Dynamics of Cycling Adoption: A Model with Social Influence

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***Abstract*— To address the consequences of climate change, policies promoting green transportation, particularly cycling, are gaining importance. To address this need, this paper introduces a novel compartmental model to analyze the dynamics of bicycle adoption. We demonstrate the existence and global asymptotic stability of a single equilibrium point using order-preserving monotonic systems theory. Furthermore, we establish the system's identifiability, ensuring unique parameter estimation from observed trajectories. A case study of Stockholm, Sweden, showcases the model's ability to accurately characterize cycling adoption dynamics, highlighting its potential for informing sustainable transportation strategies.**

I. INTRODUCTION

Promoting green modes of transportation is crucial to mitigate the environmental impact of transportation, notably in the perspective of the fight against climate change. Furthermore, active modes of transportation, such as walking and cycling, offer significant benefits beyond emission reduction, including improved public health and reduced traffic congestion [1], [2]. These modes, alongside electric vehicles, represent key enablers for sustainable urban mobility.

Transport mode choice is a complex process, influenced by interconnected variables that have been extensively studied across disciplines like social sciences, public health, economics, and transportation science. Research primarily aims to identify and analyze the key factors driving mode selection, which can be broadly categorized as: individual (personal characteristics and preferences), contextual (environmental and situational influences), objective (measurable factors like cost and travel time), and subjective (perceptions and attitudes). These factors, often interrelated, are typically examined using linear and nonlinear statistical models [3]. Traditionally, studies have emphasized the importance of demographic factors (e.g. age, ethnicity, and gender) and of contextual factors, such as availability and quality of infrastructure, cost, and geographic conditions [4]–[6]. However, recent research has emphasized the significant role of social dynamics, including peer interaction and social influence [7], [8]. For example, in [9] it was demonstrated that the dominant transport modes within someone's immediate environments, such as workplaces and schools, strongly influence personal choices.

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Another important aspect in the study of the adoption of green transport modes has been the development of dynamic models that seek to replicate (and predict) these phenomena. Several works have addressed the development of decision-making models applied to the adoption of electric vehicles, considering factors such as social impact and incentive policies [10]–[12]. In particular, models have been developed based on the well-known Bass adoption model, generally used in economics to characterize the penetration of a product in the market [13]. For instance, [14] seeks to achieve a specific greenhouse gas emission target over a time horizon through monetary incentives to purchase electric vehicles, using a Bass model and optimal policies. Less frequently, adoption models for active transport modes have also been proposed. These works have studied the bicycle-sharing market using methods from game theory [15] and Bass model [16].

This work introduces a mathematical model capable of characterizing the dynamics of adoption of green modes of transport, particularly cycling adoption. Our model incorporates elements of social influence and innovation, similar to the Bass model, while including the possibility of abandoning the new habit (cycling, in our case). Although we recognize the crucial role that contextual factors, such as infrastructure, play in facilitating or hindering cycling, in this modeling work we choose to prioritize psychosocial factors involved in the phenomenon of cycling adoption.

The proposed model belongs to the family of compartmental models, which have been historically used in the study of infectious diseases. More recently, their use has expanded to describe complex social phenomena, such as social satisfaction [17], product acquisition [18], or rumor spreading [19]. However, to the authors' knowledge, there is no previous work using compartmental models to address habit adoption in general or, in particular, the adoption of active green transportation modes such as cycling. Our new idea is therefore that adopting a habit, such as cycling regularly, can be seen as akin to spreading an infection.

After defining the model, we provide a qualitative analysis of its properties, proving the existence of a globally asymptotically stable equilibrium. In order to apply our model to a real case study, we study the identifiability of its parameters and we perform such identification on a 24-year time series from the city of Stockholm, Sweden.

Outline: The model is defined in Section II and analyzed in Section III, where emphasis is placed on the study of global stability using monotonic system theory. In Section IV, the identifiability of the system is studied and then, in Section V, the parameter estimation is proposed for

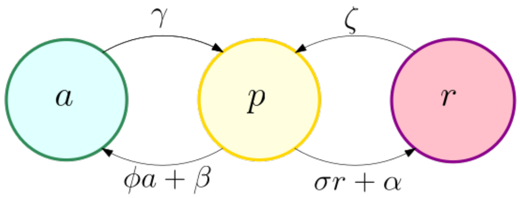

Fig. 1: PAR compartmental model (2).

a case study based on the cycling adoption in Stockholm, Sweden. Finally, Section VI presents the study's conclusions and proposes directions for future research.

II. Model Definition

We introduce the PAR (Potential-Adopter-Rejecter) model as a compartmental model to analyze the dynamics of adopting cycling among a population. As shown in Fig. 1, the model consists of three compartments: potential adopters (p), adopters (a), and rejecters (r). Also, we consider a closed population model, i.e. proportions of each compartment sum to one:

$$p(t)+a(t)+r(t)=1 \quad \text{for all times } t\geq 0. \tag{1}$$

The transitions between the compartments are due to social influence or to spontaneous transitions, per the following set of ordinary differential equations:

$$\dot{p}=-\phi pa-\sigma pr+\gamma a+\zeta r-(\alpha+\beta)p \tag{2a}$$
$$\dot{a}=\phi ap-\gamma a+\beta p \tag{2b}$$
$$\dot{r}=\sigma pr-\zeta r+\alpha p. \tag{2c}$$

The equations contain several parameters:

- $\phi\geq 0$ and $\sigma\geq 0$ represent the social influence rates for adopting and rejecting, respectively. These rates mean that social interactions increase the spread of positive or negative habits about cycling.
- $\beta\geq 0$ and $\alpha\geq 0$ are, respectively, rates of the spontaneous decisions to adopt and reject. These rates describe personal attitudinal and affective drivers based on beliefs, emotions and expectations in relation with cycling behaviour.
- $\gamma\geq 0$ and $\zeta\geq 0$ are, respectively, rates of the spontaneous surrender of adoption and rejection. These rates describe personal attitudinal and affective barriers based on beliefs, emotions and expectations in relation with cycling behaviour.

It is worth noting that the proposed PAR model has well-known compartmental models as degenerate cases:

- If $\alpha=\beta=0$, we have the case of no autonomous decision from the compartment of potential adopters. This case coincides with the bi-virus SI_1I_2S model [20]: in the case of disease, it is impossible to become infected without contact with an infected person.
- If $\gamma=\zeta=0$, we have the case of the Bass competition model [21], in which potential users can choose between two services and the market is saturated in a finite time.

In view of this literature, we restrict our study to the *non-degenerate case*, i.e. the parameters ϕ, σ, γ, ζ, β and α are assumed to be *strictly positive* from now on.

III. Model analysis

Given (1), hereafter the model analysis is restricted to the reduced equivalent model:

$$\dot{a}=\phi a(1-a-r)-\gamma a+\beta(1-a-r) \tag{3a}$$
$$\dot{r}=\sigma r(1-a-r)-\zeta r+\alpha(1-a-r) \tag{3b}$$

We first study the well-posedness of the model and then its stability properties. To this goal, let us define the convex polygon

$$X=\{(a,r)\in\mathbb{R}^2_+ : a\geq 0, r\geq 0, a+r\leq 1\} \tag{4}$$

and let $f_a(a,r)$ and $f_r(a,r)$ denote the right-hand sides of (3a) and (3b), respectively.

Proposition 1: (PAR model well-posedness). Consider the system (3). If $(a(0),r(0))\in X$, then $(a(t),r(t))\in X$ for all $t>0$ i.e. (3) is positively invariant in the set X.

Proof: It is easy to note that if $a=0$, then $\dot{a}\geq 0$ and if $r=0$, then $\dot{r}\geq 0$: therefore, trajectories $a(t)$ and $r(t)$ originating from $(a(0),r(0))\in\mathbb{R}^2_+$ remain in $\mathbb{R}^2_+$ for all $t\geq 0$. Consider now that (a,r) belongs to the other part of the boundary of X, that is, the set satisfying $a+r=1$. From (3a) and (3b), we compute

$$f_a(a,r)+f_r(a,r)=-\gamma a-\zeta r,$$

therefore, since $\gamma,\zeta>0$ and $(a,r)\in X$ then $\dot{a}+\dot{r}<0$ and solutions satisfy $a+r\leq 1$. ■

With this result, it is possible to study the equilibria of (3).

Proposition 2: (PAR model equilibrium). System (3) has a unique equilibrium $e^*=(a^*,r^*)\in X$.

Proof: Any equilibrium point $e^*=(a^*,r^*)$ has to be a solution of setting the left-hand-side of (3) equal to zero:

$$\phi a^*(1-a^*-r^*)-\gamma a^*+\beta(1-a^*-r^*)=0,$$
$$\sigma r^*(1-a^*-r^*)-\zeta r^*+\alpha(1-a^*-r^*)=0;$$

or equivalently, after $p^*=1-a^*-r^*$ due to (1),

$$\phi a^*p^*-\gamma a^*+\beta p^*=0, \qquad \sigma r^*p^*-\zeta r^*+\alpha p^* \quad =0,$$

which lead to:

$$a^*=\frac{\beta p^*}{\gamma-\phi p^*}=: q_1(p^*), \qquad r^*=\frac{\alpha p^*}{\zeta-\sigma p^*}=: q_2(p^*). \tag{5a}$$

Replacing r^* by $1-a^*-p^*$ in (5a), we get

$$a^*=1-p^*-q_2(p^*)=: q_3(p^*).$$

Therefore, any equilibrium corresponds to an intersection of the graphs of q_1 and q_3 inside the triangle $\{(p,a)\in\mathbb{R}^2 : p\geq 0, a\geq 0, p+a\leq 1\}$. As illustrated in Figure 2, both graphs consist of two branches, separated by vertical asymptotes. Elementary calculations, which we omit for the sake of brevity, lead to verify that the left branches of the graphs always intersect exactly once within the triangle, whereas

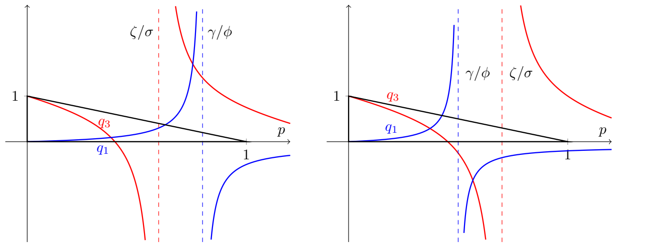

Fig. 2: Relevant cases for the study of functions q_1 and q_3.

the right branches of both graphs always lie outside of the triangle. Therefore, the existence of a unique equilibrium is proved. ■

We are now going to study the global behavior of PAR system within the framework of monotone systems (in the preserving-order sense). The theory of monotone systems [22]–[24] has been successfully applied to compartmental systems, such as [25]–[27]. Let be $z = (a, r)$, so the PAR system in (3) can be written as

$$\dot{z}(t) = f(z, t), \tag{6}$$

with Jacobian matrix

$$J(z) = \begin{pmatrix} \phi(1 - 2a - r) - \gamma - \beta & -(\phi a + \beta) \\ -(\sigma r + \alpha) & \sigma(1 - a - 2r) - \zeta - \alpha \end{pmatrix}$$

Let us consider dynamical systems in the form (6) on a domain $\mathcal{X} \subset \mathbb{R}^2$, endowed with the partial order relation induced by a positive cone $\mathcal{K}_+ \subset \mathbb{R}^2$ as follows: we write $z_1 \leq z_2$ if $z_2 - z_1 \in \mathcal{K}_+$. In this context, we say that (6) is *strongly monotone* if the order is strongly preserved, in the following sense:

$$\forall z_1, z_2 \in \mathcal{X} : z_1 < z_2 \Rightarrow f(z_1, t) \ll f(z_2, t), \forall t \in \mathbb{R}_+,$$

where $z_1 < z_2$ means that $z_1 \leq z_2$ and $z_1 \neq z_2$, and $z_1 \ll z_2$ means that $z_2 - z_1 \in \mathrm{Int}\mathcal{K}_+$. Also, if $\mathcal{X} \subset \mathbb{R}^2_+$ and $\mathcal{K}_+$ has no empty interior, we say (6) to be *strongly order preserving (SOP)*. With these definitions, we can state the following monotonicity result.

Proposition 3: (PAR model monotonicity). The system (3) is strongly monotone in X with respect to the order induced by the cone

$$K := \{(a, r) \in \mathbb{R}^2 : a \geq 0, r \leq 0\}.$$

Proof: Following [28, Theorem 1.1], a system is strongly monotone respect to the order induced by the positive orthant $\mathbb{R}^2_+$ if and only if its Jacobian is cooperative and irreducible. A Jacobian matrix is called cooperative if it has non-negative off-diagonal entries (Metzler).

In order to use these facts to prove the theorem's statement, let us introduce the diagonal matrix $Q = \mathrm{diag}(1, -1)$ with $Q = Q^{-1} = Q^T$, such that the change of variables $y = Qz = Q \begin{bmatrix} a \\ r \end{bmatrix} = \begin{bmatrix} a \\ -r \end{bmatrix}$ leads to the system $\dot{y} = g(y, t)$. Its Jacobian $\tilde{J}(y)$ is related to the Jacobian of (3) by $\tilde{J}(y) = QJ(z)Q$, implying that

$$\tilde{J}(y) = \begin{pmatrix} \phi(1 - 2a - r) - \gamma - \beta & \phi a + \beta \\ \sigma r + \alpha & \sigma(1 - a - 2r) - \zeta - \alpha \end{pmatrix} \tag{7}$$

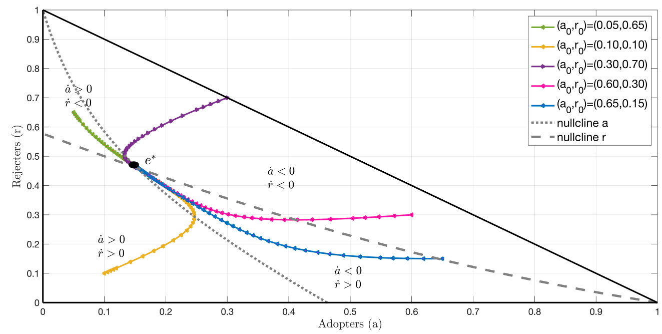

Fig. 3: Global convergence of PAR model. The figure shows solutions in arrowed solid lines, which originate from different initial conditions and converge to the equilibrium $e^* = (0.16, 0.45)$. The direction of the trajectories depends on the region in which they are located, generated by the nullclines of a (dotted line) and r (dashed line) respectively.

which is Metzler and, having no zero elements, is irreducible. Therefore, it preserves the order in $\mathbb{R}^2_+$. As a consequence, (3) preserves the order induced by the cone K. ■

From these facts, it is now possible to deduce our main result regarding the qualitative behavior of the PAR system.

Proposition 4: (PAR model global stability). The unique equilibrium $e^* \in X$ of the system (3) is globally asymptotically stable.

Proof: From [23, Theorem D], if the following conditions hold:

(i) the system (3) is SOP in $\mathbb{R}^2_+$;
(ii) every solution of (3) has compact closure in $\mathbb{R}^2_+$;
(iii) there is not more than one equilibrium point;

then, the equilibrium is globally asymptotically stable.

For the PAR system, condition (iii) holds as stated by Proposition 1. Condition (i) also holds, because by Proposition 3, the system (3) is strongly monotone, and since $X \subset \mathbb{R}^2_+$ and K has no empty interior, (3) is SOP. Condition (ii) is also satisfied by Proposition 1, because the invariant set X is compact. ■

In order to illustrate the above results and the role of the parameters, we now present relevant numerical examples.

Example 1: Consider a PAR system with parameters $\rho = [\phi, \sigma, \gamma, \zeta, \beta, \alpha] = [0.5, 0.2, 0.1, 0.1, 0.05, 0.05]$. Fig. 3 shows the convergence of some trajectories of the system to the globally asymptotically stable equilibrium $e^* = (0.16, 0.45)$. The behavior of the trajectories depends on the region in which they are located. These four regions are generated by the nullclines of a ($f_a(a, r) = 0$) and r ($f_r(a, r) = 0$), whose intersection is the unique equilibrium point e^*. □

By inspecting (3a), it is possible to derive a ratio $R(t)$ of the inflow and the outflow of the adopter compartment such that

$$R(t) = \frac{\phi a(1 - a - r) + \beta(1 - a - r)}{\gamma a}, \tag{8}$$

where the positive rates (inflows) $\phi a(1 - a - r)$ and $\beta(1 - a - r)$ tell, respectively, how many new adopters will be generated in $t > 0$ by social influence or spontaneous

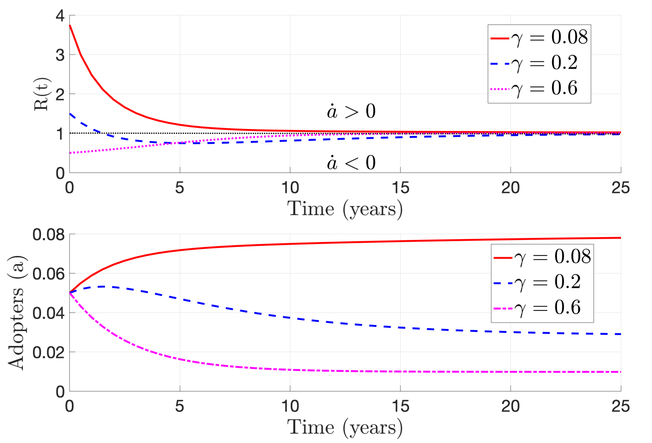

Fig. 4: Effect of parameter γ on adopter fraction $a(t)$, with $\rho = [0.2,0.15,\gamma_i,0.05,0.05,0.3]$. Top: $R(t)$ values. Bottom: $a(t)$ solutions. See legend for γ values and line styles.

decision, and the negative rate (outflow) γa tells how many adopters will surrender in $t > 0$. Consequently, $R(t) > 1$ implies a growth in the number of adopters while $R(t) < 1$ leads to a decay.

Example 2: In order to illustrate the influence on the trajectories of the parameter variations, Fig. 4 shows the influence of the γ parameter on system solutions. The parameters $\phi, \sigma, \zeta, \beta, \alpha$ are kept fixed, and the value of γ changes so that $\rho = [0.2, 0.15, \gamma_i, 0.05, 0.05, 0.3]$ with $\gamma_i = [0.08, 0.2, 0.6]$. It is possible to note that a greater γ causes a reduction in the growth rate of the adopter compartment, as shown in Fig. 4 (bottom). Fig. 4 (top) shows how the value of $R(t)$ is related to the growth ($R(t) > 1$) or the decrease ($R(t) < 1$) of the compartment a. This is evidenced in the non-monotone curve of a for $\gamma = 0.2$ (dashed blue). It can be seen that for $t < 1.5$, $R(t)$ is greater than 1, consequently there is a growth in the number of adopters. From $t > 1.5$, $R(t)$ becomes lower than one, leading to a decrease of $a(t)$ until the equilibrium is reached. □

IV. MODEL IDENTIFIABILITY

Prior to practical parameter estimation from data, it is crucial to establish the model's theoretical identifiability: the assurance that a unique parameter set corresponds to a given trajectory. System (3) is linear with respect to its parameters. Consequently, we aim to determine conditions for recovering the parameter vector $\rho = \left(\rho_a^T, \rho_r^T\right)^T$, where $\rho_a = (\phi, \sigma, \gamma)^T$ and $\rho_r = (\zeta, \beta, \alpha)^T$, from the observed variables $a(t)$ and $r(t)$, along with their respective derivatives. Let $y = (y_1, y_2)^T = (a(t), r(t))^T$ represent the observation vector. Following the approach outlined in [29], system (3) is identifiable from observation vector y if there exists $t > 0$ such that the map

$$\rho \in \mathbb{R}_+^6 \quad \to \quad y(\cdot) \in \mathcal{C}^\infty([0,t], \mathbb{R}_+^2)$$

is injective, where $y(\cdot)$ is a solution of (3). We can thus state the following proposition.

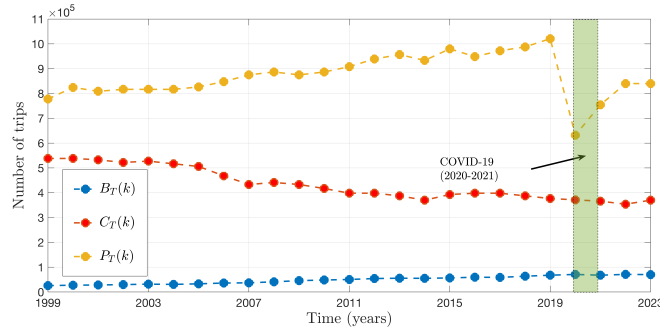

Fig. 5: Measured transportation trip data for Stockholm (1999-2023): bicycle $B_T(k)$ (blue), car $C_T(k)$ (red), and public transport $P_T(k)$ (yellow).

Proposition 5: (PAR model identifiability) The PAR model (3) is identifiable from the trajectory $y(t) = \big(a(t),\, r(t)\big)^\top$ and its time derivatives, provided that the matrices M_a and M_r are full rank. Specifically,

$$M_a = \begin{bmatrix} y_1(1-y_1-y_2) & -y_1 & 1-y_1-y_2 \\ \dot{y}_1 - 2y_1\dot{y}_1 - (y_1y_2)^{(1)} & -\dot{y}_1 & -\dot{y}_1 - \dot{y}_2 \\ \ddot{y}_1 - 2(y_1\dot{y}_1)^{(1)} - (y_1y_2)^{(1)} & -\dot{y}_1 - \dot{y}_2 & -\ddot{y}_1 - \ddot{y}_2 \end{bmatrix}$$

and the matrix M_r is obtained from M_a by interchanging y_1 and y_2.

Proof: Since right-hand sides of (3a) and (3b) are polynomials, the solution y of (3) is $\mathcal{C}^\infty$. Two differentiations of (3a) result in the linear system

$$M_a\rho_a = \left(\dot{y}_1,\ \ddot{y}_1,\ y_1^{(3)}\right)^T.$$

Similarly, from (3b) we get

$$M_r\rho_r = \left(\dot{y}_2,\ \ddot{y}_2,\ y_2^{(3)}\right)^T$$

Both systems have unique solution if and only if $\operatorname{rank}(M_a(t)) = \operatorname{rank}(M_r(t)) = 3$, ensuring that the map is injective. ■

V. CASE STUDY

This section describes a case study for the city of Stockholm to illustrate the effectiveness of the PAR model.

A. Dataset

The raw data used to characterize cycling adoption in the city of Stockholm were taken from the Stockholm Transport Department (Stockholms trafikkontoret) public data for the period between 1999 and 2023 [30]. This dataset describes the evolution of transport use by counting the number of trips per year by car, public transport and bicycle, as shown in Fig. 5.

Let $C_T(k)$, $P_T(k)$, and $B_T(k)$ be the number of trips made by car, public transport, and bicycle. In order to have values in the range $[0, 1]$, we normalize the data as follows:

$$\bar{C}_T(k) = \frac{C_T(k)}{N(k)},\ \bar{P}_T(k) = \frac{P_T(k)}{N(k)},\ \bar{C}_B(k) = \frac{B_T(k)}{N(k)},$$

TABLE I: Estimated parameter values $\bar{\rho}$.

Social influence rate to adopt	$\phi = 2.9998$
Social influence rate to reject	$\sigma = 0.1231$
Autonomous surrender rate to adopt	$\gamma = 1.7287$
Autonomous surrender rate to reject	$\zeta = 0.0758$
Autonomous decision rate to adopt	$\beta = 0.0111$
Autonomous decision rate to reject	$\alpha = 0.0024$

where $N(k)$ is the total number of trips in each year k. We relate these normalized values to the model outputs y_1 and y_2 from equation (3), which represent adopters and rejecters, respectively, under the following assumptions: (i) the number of trips is directly proportional to the number of individuals choosing a given transport mode, and (ii) car users are generally less likely to adopt alternative modes like cycling [31]. From these assumptions we consider

$$\bar{y}_1(k) = \bar{B}_T(k),$$
$$\bar{y}_2(k) = 0.7\bar{C}_T(k) + 0.3\bar{P}_T(k),$$

where $\bar{y}_1(k)$ and $\bar{y}_2(k)$ are measure data of y_1 and y_2 respectively. The factors of 0.7 for car users ($\bar{C}_T(k)$) and 0.3 for public transport users ($\bar{P}_T(k)$) are consistent with the studies [32]–[34], which show that once an individual has acquired the habit of using a car, they rarely consider other modes of transport as alternatives.

B. Parameter estimation

Let us consider $\bar{y}_1(k)$ and $\bar{y}_2(k)$ data outputs of $a(t)$ and $r(t)$ respectively, where $k = 1999, 2000, \ldots, 2023$. We formulate a nonlinear least squares problem to fit the model outputs y_1 and y_2 from equation (4) to the observed data $\bar{y}_1(k)$ and $\bar{y}_2(k)$. This approach is commonly employed for parameter estimation in nonlinear models [35], [36]. Following the procedure outlined in [37], we utilize the MATLAB Optimization Toolbox [38] to estimate the optimal parameter set. We begin by defining an initial parameter guess, ρ_0, consistent with the dynamics of adoption. Based on findings from [9], we assume a higher peer influence on active transport adoption compared to other modes. Additionally, we postulate a lower spontaneous adoption rate than the spontaneous surrender rate. These considerations lead to the initial guess $\rho_0 = [1.5, 0.5, 0.1, 0.1, 0.05, 0.05]$.

As observed in Figure 5, the data for 2020 and 2021 exhibit anomalies, likely due to COVID-19 pandemic-related measures. Since our model does not account for such exogenous events, we exclude these data points prior to parameter estimation. Subsequently, we generate $N = 20$ datasets with simulated measurement error and fit the model using the Levenberg-Marquardt algorithm (**lsqcurvefit** function in MATLAB) for the original and generated datasets, with $M = 50$ random initial conditions (**multistart** function in MATLAB). The resulting parameter set, $\bar{\rho}$, which minimizes the sum of squared residuals (SSR), is presented in Table I.

C. Model fitting

Figure 6 compares the observed ($\bar{y}_1(k)$,$\bar{y}_2(k)$) and model-generated ($y_1(t)$, $y_2(t)$) trajectories for adopters and rejecters. The model (3) with parameters $\bar{\rho}$ demonstrates a good fit, with $SSR = 8.7978 \times 10^{-4}$.

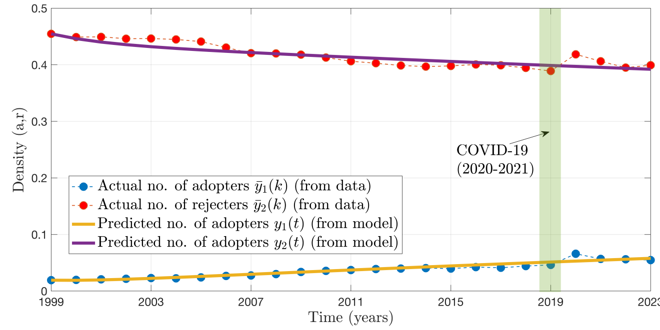

Fig. 6: PAR model fit with estimated parameters $\bar{\rho}$: Observed data (dash-dotted lines) and model solutions (solid lines) for adopters (a, blue/yellow) and rejecters (r, red/purple), 1999-2023.

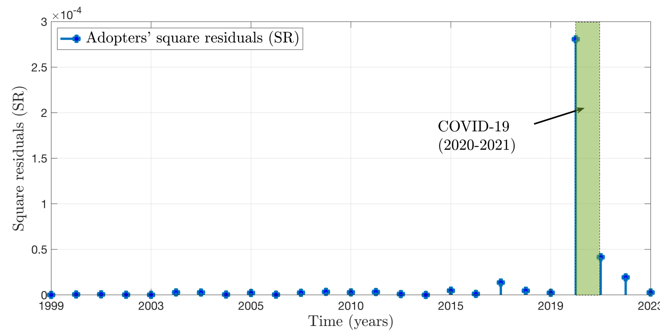

Fig. 7: Adopters' square residual (SR).

In this case study, we are particularly interested in the dynamics of the adopter compartment a (observed variable y_1). In that sense, we evaluate the residuals at time $k = 1999, \ldots, 2023$. Fig. 7 shows the adopters' squares residuals (SR); where ignoring the residual errors in the COVID-19 period, we observe small SR values. From the initial assumptions in data generation, we consider that the total number of measured trips and the total number of individuals (adopters, rejecters, and potential adopters) have the same magnitude. Furthermore, since $\bar{y}_1(k) = \bar{B}_T(k)$, we claim that the proportion of adopters is the same as the proportion of bicycle trips. From the aforementioned, we compute the maximum residual error as $\sqrt{SR(2022)} = 0.0043$, equivalent to 305 adopters out of a total of 71062. We believe that these results illustrate the effectiveness of our model in reproducing dynamics of cycling adoption.

VI. Conclusion

This paper introduced a novel compartmental PAR model of cycling behavior. We proved the existence of a unique equilibrium point and global convergence and order preservation of solutions. Through numerical simulations, we characterized the influence of parameters and initial conditions on solutions. We also established the model's identifiability, providing conditions for unique parameter recovery from persistent trajectories. To illustrate the model's effectiveness, we conducted a case study using data from Stockholm, Sweden.

While the current model assumes a homogeneous population, future research should explore the impact of demographic heterogeneity and community network interactions on adoption parameters. Furthermore, we believe that including contextual factors such as the quality of cycling infrastructure is important for a comprehensive view of the cycling adoption phenomenon. Finally, investigating optimal policy design to maximize cycling adoption under specific constraints presents a promising avenue for future work.

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