , the inertial time of that reference frame could be an appropriate time to write the quantum wave equations. Thus, in the case that a (canonical) flat background metric is present in the theory, a first possibility is to write quantum mechanics in a gravitational field as in a Minkowski space, using that flat metric and in particular using the inertial time in an inertial frame for γ^0 . Another possibility is to have recourse to generally-covariant extensions of the wave equations. This is the only possibility for GR and similar theories, for which there is not a canonical flat metric. In particular, there is the standard form of the Dirac equation in a curved spacetime (Dirac-Fock-Weyl equation). However, the Dirac-Fock-Weyl equation has a non-uniqueness problem regarding the energy spectrum, see Ref. [11] and references therein. This problem has an elegant and general solution in the case of the investigated theory, because the metric has the required “space-isotropic diagonal” form [11], but that form and hence that solution do not generally apply in GR.

2) In addition, GR itself has several serious problems, some of which are

well acknowledged by physicists, including specialists of GR: i) the unavoidable existence of singularities (in gravitational collapse and in cosmology as well) — clearly it can hardly be considered plausible that a star become a *point*. ii) The necessity, to explain motion at the scale of galaxies or galaxy clusters, of postulating dark matter, that unfortunately has not been identified, despite decades of enormous research efforts. iii) The (well-known) inherent complexity of GR, even though some may consider that it is not a scientific problem.

Besides those three well-acknowledged problems of GR, it still has two grave problems:

3) The issue of the (non-)uniqueness of the predictions of GR. It is well known that, of the ten Einstein equations, only six are independent, so that they, together with boundary conditions, cannot determine the ten independent components of the metric tensor; see e.g. Ref. [29].¹ Logunov and his collaborators have studied in great detail the example of a static, spherically symmetric, and asymptotically flat spacetime (let us name this an SSS spacetime for short); see Refs. [24, 25] and references therein. They choose, as usual, quasi-spherical coordinates — i.e., spatial coordinates r, θ, ϕ (together with the time t) in which the metric has, asymptotically at large r , the form of the Minkowski metric in spherical coordinates r, θ, ϕ plus the inertial time t . They find that, for a given mass M of the central body, the Einstein equations have an infinite set of solutions for the SSS metric in quasi-spherical coordinates, including a family depending on an arbitrary real parameter λ . They prove that not only the metric but also the associated physical predictions (e.g. for the gravitational time delay) are ambiguous: in a first step, using the statement that the coordinates (or more precisely the “arithmetization of space”, as they say, i.e. the chart) are the same for all of these solutions — that is, assuming one may choose the coordinate

¹ Contrary to an often-read argument, the indeterminacy of the Einstein equations is not a necessary consequence of the general covariance of these equations. According to that argument, the general covariance — which means the possibility of transforming the metric tensor, solution of the Einstein equations, by applying its tensor transformation after an arbitrary change of the four coordinates — would imply that the Einstein equations indeed would have to leave four degrees of freedom, i.e., would have to involve only $10 - 4$ independent equations. However, the four components of the (generally-covariant) Dirac-Fock-Weyl equation are independent, as are the four components of the flat-spacetime Dirac equation. Also, the d’Alembert wave equation, as applied to a 4-vector field in a flat spacetime, has four independent components (with four times the same operator), and the same remains true for its generally-covariant extension to a curved spacetime.

system (chart) before solving the Einstein equations [24, 25].² The criticism of their arguments does not accept that assumption, see e.g. Refs. [20, 30]. However, when one considers an SSS spacetime to describe (for example) the gravitational field created by the Sun, and its effects on the planets considered as test particles, then it is only natural to put the center of the assumed symmetry (i.e., the center of the Sun) at the origin of the quasi-spherical coordinates. This is often done in the standard general-relativistic literature, e.g. [21, 29]. The defenders of GR (e.g. Refs. [20, 30]) against Logunov and coworkers consider that the (infinitely many) different solutions of the Einstein equations found by Logunov et al. [24, 25] for the SSS metric in quasi-spherical coordinates correspond to just one and the same metric in different forms, any two of them being related together by a change of the (quasi-spherical) coordinates. For example, the SSS metric “in Schwarzschild coordinates”, Eq. (7) in Ref. [30] or Eq. (1.10) in Ref. [24], and the SSS metric “in harmonic coordinates”, Eq. (8) in Ref. [30] or Eq. (1.11) in Ref. [24], exchange by a mere change of the radial variable [30, 24], namely

$$r = \rho - GM/c^2. \quad (1)$$

We note that this change does not leave invariant the zero of the radial variable. Thus, if the symmetry center is the point $r = 0$ in the first coordinate system, it will be represented by the sphere $\rho = GM/c^2$ in the second one, which is weird and indicates that the coordinate change is singular. Anyway, with this point of view (that the correspondence between the coordinates and the physical space cannot be taken for given at the stage of solving the Einstein equations), we do not know a priori, for a given SSS solution of the Einstein equations, what is the correspondence between the quasi-spherical coordinates and the points in the static physical space. That correspondence must be different for two different forms of the SSS metric, for otherwise the predictions obtained with either of the two do differ. This can be checked e.g. by calculating the interval of proper time between departure and arrival of light traveling in a circumference around the Sun: it differs for two different forms of the metric, if the coordinate system is assumed to be the same for the two [20]. This, after all, is a confirmation of the statements by Logunov et al. Thus, with the point of view stated above, we are supposed to be allowed to choose arbitrarily any possible form of the SSS metric, and we must resolve the ambiguity in the coordinate system. This can be done only by fitting observed quantities, which really is acknowledging the ambiguity

² Note that this is not to assume that the coordinates are directly observable quantities, contrary to the statement in Ref. [30].

in the predictions. In fact, in Ref. [25], the ambiguity in the prediction of the gravitational time delay is proved also when adopting that point of view. This is the passage with “Chuk and Gek”, in which indeed each among Chuk and Gek uses his *own* system of quasi-spherical coordinates: the predictions for the gravitational time delay of two solutions for the metric, corresponding with two different values of the parameter λ introduced above, are expressed in terms of observed quantities, and are found to differ [25].

4) The issue of the post-Newtonian approximation scheme. At the beginning of the 20th century, the Newtonian celestial mechanics was able to account with a high precision for the observed motions of the planets of the solar system and their satellites, except for the so-called “residual advance” in Mercury’s perihelion. This is the advance of ca. $43''$ per century that remains for the observed angular position of Mercury’s perihelion, once the Newtonian N -body problem has been solved in the solar system, taking into account the Newtonian mutual attraction forces of the Sun and all planets. The theoreticians strived to explain this residual advance, e.g. by postulating a yet-to-be-discovered planet between the Sun and the orbit of Mercury. But the advent of special relativity (SR) included that of “a new mechanics” (Poincaré’s word in 1904), involving e.g. a velocity-dependent inertial mass. Newton’s theory had of course to be modified to become compatible with that new mechanics. In his very papers of 1905, Poincaré proposed a Lorentz-invariant modification of Newton’s attraction law. He found later that this modification led to a residual advance of Mercury of one sixth ($7''$ per century) of the observed residual advance [26]. This was obtained by assuming Mercury to be a test particle in the SSS field created by the Sun. The same assumption was made by Einstein when he found [19] that GR accounts exactly for the $43''$ per century. However, that assumption is clearly an oversimplification. One has to formulate the N -body problem in the new theory and to propose an approximation scheme to solve it, using the fact that the gravitational field is (very) weak in the solar system, so that Newton’s theory is a (very) good first approximation. Such is the aim of the so-called *post-Newtonian approximation (PNA) schemes*. The first systematic PNA schemes were formulated by Fock [21] and Chandrasekhar [18]. They are essentially equivalent and are essentially the ones used up to now for the celestial mechanics of GR in the solar system, hence we shall refer to them as “the standard PNA scheme”. In that scheme, the spacetime metric ($\gamma_{\mu\nu}$), as well as the corresponding connection coefficients and curvature tensor, are formally expanded in power series of $1/c$ (with c the velocity of light). However, the matter fields: pressure p , velocity \mathbf{u} ,

etc., are not expanded. As was first noted by Futamase & Schutz [22], that point makes it difficult to understand the meaning of the standard PNA scheme. If one searches for asymptotic expansions in the mathematical sense, then all fields have to be expanded, moreover one has to properly define a small parameter ϵ and to define a *family* (S_ϵ) of systems, in order that ϵ can (conceptually) be made arbitrarily small — with the real system of interest S corresponding to a small value ϵ_0 of ϵ , so that one may use the expansions for S . Futamase & Schutz [22] were the first ones to propose a PNA scheme consistent with the method of asymptotic expansions, and this was for GR. But they derived only local equations, not equations for the motion of the mass centers; in fact, their local equations were hardly explicit enough for further use. An asymptotic PNA scheme, together with the corresponding explicit local equations, was proposed independently for the investigated scalar theory, and was developed up to the global equations of motion for the mass centers, using also an asymptotic scheme for the “good separation” between the celestial bodies; see Ref. [6] and references therein. A similar asymptotic PNA scheme was then proposed for *GR in the harmonic gauge* [9].³ It was compared with the standard PNA scheme, noting that in particular the source term on the r.h.s. of the Poisson equation for the Newtonian potential does differ between the standard scheme and the asymptotic scheme [9]. It was developed up to the global equations of motion for the mass centers [9]: it was found that the acceleration of a mass center, given by these equations, is exactly the standard Lorentz-Droste acceleration (usually named the Einstein-Infeld-Hoffmann acceleration), *plus* a 1PN correction that vanishes if all bodies are exactly spherically symmetric (this term is likely to be extremely small in the solar system), *plus* a 1PN correction depending on the rotation velocity of the body and on its internal structure. The latter correction to the acceleration does not seem to be negligible for the giant planets, thus threatening the agreement with celestial-mechanical observations.

Other problems of GR will be mentioned at §§4.1 and 5 below.

³ The application of this scheme to the equations of motion (local and global) has been limited, up to now, to the *first* PNA (“1PN approximation”), i.e., the approximation that immediately follows the Newtonian approximation in the iteration scheme. The same is true for the PNA scheme for the investigated scalar theory.

3 First Steps in the Construction of the Investigated Theory

3.1 Gravity as Archimedes' Thrust

Consider a perfect fluid and, immersed in it, an object occupying a small spatial domain ω , so that the gradient $\text{grad } p_e$ of the fluid pressure p_e may be considered uniform in the neighborhood of ω . Denoting its volume $V(\omega)$ by δV , the resultant of the pressure forces exerted by the fluid on the object (Archimedes' thrust) is then:

$$\mathbf{F}_A = - \int_{\partial\omega} p_e \mathbf{n} dS = -\delta V \text{grad } p_e. \quad (2)$$

(Here $\partial\omega$ is the boundary of ω , \mathbf{n} is the outward normal to $\partial\omega$, and dS is the area element.) If the elementary particles are such small objects immersed in an imagined perfect fluid or ether, and if they all have the same mass density $\rho_p = \delta m / \delta V$, then (2) rewrites as

$$\mathbf{F}_A = -\delta m \frac{\text{grad } p_e}{\rho_p}. \quad (3)$$

Thus, in that case, the pressure force is a mass force, and we can read the corresponding acceleration — the gravity acceleration — in Eq. (3)

$$\mathbf{g} = -\frac{\text{grad } p_e}{\rho_p}. \quad (4)$$

This concept [1] turns out to be equivalent to Euler's proposal in 1746. See Ref. [8] for a summary of that proposal by Euler and a comparison with this concept.⁴ The latter is further developed, in our attempt, by taking inspiration from the idea, developed in detail by Romani [27], according to which the elementary particles themselves, e.g. the electrons, could be vortices or combinations of several vortices, thus self-organized local flows in the ether. (At a time where the constituents of the atoms had not been identified, William Thomson had postulated

⁴ It is also discussed in Ref. [8] that the ether which would exert this pressure cannot be in equilibrium, and in fact can hardly obey classical mechanics. Actually the ether of gravity must be the macroscopic ether or "macro-ether" obtained by averaging the pressure and density fields in the "micro-ether". Because the macro-ether defines the reference frame in which mechanics is primarily defined, the motion of that reference frame itself does not obey mechanics. And gravitation is assumed to result from the *macroscopic* part of the pressure gradient in the micro-ether.

that the atoms would be vortices, but it turned out that there were too many different atoms.) This idea is interesting to understand the creation/annihilation processes and the instable “resonances” observed in particle physics. If the particles subjected to the gravitation/pressure force are indeed made of the same fluid that exerts that pressure, then we have in Eq. (4): $\rho_p = \rho_e$, the “ether density”, thus [1]

$$\mathbf{g} = -\frac{\text{grad } p_e}{\rho_e}. \quad (5)$$

Moreover, this idea of a universal fluid, of which the matter particles would be local flows, means that this fluid is totally continuous and is not made of (discontinuous) constituents — unlike usual fluids, which are made of molecules, atoms, or ions. In that case, no temperature can be defined for that fluid, so that the state equation of the fluid has the simple form [27]

$$\rho_e = \rho_e(p_e). \quad (6)$$

(This is called a barotropic fluid.)

3.2 First Consequences for the Description of Gravity

Equation (5) is taken as starting point: it replaces the equation $\mathbf{g} = \text{grad } U$ of Newtonian gravity. The latter propagates instantaneously, hence it should correspond with the limiting case of an incompressible fluid: $\rho_e = \text{Constant}$. Now, the vector field of the Newtonian gravity attraction is characterized by the Poisson equation

$$\text{div } \mathbf{g} = -4\pi G\rho \quad (7)$$

where ρ is the volume density of mass and G is Newton's gravitation constant. Thus, for the incompressible case, we must have by (5) and (7) [1]

$$\Delta p_e = 4\pi G\rho\rho_e. \quad (8)$$

(Here Δ is the Laplacian.) In the real (compressible) case, the modifications of p_e (or of $\rho_e = \rho_e(p_e)$) should propagate according to the wave equation involving the “sound” velocity in the fluid [1]

$$c_e = \left(\frac{dp_e}{d\rho_e} \right)^{1/2}. \quad (9)$$

The matter particles being assumed to be self-organized flows in the ether, that velocity c_e should be the upper limit for their velocity v :

$$v \leq c_e. \quad (10)$$

3.3 Accounting for SR: Effects of Gravity on Clocks and Rods

In SR, space and time are homogeneous, as can be seen on the expression of the Lorentz transformation. On the contrary, in a gravitational field, and if gravity is due, as hypothesized above, to the *gradient* of the density of a universal fluid, then *heterogeneous* space and time are expected instead. According to Eq. (5), the “ether density” ρ_e is decreasing in the very direction of the gravitational attraction. Now, in the situation of SR, the Lorentz contraction implies that the ether density is smaller in a frame moving at velocity u , by precisely the Lorentz factor $\gamma_u = (1 - u^2/c^2)^{-\frac{1}{2}}$: $\rho'_e = \delta m_e / \delta V' = \rho_e / \gamma_u$ — because the volume element in the moving frame is larger, $\delta V' = \gamma_u \delta V$, due to the Lorentz contraction of the moving measuring rod, whereas the “mass” or rather the amount of the fluid ether is unchanged: the mass increase concerns material particles, not the fluid itself. This leads to postulate gravitational rod contraction and clock slowing [2], both in the ratio

$$\beta(\mathbf{x}, T) := \rho_e(\mathbf{x}, T) / \rho_e^\infty(T) \leq 1 \quad (11)$$

where $\rho_e^\infty(T)$ is the supremum of $\rho_e(\mathbf{x}, T)$ over the spatial position \mathbf{x} . $\rho_e^\infty(T)$ is asymptotically reached as one goes very far from any massive body. To make sense, these assumptions need that one has a reference, with respect to which these effects occur. Thus, we assume that the rest frame of the ether is an Euclidean space M , endowed with an Euclidean spatial metric \mathbf{h}^0 , and equipped with a uniform time T . In other words, we assume that the reference frame of the ether is an inertial frame \mathcal{E} for a Minkowskian spacetime metric γ^0 , with the corresponding Euclidean spatial metric \mathbf{h}^0 and the corresponding inertial time T . The rod contraction and clock slowing occur with respect to γ^0 . Thus, the time $t_{\mathbf{x}}$ measured at a spatial position \mathbf{x} in M , flows more slowly than the inertial time T in \mathcal{E} , in the ratio

$$\frac{dt_{\mathbf{x}}}{dT} = \beta(\mathbf{x}, T) \quad (12)$$

and, due to the rod contraction, a length dl measured at \mathbf{x} is dilated, as compared with the length dl^0 that would be measured by an “unaffected” measuring rod (that would measure the Euclidean distance defined with \mathbf{h}^0), in the ratio

$$\frac{dl}{dl^0} = \frac{1}{\beta(\mathbf{x}, T)}. \quad (13)$$

However, the gravitational rod contraction and the corresponding measured length dilation (13) can take place

- either (**v1** of the theory [2]), as in SR, in one direction only, which then can be only that of the gravity acceleration \mathbf{g}
- or (**v2** of the theory [8, 10]) in a (locally) *isotropic* way.

Independently of that, the assumed time slowing (12) implies that

$$\beta = \sqrt{\gamma_{00}} \quad (14)$$

where $\gamma = (\gamma_{\mu\nu})$ is the physical spacetime metric. These two assumptions are used to define the latter: for v2, this is [8] (in Cartesian coordinates for the flat metric γ^0 that are adapted to the preferred reference frame \mathcal{E})

$$ds^2 := \gamma_{\mu\nu} dx^\mu dx^\nu = \beta^2 (dx^0)^2 - \beta^{-2} dx^i dx^i. \quad (15)$$

This form involves an additional assumption of “global synchronization”, i.e., $\gamma_{0i} = 0$ ($i = 1, 2, 3$) in such coordinates. Note that the flat metric γ^0 writes

$$(ds^0)^2 = (dx^0)^2 - dx^i dx^i \quad (16)$$

in such coordinates. The field β , or the field $\psi = -\text{Log } \beta$, is the gravitational field of the theory.

Since SR implies the upper limit c to the velocity of any matter, and since the upper limit is unique, (10) implies that $c_e = c$. Owing to the definition (9), this in turn implies that the barotropic state equation has the simple form

$$p_e = c^2 \rho_e. \quad (17)$$

Accounting for (11), the gravity acceleration (5) rewrites therefore as

$$\mathbf{g} = -c^2 \frac{\text{grad}_{\mathbf{h}} \beta}{\beta} \quad (18)$$

where $\text{grad}_{\mathbf{h}}$ means that we now interpret the gradient operator in Eq. (5) as being the one defined with the physical (Riemannian) space metric \mathbf{h} (see before Eq. (21) below), i.e. [2]

$$(\text{grad}_{\mathbf{h}} \beta)^i := h^{ij} \beta_{,j} \quad (19)$$

$(h^{ij}) := (h_{ij})^{-1}$ being the inverse matrix of matrix (h_{ij}) .

4 Dynamics

4.1 Relativistic Dynamics and Newton's Second Law

In SR, Newton's second law is modified to the form first proposed by Planck, that involves a velocity-dependent inertial mass $m(\mathbf{v})$, and that, rather than equating the force to the product of the mass by the acceleration, equates the force to the time derivative of the momentum (thus allowing to avoid the introduction of two different masses in SR, longitudinal and transversal)

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} \quad (20)$$

where

$$\mathbf{P} := m(\mathbf{v})\mathbf{v}, \quad m(\mathbf{v}) := m(0) \cdot \gamma_{\mathbf{v}}, \quad \gamma_{\mathbf{v}} := \left(1 - \frac{\mathbf{v}^2}{c^2}\right)^{-\frac{1}{2}}.$$

In general relativity (GR), one does not try to use Newton's second law: instead, one assumes that free particles follow spacetime geodesics. In that way, instead of being merely the useful mathematical concept "space \times time", spacetime becomes the physical arena in GR, in which past, present and future have no absolute meaning. And indeed, it turns out that "time travels" (defined as closed time-like curves in spacetime, CTCs) are possible according to GR: specifically, some exact solutions of the Einstein equations do contain CTCs. Beginning with Gödel's universe, the known solutions containing CTCs have long been restricted to physically very unpalatable situations. However, new solutions have been found by Bonnor, who has argued that "Some of these solutions refer to ordinary materials in situations which might occur in the laboratory, or in astrophysics" [16]. As is well known, time travel leads to causality breaking and to paradoxes.

The investigated theory has "preferred" and separated space and time, and postulates a gravitational force. Therefore, Newton's second law is needed in that theory.

4.2 Newton's Second Law in a "Curved Spacetime"

We proposed an extension to a general curved spacetime [3] of Planck's modification for SR, Eq. (20), of Newton's second law. To define that extension, a

given reference frame \mathcal{F} is considered. (A reference frame is essentially a reference body made of observers equipped with meters and clocks, and formally it can be defined as a congruence of world-lines [3, 17].) The data of a reference frame \mathcal{F} contains that of the associated *space* $M_{\mathcal{F}}$, that is essentially the set of the points in \mathcal{F} : $M_{\mathcal{F}}$ is a 3D manifold endowed with a Riemannian metric \mathbf{h} (i.e., simply a scalar product mapping depending on the spatial position \mathbf{x}), also depending in general on the *time* t , thus $\mathbf{h}_{\mathbf{x},t}$, see e.g. Ref. [23]. In order to respect the identity between inertial mass and passive gravitational mass, the gravity force must have the form

$$\mathbf{F}_g = m(\mathbf{v})\mathbf{g} \quad (21)$$

while the momentum is defined as

$$\mathbf{P} := m(\mathbf{v})\mathbf{v} \quad (22)$$

where \mathbf{v} is the velocity in \mathcal{F} , measured with local clocks and rods, and $v := [\mathbf{h}(\mathbf{v}, \mathbf{v})]^{1/2}$ is its modulus. The difficulty is to consistently define the time derivative of the momentum, say $D\mathbf{P}/Dt$. For a constant gravity field, the spatial metric $\mathbf{h} = \mathbf{h}_{\mathbf{x}}$ is independent of t . Then the standard “absolute derivative” of the vector \mathbf{P} imposes itself, i.e., $D\mathbf{P}/Dt$ has to be defined [2] as the covariant directional derivative of \mathbf{h} along \mathbf{P} . In the general case, it can be shown that Leibniz’s rule plus natural requirements imply that only one consistent definition exists for $D\mathbf{P}/Dt$ [3]. Finally, the time derivative must obviously be with respect to the physical, local time of the observers of the preferred frame \mathcal{E} , Eq. (12), instead of the inertial time T of \mathcal{E} . Thus, in the presence of a non-gravitational force \mathbf{F}_0 , Newton’s second law is defined as:

$$\frac{D\mathbf{P}}{Dt_{\mathbf{x}}} = \mathbf{F}_0 + m(\mathbf{v})\mathbf{g}. \quad (23)$$

Consider a “free particle” ($\mathbf{F}_0 = \mathbf{0}$) and a *constant* gravitational field ($\partial\beta/\partial T = 0$). Then, the gravity acceleration \mathbf{g} being defined by (18) with (14), it has been proved [3] that *this motion follows the geodesics of the spacetime metric* γ . This is valid for a general form of the metric, not necessarily (15). Moreover, this result applies not only to mass particles, but also, at least for an important case of metric [4], to *light-like particles (photons)* — replacing in Eqs. (22) and (23) the velocity-dependent mass $m(\mathbf{v})$ by the photon energy $E = h\nu$, the frequency ν being defined in terms of the local, physical time, $\nu := dn/dt_{\mathbf{x}}$ with n the number of oscillations. Thus, according to the natural extension

(23) of Newton's second law with the gravity acceleration (18), the motion along spacetime geodesics is valid, but only in the particular case of a constant gravitational field.

4.3 Dynamics of a Continuous Medium

Consider first a "dust", i.e., a continuous medium made of non-interacting particles, subjected to the gravity force. In that case, Newton's second law without non-gravitational force (23), which can be rewritten as an equation for the four-acceleration of a test particle [3], may be used pointwise in the medium. That expression of the 4-acceleration contains the 4-velocity U^μ and, multiplying it by the proper rest-mass density ρ^* , one sees the energy(-momentum-stress) tensor of the dust appear, $T^{\mu\nu} = \rho^* U^\mu U^\nu$. One thus gets, in that case of a dust, an equation for \mathbf{T} [5]

$$T_{\mu;\nu}^\nu = b_\mu \quad (24)$$

with

$$b_0(\mathbf{T}) := \frac{1}{2} h_{jk,0} T^{jk}, \quad b_i(\mathbf{T}) := -\frac{1}{2} h_{ik,0} T^{0k}$$

where (h_{ij}) is the (curved) spatial metric. (This equation is valid in any coordinates (y^μ) that are adapted to the frame \mathcal{E} and such that $y^0 = \phi(T)$ for some function ϕ .) The universality of gravity means that *the same equation must hold true for any continuous medium*, including the case of an electromagnetic field.

5 Energy Conservation and Field Equation

As we saw, the logic of the theory leads to the demand that the Newtonian gravitation should be recovered in a limiting case ("incompressible ether"), which gives Eq. (8), and that, in the general case, time derivatives of the field p_e (or equivalently the field $\rho_e = p_e/c^2$, or $\beta = \rho_e/\rho_e^\infty = \sqrt{\gamma_{00}}$) should be added to give some wave equation. Continuing to consider the curved metric as the physically relevant one, this means an equation of the form

$$\Delta_{\mathbf{h}} \rho_e + (\text{time derivatives of } \rho_e) = \frac{4\pi G}{c^2} \sigma \rho_e F(\beta) \quad (25)$$

with $\Delta_{\mathbf{h}} = \text{div}_{\mathbf{h}} \text{grad}_{\mathbf{h}}$ the Laplace(-Beltrami) operator based on the curved spatial metric \mathbf{h} , and

$$F(\beta) \rightarrow 1 \text{ as } \beta \rightarrow 1$$

the “time derivatives” term being such that the operator on the l.h.s. of (25) is a wave operator. On the r.h.s. of (25), σ denotes the relevant mass-energy density, which has to be defined in terms of the energy tensor \mathbf{T} . Using the relation between the flat and curved metrics, Eqs. (15) and (16), one gets [10]:
⁵

$$\Delta_{\mathbf{h}} \rho_e = \rho_e^\infty \beta^3 \Delta (\text{Log } \beta) \quad (26)$$

which leads us to take

$$\psi := -\text{Log } \beta$$

as the scalar gravitational field, and to try to formulate the equation for ψ in terms of the flat metric, which of course is simpler.

On the other hand, we want that a local energy conservation equation take place in that theory, by virtue of (i) the dynamical equation (24) and (ii) the sought-for equation for the field ψ . Using the form assumed for the spatial metric, that is obvious in Eq. (15), we can rewrite the time component of (24) as [10]

$$(\beta^{-2} T_0^j)_{,j} + (\beta^{-2} T_0^0)_{,0} = -\psi_{,0} T^{00} := \alpha. \quad (27)$$

In order to rewrite (27) as a local equation of conservation, thus as the vanishing of a 4-divergence, it is necessary and sufficient that also its r.h.s. α be a 4-divergence, and this has to happen by virtue of the sought-for equation for ψ . This does happen if we take the gravitational source σ to be the energy component T^{00} , while postulating for the scalar field ψ the flat wave equation

$$\square \psi := \psi_{,0,0} - \Delta \psi = \frac{4\pi G}{c^2} \sigma \quad (28)$$

$$\sigma := T^{00}, \quad x^0 := cT. \quad (29)$$

(This corresponds to $F(\beta) = \beta^2$ in Eq. (25).) Then we do obtain α as a 4-divergence

$$\alpha = \frac{c^2}{4\pi G} \left(-\frac{1}{2} \partial_0 (\psi_{,0}^2 + (\text{grad } \psi)^2) + \text{div} (\psi_{,0} \text{grad } \psi) \right) \quad (30)$$

and Eq. (27) rewrites as the following *conservation equation for the energy* [10]

$$\partial_0 (\varepsilon_m + \varepsilon_g) + \text{div} (\Phi_m + \Phi_g) = 0 \quad (31)$$

⁵Henceforth, div , grad and $\Delta := \text{div grad}$ are the usual operators, defined with the Euclidean metric \mathbf{h}^0 .

where the material and gravitational energy densities are given (in mass units) by

$$\varepsilon_m := T^{00}, \quad \varepsilon_g := \frac{c^2}{8\pi G} (\psi_{,0}^2 + (\text{grad}\psi)^2) \quad (32)$$

and the corresponding fluxes are

$$\Phi_m := (T^{0j}), \quad \Phi_g := -\frac{c^2}{4\pi G} (\psi_{,0} \text{grad}\psi). \quad (33)$$

This result is in contrast with GR, in which (to say it shortly in spite of the enormous literature) there is not such a local conservation equation for the energy. Indeed the so-called “covariant conservation equation” $T_{;\nu}^{\mu\nu} = 0$ “does not generally express any conservation law whatever” [23], or in other words these four equations “do not by themselves lead to conservation laws” [21]. See Ref. [12], §4.4, for a more detailed discussion.

6 Observational Tests of the Theory

In order to test that theory in celestial mechanics, an asymptotic PNA scheme was developed, see Point 4 in §2.2 above. However, this scheme was fully developed for v1 [6] (see §3.3 above for the distinction between v1 and v2): for v2, the local equations and the general form of the equations of motion of the mass centers have been obtained [10]. A first and very important point is that *Newton’s theory is recovered as the first approximation*. The first corrections to Newton’s theory occur at the second approximation, i.e. the first PNA, thus they are $O(1/c^2)$, hence very small e.g. in the solar system. The main results of the first PNA are as follows:

- The *internal structure* of the bodies influences celestial mechanics, for v1 [6] and for v2 as well [10]. This is in fact also the case for GR [9], see again Point 4 in §2.2 above. For v1, that influence subsists at the point-particle limit (which is a fatal feature), and this is due to the anisotropic gravitational contraction postulated in that first version: it is (therefore) not the case for v2 [10].
- For v2, it has been obtained the acceleration $a_{S \vee}$ of a test particle in the field of a unique massive body S (the Sun, say) that has spherical symmetry in its own rest frame and that moves at a uniform speed \mathbf{V} with respect

to the preferred frame \mathcal{E} [13]. In the particular subcase that $\mathbf{V} = 0$, this is the same acceleration as the acceleration $\mathbf{a}_{S\text{ GR}}$ of standard GR, i.e., that derived in GR from the spatially-isotropic SSS solution of GR, Eqs. (9.5.3) and (9.5.14) in Ref. [29]. Now the latter acceleration, to which one adds the Newtonian acceleration $\mathbf{a}_{P\text{ N}}$ due to the planets, is the one used in several reference ephemerides, see e.g. Ref. [28] and references therein. The celestial mechanics of that theory is currently based on the same acceleration, except for substituting $\mathbf{a}_{S\text{ v}}$ for $\mathbf{a}_{S\text{ GR}}$ — thus $\mathbf{a} = \mathbf{a}_{S\text{ v}} + \mathbf{a}_{P\text{ N}}$. Thus, if $\mathbf{V} = 0$, that celestial mechanics is indistinguishable from the standard celestial mechanics of GR. To determine \mathbf{V} , we built a software that loops on the integration of the equations of motion in order to find values of the parameters (including \mathbf{V} and the masses of the planets) that minimize the residual with observations. However, currently, instead of observations, the data of an ephemeris based on standard GR are used [13]. As discussed there in some detail, in that case the fact that $\mathbf{a}_{S\text{ v}} = \mathbf{a}_{S\text{ GR}}$ if $\mathbf{V} = 0$ makes it expectable that the minimization program velocity finds a very low value of $V = |\mathbf{V}|$ (ca. 40 km/h in fact), as it does. In the future, the optimization should be made with primary observations instead of ephemerides based on GR, and using complete equations of motion for v2 accounting for structure effects, as were obtained for v1 [6].

- *The same effects are predicted as the standard predictions of GR for electromagnetic rays*, independently of the velocity \mathbf{V} (assumed small with respect to c and constant) of the massive body with respect to the preferred frame; see Ref. [10] and references therein. This is of course a very important point, as the gravitational effects on light rays are the most important predictions of GR from a practical point of view.
- The data of binary pulsars should be nicely fitted, as is the case in GR, because the energy loss by gravitational radiation has the same structure as in GR, in particular there is only a quadrupole radiation. Actually the latter point has been proved only for v1 [7]. However, for v1, this conclusion depended on the fact that the usual wave equation (with an energy source on the r.h.s.) was got at the relevant post-Minkowskian approximation [7]. Since in v2 the exact field equation is the usual wave equation with an energy source, Eq. (28), one expects that similar results will be obtained for v2 regarding the binary pulsar energy loss as with v1.

7 Maxwell Equations in a Gravitational Field and Interaction Energy

In this theory, the first (homogeneous) set of the Maxwell equations is unchanged. However, in the presence of the gravitational field, the second (inhomogeneous) set cannot be obtained simply by substituting covariant derivatives for partial derivatives, as is done in GR, because this would mean that the dynamical equation for the energy tensor \mathbf{T} is $T^{\mu\nu}_{;\nu} = 0$, whereas it is Eq. (24). Therefore, a different and more difficult path has to be traced. It turns out that this leads to introduce an additional energy tensor in the presence of both an electromagnetic (EM) field and a gravitational field: the “interaction” tensor $\mathbf{T}_{\text{inter}}$, which might contribute to the dark matter. This research, which makes a somewhat voluminous work, has been reviewed in a recent paper [14], so we will content ourselves here with a brief summary.

7.1 Maxwell Second Set and the Interaction Energy

The second set is got by applying the Dynamical Equation (24) to a charged medium in the presence of Lorentz force. However, one has to assume an additional “interaction energy tensor” $\mathbf{T}_{\text{inter}}$

$$\mathbf{T}_{(\text{total})} = \mathbf{T}_{\text{charged medium}} + \mathbf{T}_{\text{field}} + \mathbf{T}_{\text{inter}}. \quad (34)$$

Indeed, without $\mathbf{T}_{\text{inter}}$, charge is (violently) not conserved. The form of $\mathbf{T}_{\text{inter}}$ is got by asking that it be Lorentz-invariant in the situation of SR, i.e. when the metric γ is Minkowski’s metric γ^0 . This leads uniquely to the following definition

$$T_{\text{inter } \nu}^{\mu} := p \delta_{\nu}^{\mu}, \quad \text{or} \quad (T_{\text{inter}})^{\mu\nu} := p \gamma^{\mu\nu} \quad (35)$$

with some scalar field p , not to be confused with the scalar gravitational field ψ . The corresponding interaction energy is $E_{\text{inter}} := T_{\text{inter}}^{00} = p\gamma^{00}$. The medium with energy tensor (35) can be counted as “dark matter”, because: (i) it is not localized inside usual matter: $p \neq 0$ at a generic point. (ii) It is gravitationally active, for T^{00} is the source of gravitational field, cf. Eq. (28). (iii) it is not usual matter, e.g. no velocity can be defined for it, because its expression (35) is invariant under local Lorentz transformations.

7.2 Equation for the Field p

With the interaction energy tensor (35), we have just one unknown more: the scalar field p . The additional equation is simply *charge conservation*. Then the system of equations of electrodynamics of the theory is closed, and satisfies charge conservation. In a given general EM field (\mathbf{E}, \mathbf{B}) and in a given weak gravitational field with Newtonian potential U , the charge conservation rewrites as

$$k^\nu p_{,\nu} = f, \quad \text{or} \quad \mathbf{k} \cdot \nabla p = f, \quad \nabla := \nabla_4 \quad (36)$$

with f containing derivatives of \mathbf{E}, \mathbf{B} and U , and

$$k^\nu := G^{\mu\nu}_{,\mu}. \quad (37)$$

Here, $G^{\mu\nu} = -G^{\nu\mu}$ is the inverse tensor of the EM field tensor $F^{\mu\nu}$ of a first approximation, in an asymptotic expansion with respect to the gravitational field strength.

7.3 Averaging the PDE for p

In order to check if the interaction energy might contribute to dark matter, we need to integrate *on a galactic scale* $r \sim 10^{19}\text{m}$ the PDE (36) for the scalar field p . However, the given fields \mathbf{k} and f in (36) vary on a scale $r \sim \lambda \simeq 10^{-6}\text{m}$ and $t \sim \lambda/c$, like \mathbf{E} and \mathbf{B} . Therefore, some consistent averaging is absolutely needed. After several attempts based on homogenization theory, we use the theory of macro-homogeneous fields: we assume that the fields \mathbf{k} and ∇p are “locally macro-homogeneous”. I.e., they are *slow variations of macro-homogeneous fields*. Also, because $G^{\mu\nu} = -G^{\nu\mu}$, we have $\text{div } \mathbf{k} = G^{\mu\nu}_{,\mu,\nu} = 0$. It follows [14] that the PDE (36) averages to

$$K^\nu P_{,\nu} = F \quad (38)$$

or

$$\mathbf{K} \cdot \nabla P = F \quad (39)$$

with

$$\mathbf{K} := \bar{\mathbf{k}}, \quad P := \bar{p}, \quad F := \bar{f}$$

where the upper bar means spacetime average over a representative volume. Thus, the same equation is obtained as (36), but with *spacetime-averaged* fields.

7.4 Solving the PDE for P

The PDE (38) for P rewrites as an advection eqn with source:

$$\partial_T P + U^j \partial_j P = S \quad (40)$$

where

$$S := cF/K^0, \quad U^j := cK^j/K^0. \quad (41)$$

Therefore, on any integral curve $T \mapsto \mathbf{x}(T)$, $\mathbf{x}(T_0) = \mathbf{x}_0$ of the velocity field U^j , we have

$$P(T, \mathbf{x}(T)) = P(T_0, \mathbf{x}_0) + \int_{T_0}^T S(t, \mathbf{x}(t)) dt. \quad (42)$$

7.5 Calculating the Micro-Field k^ν and the Averaged Field K^ν

The 4-vector “micro-field” k^ν depends only on the (microscopic) EM field (\mathbf{E}, \mathbf{B}) . To compute k^ν , we built the “Maxwell model of the interstellar radiation field (ISRF)”, based on axial symmetry (of the galaxy and the ISRF inside it) as a relevant approximation. The axial symmetry implies that the independent variables, say x^α ($\alpha = 0, 1, 2$) are the time t , the distance ρ to the axis of symmetry, and the altitude z over the plane of the galaxy. Thus, the micro-field k^ν is computed with the said model on a 3D spacetime “fine” grid, with steps δ_α . Then we take the local spacetime average K^ν of the field k^ν on a “rough” grid with steps $\delta_{\alpha g} = g\delta_\alpha$ (g is an integer, e.g. $g = 6$ in an example computation [14]). The average is done by considering, for each point of the rough grid, its $(2g+1)^3$ nearest neighbours of the fine grid, thus a discrete averaging. Then we calculate $U^j = cK^j/K^0$, whose integral lines are the characteristics. Currently we can numerically compute some integral lines of U^j in a model of the Milky Way, at a sub-kpc scale [14].

8 Conclusion

The investigated theory extends Lorentz-Poincaré SR to gravity. Its main points are as follows:

- It is a preferred-frame theory starting from a mechanism for gravity: gravitation is interpreted as Archimedes’ thrust in a fluid ether.

- Motion is defined by a unique extension of Newton's second law to a curved spacetime, which implies geodesic motion for a static field.
- The scalar gravitational field ψ is subjected to the usual wave equation, with the material energy density on the r.h.s.. Together with Newton's second law for the material continuum, this implies an exact local conservation equation for the material plus gravitational energy.
- The theory has a correct Newtonian limit. The gravitational effects on light rays are identical with the standard predictions of GR. There is quadrupole gravitational radiation only.
- The same motion is predicted as in standard GR for a test particle in the static, spherically symmetric situation. Celestial mechanics currently is based on adding the acceleration due to the Sun, assumed spherical and having a uniform motion, to the Newtonian acceleration generated by the planets. The absolute velocity of the solar system in that theory remains an open question.
- To formulate the Maxwell equations in a gravitational field, it is necessary to postulate a definite interaction energy tensor in the presence of both an EM field and a gravitational field. The medium with that energy tensor has a special nature, since no velocity can be defined for it. Being present virtually everywhere and gravitationally active, it is a candidate for dark matter. Calculations attempt at checking the amount and the distribution of that energy in a galaxy. They are difficult but progress is being done.

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