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# Multilingual Mathematics Education: an analysis of a Mexican research-based Master's degree course developed in the Questioning the World Paradigm

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## Abstract

Research in Latin America has made significant advances in multilingual approaches to mathematics education, yet this work remains underrepresented in researcher education. To address this gap, we explored how Master's courses enable student-researchers to design Multilingual Instructional Resources for mathematics teaching in indigenous schools. Using an intrinsic case study, we examined a three-course sequence in a research-based Master's program in Mexico, developed in the *Questioning the World Paradigm*. Across the sequence, each course established, and progressively enriched, didactic milieus by integrating three essential dimensions: mathematical, cultural, and linguistic. Through a media-milieu study dialectic we analyze how media like academic papers and educational resources focused on indigenous education shaped –and were shaped by– the evolving milieus to support the design, implementation, and evaluation of bilingual (Otomi-Spanish) instructional resources. Two resources created by a student-researcher, Atl, are presented. Implemented in Otomi classes, they engaged pupils in a new teaching approach framed by two fundamental mathematical activities: counting, through explorations of Otomi numbers and their arithmetic composition; and designing, through a spatial structuring task related to manufacturing objects. Cultural dimensions are promoted through co-design with indigenous teachers to explore Otomi mathematical meanings, while linguistic dimensions are established through the explicit use of different languages. Findings highlight how Master's level researcher education can cultivate inquiry, co-design, and bilingual implementation practices to contribute to revitalizing indigenous languages through mathematics education.

**Keywords** Multilingual Mathematics Education · Researcher education · *Questioning the World Paradigm* · Multilingual Instructional Resource(s) (MIR)

## 1 Introduction

Indigenous peoples make up 9.8% of Latin America's population (CONEVAL, 2022) with 80% concentrated in Mexico, Chile, Colombia, Guatemala, and Peru. Mexico, a multilingual, multicultural country, recognizes Spanish and 68

indigenous languages. Among the most widely spoken we find Nahuatl, Maya, Tseltal, Mixteco, Tsotsil, Zapoteco, and Otomi. These languages are central to cultural identity and community cohesion but are increasingly endangered. Although Mexico established an indigenous education system in 1921, instructional materials for teaching mathematics in these languages remain largely absent. Consequently, most bilingual teachers rely on Spanish for mathematics instruction, often marginalizing students' linguistic and cultural resources. This issue has been documented in studies like Avila (2014), Cortina (2013), and Cortina and Rojas (2016), which highlight the potential of Multilingual and Multicultural Approaches to Mathematics Education. However, these forms are rarely incorporated into Master's level research education programs. This study, then, explores how researcher education in a Master's program can become a space for developing pedagogical strategies that engage lan-

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guage and culture in mathematics education. We argue that such programs offer an opportunity to rethink classroom practice in ways that draw on students' everyday experiences to promote linguistic awareness and cultural knowledge.

We model and analyze the design and implementation of three Master's courses that prepare future researchers to create instructional mathematics resources that integrate indigenous languages. After presenting the literature review, theoretical framework, research question, and methodology, we examine how these courses enabled students to design and test resources in an indigenous primary school. To illustrate the process, we focus on two examples created by a student-researcher named Atl. One involves the Otomi number system, the other focuses on Otomi geometrical design. The paper concludes with a discussion of key insights and implications for further research and researcher education.

## 2 Literature review

This section examines the literature that frames our research problem, focusing on three intersecting domains: cultural diversity in mathematics education, language awareness and pluralistic approaches, and plurilingual competence and intercultural education.

### 2.1 Diversity in mathematics education

In an increasingly multilingual, multicultural world, research has demonstrated that diversity offers significant opportunities to enrich the teaching and learning of mathematics across levels (Halai & Clarkson, 2016). Following Barwell et al. (2016), we identify three key dimensions of diversity in mathematics education: indigenous mathematical knowledge, culture and ethnomathematics, and language and multilingualism. The first has been explored through analyses of mathematical practices and number systems rooted in indigenous cultures (Balderas, 2023; Barquera, 2003; Bishop, 1988; Gilsdorf, 2009). Avila (2014), however, holds that much of this knowledge remains unstudied and unsystematized. The second, defined by D'Ambrosio (1985) and developed by Rosa et al. (2017), refers to the diverse ways in which cultural groups understand, use, and express mathematical ideas. Barton (1996, p. 214) stresses that such knowledge exists independently of whether a culture formally defines it as "mathematics". The third dimension has long been of interest in mathematics education (Adler, 2002). More recently, language diversity has been promoted as both a pedagogical resource and a medium for constructing mathematical meaning (e.g., Barwell et al., 2016; Cortina, 2013; Erath et al., 2021; Hunter & Hunter, 2024). In bilingual and multilingual classrooms, language

is not merely a tool for communication but also a central element in meaning-making (Barwell, 2018; Planas, 2018). This perspective has expanded to include the cultural and linguistic contexts of students' homes and communities by recognizing them as valuable resources for mathematical learning (Trouche et al., 2023).

Despite growing interest in multilingual and multicultural mathematics education, relatively few studies have explored how these dimensions can be systematically integrated into research education programs, especially considering indigenous contexts. The present study seeks to address this gap.

### 2.2 Language awareness and pluralistic approaches

Our work also draws on broader language-education frameworks, particularly those that emphasize language awareness and pluralistic approaches. Language awareness, as articulated by Hélot (2008, p. 1), aims "to support multilingualism at both institutional and individual levels by educating all pupils together (monolinguals and bi/multilinguals) to respect linguistic and cultural diversity." This includes recognizing all the languages present in a classroom, even ones that are not taught formally. Closely tied to multilingualism, Pluralistic Approaches to Languages and Cultures (Candelier et al., 2012) encourage teaching and learning activities that integrate several languages or cultures. Two approaches are especially relevant: awakening to language by engaging students with the wide variety of languages spoken at home, in the community, or globally, regardless of whether they are part of the school curriculum (Poisard et al., 2018), and integrated didactic approach to create links among the languages taught in school and support mutual reinforcement and deeper understanding. These approaches are compelling for the revitalization of local minority languages, whether understood as indigenous, regional, or stigmatized varieties of dominant or immigrant languages (Blanchet & Kervran, 2016).

These perspectives provide a foundation for framing the revitalization of indigenous languages, not as isolated initiatives but as parts of a broader orientation toward multilingual and multicultural mathematics education. This orientation resonates with cultural diversity (de Abreu, 2014) and the participation of indigenous students in mathematics education (Meaney & Trinick, 2020).

### 2.3 Plurilingual competence and intercultural education

Our study also engages with plurilingual and pluricultural competence (Coste et al., 1997/2009); that is, the ability to mobilize multiple languages and cultures as a single, composite resource, rather than as separate proficiencies.

This notion differs from multilingualism and multiculturalism, which tend to consider languages and cultures separately. Plurilingual and pluricultural competence, in contrast, emphasize their interconnections and the fluid ways in which learners draw on their full repertoires in communicating learning. In this framework, multilingual classrooms can evolve into formative spaces when languages are explicitly compared, connected, and mutually reinforced to move beyond the simple juxtaposition of several monolingual systems. This vision aligns with intercultural education models in Latin America (Parra et al., 2016) that place endangered and underrepresented languages, including indigenous languages in Mexico, regional languages in France, and immigrant languages, at the center of equity-oriented educational projects (Gates, 2020).

In mathematics education, these perspectives strengthen approaches that recognize students' linguistic repertoires and cultural practices as valuable resources for participation and meaning-making (Halai & Clarkson, 2016; Rosa et al., 2017; Hunter & Hunter, 2024). By linking plurilingual competence and intercultural education with mathematics education, our work highlights how researcher education can prepare future scholars to design Multilingual Instructional Resources that foster mathematical learning and contribute to the revitalization of indigenous languages.

### 3 Theoretical framework

The study is grounded in the Anthropological Theory of the Didactic (Chevallard, 1999, 2015), with specific emphasis on the concept of *institution* and the *Questioning the World Paradigm*.

#### 3.1 The concept of institution: an example between primary school and university

According to this theory, human activities occur within the framework of *institutions*, understood as stable social structures that provide material and intellectual resources for performing specific activities (e.g., teaching, medical care). Each institution (e.g., a school or hospital) establishes its own conditions, constraints, and norms that determine which practices are possible and permissible. As individuals carry out these activities and internalize these norms they become members of the institution, adopt specific roles (e.g., teacher, doctor), and participate in its shared culture. In this way, knowledge and practices are shaped by institutional rationales. A specific institution shares the same body of knowledge with all its members. We chose to focus on the teaching of geometry, acknowledging that the shared knowledge about how to prove that a figure is a square depends on the institution involved. In primary school, pupils might

validate a figure as a square by measuring its sides with a ruler or overlapping it on another. Students in secondary school, however, are expected to rely on geometric properties for validation, while at university formal demonstration becomes the primary means of validation. These differences reflect institutional norms determined by educational levels with distinct epistemological expectations at each stage.

Aligned with the multilingual mathematics education approach and “the inseparability of identity, language, and culture” (Parra et al., 2016, p. 76), schools or teaching institutions that operate in more than one language are recognized as “multicultural”. Teaching mathematics there and promoting multiculturalism through language awareness require a tailored didactics approach, such as *awakening to language* and *integrated didactic approach*, which entail using two or more languages during instruction. It also acknowledges that designing mathematical objects in different languages inherently imbues them with cultural significance. Thus, this approach not only supports mathematical activity but also fosters the development of students' identities and cultural awareness. One key component is the use of Multilingual Instructional Resources (MIR); that is, materials that integrate two or more languages to study a mathematical subject and help develop plurilingual and pluricultural competence.

Regarding this framework, we involved two teaching institutions in our study: a Master's degree program and a bilingual primary school. The main members at the university are student-researchers and researchers, while at the primary school they are pupils and indigenous teachers. A key distinction is that each institution (university, primary school) operates within a distinct pedagogical model and knowledge framework.

#### 3.2 The *Questioning the World Paradigm* and the media-milieu dialectic

The *Questioning the World Paradigm* is an inquiry-based approach that emphasizes generating meaningful questions regarding predefined content. One goal is to transform the classroom into a kind of research laboratory where students and teachers collaboratively construct and validate meaningful responses to a guiding question (denoted as *Q*). This process involves engaging with various resources, including questions derived from *Q*, existing knowledge (answers), information sources (media), data, and theoretical, discursive, and methodological tools. Collectively, this set is referred to as a *didactic milieu* (Bosch et al., 2020). *Milieu* a concept introduced by Brousseau (1997) and adapted in this framework to encompass all elements involved in the inquiry (Kidron et al., 2014; Barquero et al., 2022). The process can be described partially by showing the interaction among these elements using dialectical relations, such as the *media-milieu* dialectic (Bosch et al., 2020). It illustrates how

answers (e.g., pieces of knowledge) found in various media (academic papers, websites, indigenous teachers) are analyzed and reconstructed to develop a synthesized response, which is then integrated into the didactic milieu.

This paradigm is particularly well-suited to research-oriented education, as it supports teaching research by engaging students in the practice of actually conducting research. For this reason, it was adopted as the guiding framework for the design and analysis of three Master's courses in multilingual mathematics education. Our study community comprises student-researchers, researcher-teachers, and the guiding question; in this case, a multilingual question in mathematics education. The research question is: how do the three Master's courses enable student-researchers to design MIR for teaching mathematics in indigenous schools? To address it, each course was structured around a specific guiding question in order to progressively introduce the students to the principles of multilingual mathematics education and the design of MIR. The initial *didactic milieu* within each course included specific teaching activities, research tools, and media, such as academic papers and educational resources focused on indigenous education.

Following Barquero and Romo-Vázquez (2022), the inquiry emerged from examining multiple dimensions of researcher education, which can be categorized as follows:

- Epistemological: examining how the languages used to define mathematical knowledge give it a cultural meaning;
- Economic: analyzing how languages are used to study mathematical concepts in indigenous primary schools; and
- Ecological: considering the conditions and constraints inevitably involved in implementing MIR in indigenous schools.

## 4 Methodology

### 4.1 Research design and data sources

To address our research question, we conducted a qualitative analysis based on an intrinsic case study approach (Stake, 1995) that focused on the unique structure and aims of a researcher education program that comprises a three-course sequence embedded in a research-based Master's program in Mexico. The three courses were explicitly designed and implemented by researchers as part of this study. The main objective was to engage student-researchers in designing and implementing MIR for an indigenous primary school, in collaboration with bilingual indigenous teachers.

The research design draws on the *Questioning the World Paradigm*, particularly the *media-milieu dialectic*. We examine three main elements: how the initial didactic milieus developed in each course supported MIR design; how

these milieus evolved through contributions by the student-researchers; and the objectives that these transformations served. To illustrate these dynamics, we selected examples of MIR that demonstrate integration of plurilingual and pluricultural competences in relation to two mathematical activities—counting and designing (Bishop, 1988) and traced the developmental trajectory of the designs based on documentation from students' reports and classroom activities.

We highlight the work of one student-researcher, Atl, as an illustrative case, selected through a convenience sample (Cohen et al., 2009) due to the richness of her designs. We adopted two complementary triangulation strategies to enhance the trustworthiness of the findings: methodological triangulation and investigator triangulation (Cohen et al., 2009). The former drew on multiple primary data sources: written documents from the Master's courses (syllabi, student tasks, interim and final reports), participants' observations in both institutions, and field notes produced by the research team. Semi-structured interviews were conducted to further enrich and validate the primary data. Investigator triangulation involved several researchers in three processes: observation, data interpretation, and discussion of findings. Initially, each course leader analyzed her own course independently, focusing on its design and the associated student outcomes (e.g., activities, interim reports, final MIR). Subsequently, the team engaged in collaborative discussions to identify points of convergence and divergence, and to reach consensus on the key features of each course. This process allowed us to select the illustrative MIR examples for in-depth analysis to ensure coherence in the interpretation of findings.

We followed a direct interpretation of the data strategy (Stake, 1995) to understand how the student-researchers progressively transformed their MIR designs. The analytical categories were derived deductively from the theoretical framework and refined through inductive analyses of their reports. Each MIR was analyzed collaboratively by at least two researchers, who discussed interpretations and cross-validated findings in weekly meetings. Divergences were resolved through negotiated consensus.

Three interconnected analytical levels guided our analysis. First, we identified the theoretical, methodological, and cultural resources drawn from the Master's course from the initial *didactic milieu* present in each MIR. Second, we examined how the design evolved across successive versions of the MIR, highlighting the integration of new insights, contextual knowledge, and collaborative experiences. Third, we analyzed how each MIR fostered plurilingual and pluricultural competence through its internal structure, specifically within the three core didactic phases of the activity: launch, development, and closure. At this level, special attention was paid to how each phase encouraged pupils to engage with multiple languages, reflect on linguistic and cultural diversity, and draw connections between mathematical

**Table 1** Presentation of the three Master's courses

Course	Course 1	Course 2	Course 3
Participants	Student-researchers		
Subject	Introduction to Multilingual Mathematics Education: literature review, theoretical concepts, and methodology for collecting data. Solving worksheets on Multilingual Mathematics Education.	Design of bilingual resources for teaching mathematics Classroom implementation Data collection Analysis of data and results	
Sessions	16 sessions (48 hours) /	Includes school visits and discussions with indigenous teachers	
MIR created	Review and working on samples of MIR	Six MIR focused on the Otomi number system	Seven MIR that address spatial structuring

activities (e.g., counting and designing) and local cultural practices. This multilevel analysis enabled us to assess each MIR not only as a pedagogical tool but also as a means to support the revitalization of indigenous languages and foster multilingual awareness in mathematics education.

## 4.2 Participants: members of the two institutions

The Master's program involved three course instructors and six student-researchers (five women, one man) who completed the three-course sequence. Their professional backgrounds were diverse: one had taught at the kindergarten level, four in secondary school, and one at university. Several had indigenous ancestry. Atl, for instance, shared that her grandparents were Mayan and spoke Ta Yool Mam, a language they chose not to transmit due to fears of discrimination. Three women researchers served as course instructors: a French specialist in MIR design (first course, Poisard et al., 2014; Poisard, 2022), a Mexican with expertise in indigenous education (all three courses), and a Colombian-Mexican expert in indigenous teacher education and geometry (third course). During field visits, 14 teachers at an indigenous primary school collaborated with the student-researchers. Eleven were Otomi speakers, representing two dialectal varieties (Valle del Mezquital, Sierra de Santa Ana), one spoke Nahuatl, and two were Spanish monolinguals with basic knowledge of Otomi. The school, located in Hidalgo, the state with the largest Otomi-speaking population in Mexico, serves around 400 students from seven nearby communities. While the pupils primarily speak Spanish, many come from Otomi-speaking households. Teachers are responsible for offering a mandatory weekly, one-hour Otomi class, typically organized around an annual theme (e.g., animals, numbers, plants), accompanied by self-made cardboard books used as instructional materials. Although mathematics topics are not currently included in these Otomi-language sessions, teachers expressed

strong interest in incorporating them, provided that appropriate and culturally-relevant materials became available.

## 4.3 Our choices in designing the Master's courses

The Department of Mathematics Education of CINVESTAV, Mexico, offers a two-year M.Sc. degree with specialization in mathematics education that prepares students to become researchers and continue toward a Ph.D. program. This paper focuses on students enrolled in the first three semesters (September 2022–February 2024; Table 1).

Course 1 introduced key theoretical and methodological concepts related to multilingual mathematics education. Its initial didactic milieu centered on four foundational papers (Barton et al., 1998; Cortina, 2013; Cortina & Rojas, 2016; Planas et al., 2018), presentations by invited researchers, and samples of MIR. Students engaged with these media through readings, oral presentations, critical reflection, and writing syntheses. The articles covered diverse contexts, from Māori Mathematics Education to the development of oral number systems in Tu'un savi (Mixtec) (Cortina & Rojas, 2016), and an overview of multilingual education research (Planas et al., 2018). The resulting discussions allowed students to examine how mathematical concepts are embedded in cultural and linguistic systems, and to reflect on the epistemological implications of designing MIR. In workshop-style sessions they analyzed the structure of oral number systems in Spanish, followed by exposure to Breton, French, Māori, and English systems (Poisard et al., 2018), as part of an awakening to language approach. Exploring how base-20 structures in indigenous and Celtic languages contrast with base-10 in Spanish raised awareness of how mathematical meanings are shaped by language (Table 2). These tasks explicitly fostered plurilingual/pluricultural competence by encouraging comparisons across linguistic systems. Students also attended two talks: one on revitalization of Zapotec through

**Table 2** Analysis of the Tu'un savi oral number system

From 0 to 9			From 10 to 19, referring to 10 and 15			Tens from 20 to 90, base 20			
	In letters		In letters	What we hear	Notation		In letters	What we hear	Notation
0		10	<b>uxi</b>						
1	in	11	<b>uxi</b> in	∪	10+1				
2	uu	12	<b>uxi</b> uu	∪	10+2	20	<b>oko</b>		
3	uni	13	<b>uxi</b> uni	∪	10+3	30	<b>oko uxi</b>	∪	20+10
4	kumi	14	<b>uxi</b> kumi	∪	10+4	40	uu <b>xiko</b>	∪	2×20
5	u'un	15	<b>xe'o</b>			50	uu <b>xiko uxi</b>	∪	2×20+10
6	iñu	16	<b>xe'o</b> in	∪	15+1	60	uni <b>xiko</b>	∪	3×20
7	uja	17	<b>xe'o</b> uu	∪	15+2	70	uni <b>xiko uxi</b>	∪	3×20+10
8	una	18	<b>xe'o</b> uni	∪	15+3	80	kumi <b>xiko</b>	∪	4×20
9	iin	19	<b>xe'o</b> kumi	∪	15+4	90	kumi <b>xiko uxi</b>	∪	4×20+10

∪ = existing number-words

**Table 3** Analysis of the Otomi oral number system

From 0 to 9 From 6 to 9, <b>-to</b> = +5, referring to 5				From 10 to 19 (10+units) with the linking word <i>ma</i>			Tens from 20 to 90, base 20			
	In letters	What we hear	Notation	In letters	What we hear	Notation		In letters	What we hear	Notation
0				10	<b>r'et'a</b>					
1	n'ha			11	<b>r'et'a</b> ma r'a	n'ha → r'a	10+1			
2	yoho			12	<b>r'et'a</b> ma yoho	∪	10+2	20	<b>n'ate</b>	
3	hñu			13	<b>r'et'a</b> ma hñu	∪	10+3	30	<b>n'ate</b> ma <b>r'et'a</b>	∪
4	goho			14	<b>r'et'a</b> ma goho	∪	10+4	40	yo <b>r'ate</b>	yoho → yo-n'ate → r'ate
5	kut'a			15	<b>r'et'a</b> ma kut'a	∪	10+5	50	yo <b>r'ate</b> ma <b>r'et'a</b>	yoho → yo-n'ate → r'ate
6	<b>r'ato</b>	n'ha → r'a-	1+5	16	<b>r'et'a</b> ma <b>r'ato</b>	n'ha → r'a-	10+(1+5)	60	hñu <b>r'ate</b>	n'ate → r'ate
7	<b>yoto</b>	yoho → yo-	2+5	17	<b>r'et'a</b> ma <b>yoto</b>	yoho → yo-	10+(2+5)	70	hñu <b>r'ate</b> ma <b>r'et'a</b>	n'ate → r'ate
8	<b>hña'to</b>	hñu → hña'-	3+5	18	<b>r'et'a</b> ma <b>hña'to</b>	hñu → hña'-	10+(3+5)	80	goho <b>r'ate</b>	n'ate → r'ate
9	<b>güto</b>	goho → gü-	4+5	19	<b>r'et'a</b> ma <b>güto</b>	goho → gü-	10+(4+5)	90	goho <b>r'ate</b> ma <b>r'et'a</b>	n'ate → r'ate

∪ = existing number-words

Otto-Manguen linguistics (Vázquez-Martínez, 2024), the other on awakening to language and the development of number sense (Poisard et al., 2022). Those events emphasized how linguistic diversity and cultural context can serve as didactic resources in mathematics education.

Course 2 focused on the design methodology of MIR, applying the design-based research methodology used in indigenous education (Cortina, 2013; Cortina & Rojas, 2016). Its initial didactic milieu included these references, the study of design-based research with its four phases (preparation, design, teaching experience, retrospective analysis), field visits to an indigenous primary school, and a morphological analysis of the Otomi number system (Table 3). Students collaborated with Otomi teachers to co-design and implement the MIR, drawing on insights from linguistic, cultural, and mathematical perspectives. This design pro-

cess involved three versions of each MIR, culminating in a co-developed and field-tested version. Each student submitted two reports (interim and final) that documented the design, implementation, and retrospective analysis. These tasks required them to integrate their growing plurilingual competence through explicit use of Otomi and Spanish and pluricultural competence through collaboration with indigenous teachers to explore mathematical significance in Otomi communities. Six MIR were created in Course 2, focused mainly on the Otomi number system. They included worksheets and manipulatives that made cultural and linguistic features of Otomi visible in the mathematics classroom.

Course 3 addressed spatial structuring, reflecting on three key mathematical activities (Bishop, 1988): locating (in space), measuring, and designing. The didactic milieu combined five research papers (Gal & Linchevski, 2010; Gutiérrez

rez, 1991; Hershkowitz, 2020; Hunter & Hunter, 2024; Jones & Tzekaki, 2016), Mexican curricular materials, a research presentation by an expert, and two school visits to the same indigenous school as in Course 2. Through these materials, students explored spatial structuring processes in elementary school. Hershkowitz (2020) and Jones and Tzekaki (2016) provided broad overviews of geometry learning, while Gutiérrez (1991), and Gal and Linchevski (2010) highlighted theoretical and cognitive perspectives. Drawing on Gutiérrez' (1991) framework, students analyzed activities from existing Spanish-language educational resources to critically evaluate the spatial reasoning they required and how language shaped their accessibility. They also attended a talk on a measuring activity conducted in the Tenek indigenous community (Balderas, 2023) that illustrated how mathematical learning can be interwoven with cultural knowledge, language, and identity. That analysis reinforced the significance of designing instructional resources that not only address mathematical goals but also cultivate plurilingual and pluricultural competence to help learners navigate and draw on multiple linguistic and cultural repertoires in constructing mathematical meaning. The student-researchers first developed preliminary versions of their MIR, which they presented to indigenous teachers on the first school visit. The initial designs were then reworked through collaborative dialogue to integrate the teachers' knowledge of Otomi linguistic and cultural practices. The resulting, co-designed, versions explicitly drew on both Otomi and Spanish to promote plurilingual communication and affirm the cultural relevance of mathematical activities through local practices, objects, and narratives (oral and written). The second visit focused on implementing these MIR in the classroom. As in Course 2, students submitted interim and final reports that analyzed the development and classroom use of their MIR.

In total, seven MIR addressing spatial structuring were implemented: two on spatial orientation using school vocabulary and giving directions; four on shapes and medicinal plants, symmetry using embroidery and cubes, and animal construction with tangrams; and one on measuring area. These MIR provided authentic opportunities for pupils to engage with mathematics through their linguistic and cultural identities, thus supporting the development of plurilingual and pluricultural competences as essential components of meaningful mathematical learning.

## 5 Enriching the didactic milieu: findings from two MIR examples designed by Atl

This section presents the two MIR designed by Atl, selected for their integration of mathematical, cultural, and linguistic elements, as well as the quality of their design-based re-

search reports. Her school visits, in collaboration with teachers, enabled Atl to assess and refine the initial MIR for fifth- and third-grade groups. We examine how she enriched and extended elements of the *didactic milieu*, especially by incorporating plurilingual and pluricultural competence into the structure and content of each task.

### 5.1 An MIR to teach the Otomi oral number system in fifth grade

During the co-design process, this MIR focused on teaching the numbers from 1 to 30 in Otomi by relating them to Mayan numerals, a culturally meaningful element, as suggested by a teacher who was teaching the Maya number system. The three main activities for pupils were a launch task (*Who with whom*), some development tasks (*The rally*), and a closure task (*Join and discover*). These choices built directly on Course 1, which had prepared Atl to compare oral number systems and structure tasks around plurilingual reasoning. Following the activities and analyses of oral number systems in Course 1, Atl began by designing a task that connected three representations: numbers written in Spanish, their equivalents in Otomi, and their arithmetic compositions (Fig. 1). This triadic comparison highlights the objectives of Course 1 since it not only fosters plurilingual competence but also engages pupils in metalinguistic and cross-cultural mathematical reasoning by positioning both indigenous and national languages as legitimate vehicles for expressing mathematical ideas. This task has ten numbers between 1 and 33; '33 is greater than 30' was chosen because of its arithmetic composition in Otomi  $20+(10+3)$ , which differs from Spanish ( $30+3$ ). This was studied in Course 1 by comparing Spanish to base-20 number systems (Breton, partially French, and Tu'un savi). This allowed the pupils to compare these two languages by supporting the arithmetic composition, which is rarely used in schools to teach counting; that is, *n'ate* (20) *ma* (+) *r'eta* (10) *ma* (+) *hñu* (3), due to the regularity of that number system. To perform the task, pupils could see a table with the numbers 1-40 in Otomi and visualize the arithmetic operations to relate them to their names in Otomi and Spanish.

The task design was based on the morphological analysis of the Otomi number system (up to 100) in the Valle del Mezquital variety. It followed the six steps suggested by a paper read in Course 1 (Cortina & Rojas, 2016). The activity during this course, using Tu'un savi, induced Atl to proceed similarly with Otomi in Course 2. Hence, she searched for studies on the Otomi number system and based her analysis on the paper by Gilsdorf (2009). Atl identified the lexemes (1-5, 10, 20, 100), three additive bases (5, 10, 20), and a vigesimal multiplicative, then created a table that presented

### ¿QUIÉN CON QUIÉN?

En las siguientes columnas encontrarás números en español, hñähñu y la forma en la que se construyen. Relaciónalas de manera que encuentres las tres representaciones del mismo número. Guíate en el ejemplo:

Veinticinco	N'ate ma yoho	10+5
Siete	R'et'a ma r'ato	7
Quince	R'et'a ma hñato	10+8
Veintidós	N'ate ma k'uta	20+4
Trece	R'et'a ma hñu	3
Veinticuatro	Yoto	20+5
Treinta y tres	R'et'a ma k'uta	10+6
Dieciséis	N'ate ma r'et'a ma hñu	20+10+3
Dieciocho	Hñu	10+3
Tres	N'ate ma goho	20+2

**Fig. 1** A *Who with whom* task. Translation: (above) “In the following columns you will see numbers in Spanish and hñähñu<sup>1</sup> and how they are constructed. Relate them to find the three representations of the same number. Follow the example”

the same process as in Course 1, highlighting the arithmetical composition (Poisard, 2022).

Atl recognized linguistic and mathematical regularities; for example, that *n'ate* is not used to express 20 when it functions as a multiplicative base, where *r'ate* is preferred. These distinctions became part of the MIR's explicit instructions to reinforce meta-linguistic awareness and promote culturally grounded reasoning. In the preparation phase of design-based research, studied in Course 2, Atl engaged with course literature (e.g., Barton et al., 1998; Cortina & Rojas, 2016) and enriched the didactic milieu by incorporating new sources, such as Gilsdorf (2009) and Campbell and Cruz (2009). This illustrates how the framework of the courses encouraged her to expand the milieu with additional references and tools. This selective expansion of theoretical and empirical references illustrates her active role in co-constructing the research-based learning environment with the indigenous teachers. In doing so, Atl not only designed an effective multilingual task but also helped reposition indigenous language structures as valuable tools for mathematical meaning-making.

The development tasks consisted of a three-station activity referred to as *The rally*, that Atl described in her Course 2 report (authors' translation):

**Build the number:** this involved building a number indicated by five dice. First, pupils formed the name of the number in Otomi using cards, then represented the number in Mayan using foam rectangles and circles.

**Make up a story:** using cards with images and names in Otomi, pupils composed mathematical problems of addition and subtraction, writing the numbers and nouns in Otomi.

<sup>1</sup>Hñähñu is the endonym used by indigenous communities to refer to the Otomi language.

**Jump, jump!:** pupils took turns skipping rope while reciting the numbers 1 to 36 in Otomi (Atl's Course 2 report, 2023, p. 9).

Each task was designed to reinforce plurilingual competence by engaging pupils in mathematical reasoning across languages and representations, as seen in Course 1. This was made more precise through the co-design process. The *Build the number* task facilitated memorizing and recognizing Otomi number names while connecting them to Mayan numerals. The language cards allowed pupils to construct number expressions in Otomi through arithmetic composition; for example, the number 12 was written as *r'et'a mayojo* (10+2). In the *Make up a story* activity, the use of culturally relevant vocabulary drawn from themes covered in the Otomi class and materials such as the cardboard books created by the indigenous teachers, promoted pluricultural awareness. The *Jump, jump!* task, finally, supported memorization through physical engagement and rhythmic repetition, helping pupils internalize the Otomi number sequence from 1 to 36.

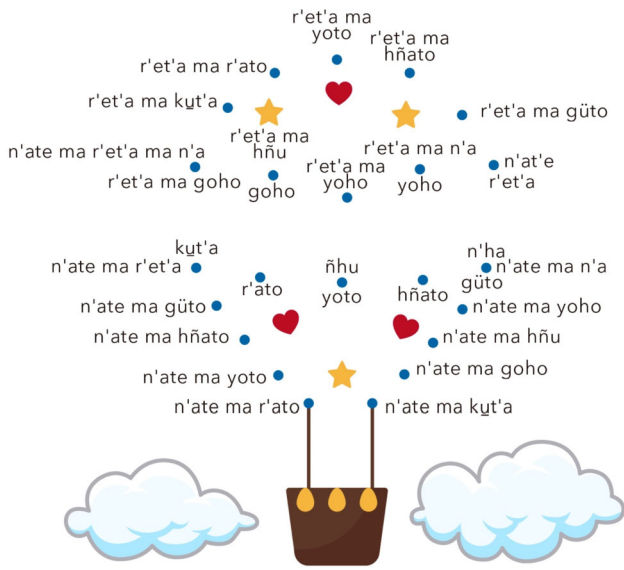
While designing these activities, Atl engaged deeply with both instructional and cognitive considerations for teaching oral number systems in indigenous languages. To do so, she drew on Gelman and Gallistel's (1978) five counting principles (one-to-one correspondence, stable-order, cardinality, abstraction, order irrelevance) to structure tasks and support conceptual development. In *Who with whom* and *Build the number*, pupils practiced one-to-one correspondence by mapping number names and numerals across languages. In *Jump jump!*, they reinforced stable order by reciting the Otomi sequence. In *Make up a story*, they explored cardinality by combining sets. Throughout these tasks, abstraction and order irrelevance supported a flexible transfer of strategies across contexts.

The closure task, *Join and discover*, invited the pupils to connect dots on a worksheet to reveal a hidden figure (Fig. 2), similar to the activity (in base-6) proposed to student-researchers to draw a flower in Course 1. Unlike traditional versions using Arabic numerals, this task presented the number names in Otomi. As Atl explained: “It's easy to identify when a shortcut was taken, so it's necessary to have good knowledge of the sequence of numbers to complete [the task]” (Atl's Course 2 report, 2023, p. 39). This task further consolidated pupils' understanding of the stable-order principle, while providing a playful, summative assessment of their engagement with the Otomi number sequence.

In Course 1, the university provided students with examples of mathematical activities on numbers, comparing oral number systems, and key references. Combined with the co-design process in the primary school, in Course 2, this enabled the creation of a relevant MIR, whose design exhibited a sophisticated integration of mathematical structure (arithmetic composition), cultural context (indigenous

**UNE Y DESCUBRE**

Los puntos siguiendo la numeración en hñähñu y descubre la figura.



**Fig. 2** The *Join and discover* task. Translation: (above) “Join the dots following hñähñu numeration to discover the hidden figure”

school), and plurilingual competence (comparison of Spanish and Otomi). This shows how Atl significantly enriched the didactic milieu. Interestingly, Gelman and Gallistel’s (1978) counting principles were not part of the media introduced initially in the Master’s courses, so by incorporating them Atl extended the didactic milieu to underscore how student-researchers can contribute to its evolution by integrating relevant theoretical tools that support mathematical learning and the revitalization of indigenous languages. In this way, the MIR demonstrated not only the product of design, but also how the interplay between course preparation and school collaboration enabled Atl to enrich the didactic milieu with plurilingual and pluricultural dimensions.

**5.2 An MIR for teaching Otomi geometrical design in third grade**

This MIR, also designed by Atl, focused on creating an animal-shaped figure (a local animal) composed of tangram puzzle pieces (geometric shapes). The instructional goal was to teach the names and characteristics of geometric shapes through the Otomi language by integrating vocabulary related to animals (non-geometric forms), the number of sides, colors, and cultural references, using various concrete materials (Fig. 3). These choices were directly supported by Course 3, which emphasized spatial structuring and provided analytical tools for studying pupils’ difficulties with visualization.

Atl’s approach explicitly sought to foster plurilingual competence by promoting comparisons of geometric shapes between Spanish and Otomi vocabulary. By integrating cultural references (e.g., local animals, Otomi names, classroom resources), she simultaneously activated pluricultural competence, affirming that “[the] pupils’ context and culture must permeate the [proposed] didactic activities.” This intention revealed ideas from Sharma (2023) and Barquera (2003) that Atl integrated beyond the didactic milieu of Course 3. Her autonomous decisions were shaped by collaborative course discussions with all participants, where the influence of the indigenous language on the design process was analyzed carefully. The sequence of activities was: a launch task (recognizing shapes, *Why do we call it that?*), some development tasks (*Construct a shape*), and a closure task (*Glue your figure*).

On the launch task, Atl introduced a triangle, a square, and a parallelogram, “each one positioned differently from how they are usually presented, without repeating the same position” (Atl’s Course 3 report, 2024, p. 5). In that way, she encouraged the pupils to rotate the shapes. Names in both Spanish and Otomi using the vocabulary proposed by Barquera (2003) and validated by the schoolteachers (Fig. 4). While triangle and square had Otomi equivalents, ‘parallelogram’ required using a Spanish loanword that provided an opportunity to highlight language gaps and creative translanguaging practices.

Atl conducted a linguistic analysis that linked geometric shapes in both languages to their defining properties, such as the number of sides and angles, using familiar number words from both vocabularies to support conceptual understanding. This strategy fostered metalinguistic awareness to allow the pupils to recognize how different languages encode mathematical ideas. Rather than treating the two languages as parallel codes, the task positioned them as mutually reinforcing resources to support the students’ understanding of geometric shapes (Jones & Tzekaki, 2016; Hershkowitz, 2020) while simultaneously encouraging intercultural reflection (Bishop, 1988). Two mathematical activities were emphasized: a focus on ‘objects compared by properties of form’ that involved interpreting figural information and manipulating and transforming visual representations (Bishop, 1983); and introducing ‘congruence’ intuitively through superposition and movement. The latter is central to designing activities that promote interaction in both mathematical and environmental contexts. Instructors encouraged Atl to engage in a careful analysis of the geometric notions and potential difficulties highlighted in the literature studied during the course, such as the static *versus* dynamic position of shapes for recognizing invariant properties. Here, discussions in Course 3 and tutoring enabled Atl to reorganize the sequence of subtasks by degree of conceptual complexity to ensure progression from recognition to abstraction while maintaining the focus on cultural references.



Fig. 3 Examples of the concrete materials used (Alt’s Course 3 report)

Spanish	Otomí
Triángulo ▶ tri-ángulo ▶ tres ángulos * [triangle ▶ tri-angle ▶ three angles]	Hñu hmi ▶ tres caras * [three faces]
Cuadrado ▶ cuadra-do ▶ cuatro * [square]	goho hmi ▶ cuatro caras * [four faces]
Paralelogramo - Romboide Lados opuestos paralelos * [parallelogram - opposite sides that are parallel]	Paralelogramo * [parallelogram]

Fig. 4 The geometric shapes (2D) in Spanish and Otomi (authors’ own work based on Alt’s Course 3 report data)

The development tasks included constructing increasingly complex figures of local animals using tangram pieces (Fig. 5). *Assemble the animal* required reproducing a 1:1 color photograph composed of visible tangram shapes. *Create your dark animal* entailed reconstructing the same animal from a 1:4 solid black silhouette using all pieces. Two intermediate scaffolding activities were provided for the pupils who needed additional support (a 1:1 scale showing the component pieces).

These instructional decisions were guided by Gutiérrez’ (1991, p. 5) research on visual discrimination: that is, “comparing several objects by identifying their visual similarities and differences”, and visual memory, understood as “the ability to recall visual features and spatial positions of objects no longer in view”, accompanied by the presence, or absence, of these skills in activities adapted from Mexican curricular materials, a series of topics discussed in Course 3. Based on those classes, challenges in students’ visualization skills in learning geometry were identified. In response, Atl independently integrated additional research on how pupils transition from a complete geometric configura-

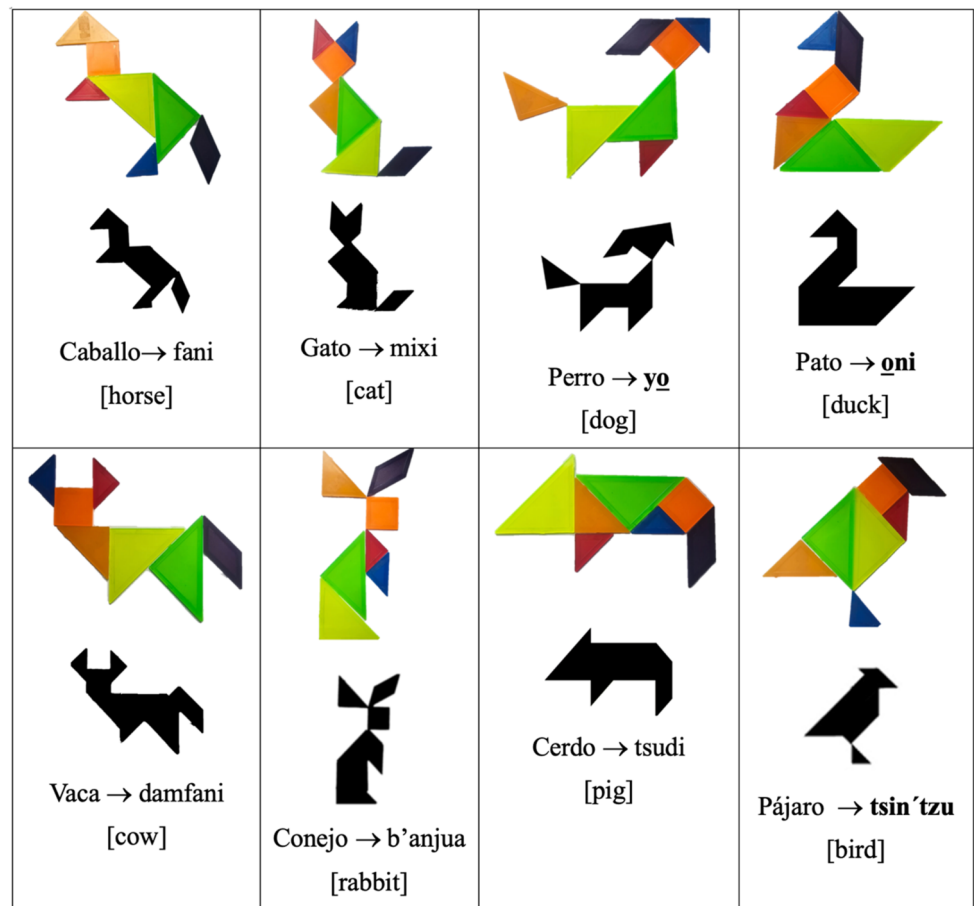
tion to its constituent parts. Here, she incorporated elements from Sarama and Clements’ (2019) learning trajectory for bidimensional shape development, as she mentioned in her final report.

The sequencing of Atl’s subtasks fostered increasing the complexity of the pupils’ actions (such as scaling, color, visibility, and maintaining shape) as they developed skills aligned with the trajectory proposed by Sarama and Clements (2019). These activities highlighted her intention to promote movements on a plane (rotation, reflection), (de)composition, and spatial relations. Atl encouraged the students to design mathematical tasks related to “manufactured” objects, which all cultures create for various purposes, with a focus on abstracting shapes from the natural environment (Bishop, 1988, p. 39). Her integrated vocabulary linked to local fauna, the properties of shape, and Otomi numbers, created a rich multicultural environment where cultural knowledge and language were positioned as assets in the mathematics classroom.

For the closure task, Atl created three subtasks to promote a final reflection: (i) reassemble and glue the animal using colored tangram stickers that varied in color and size from the original puzzle; (ii) write a descriptive sentence using both Spanish and Otomi vocabulary to name a geometric shape (e.g., “The horse has a purple parallelogram as its tail”); and (iii) compare the assembled figures with those of their peers and discuss which one covers a larger area and why. In this final task, pupils deepened their plurilingual competence by constructing bilingual mathematical sentences that combined Otomi and Spanish vocabulary with geometric reasoning. Atl chose sticker colors based on Otomi terms studied previously. By describing and comparing shapes using both languages, pupils activated their plurilingual resources as a part of their geometrical reasoning to emphasize the connection among language, shape recognition, and comparative reasoning.

Pluricultural competence was further modeled through the respectful and resourceful use of the Otomi language

**Fig. 5** Representations of the tangrams of local animals (authors' own work based on Atl's Course 3 report data)



—though it is not Atl's mother tongue— to demonstrate a commitment to inclusive, intercultural teaching practices. Tutoring and collaborative discussions around each MIR activity in Course 3 helped Atl reorganize her tasks in terms of their conceptual complexity, strengthen geometric reasoning, and create opportunities for visual abstraction and geometric generalization (e.g., focusing on shape, rather than color or orientation). In these ways, her work supports both mathematical learning and intercultural sensitivity (Avila, 2014; Sharma, 2023). The co-design with the indigenous teachers ensured cultural and linguistic validity, adapted the content to the pupils' context, made indigenous knowledge visible, and legitimized indigenous languages as tools for geometric reasoning. In this process, global course exchanges and Balderas' experiences in bilingual material design further shaped Atl's reflective decisions as she developed activities that focused on geometry. She emphasized the role of language in mathematical tasks, noting that “using language to communicate mathematical ideas, to name and describe the characteristics of geometric figures, is a challenge” (Atl's Course 3 report, 2024, p. 12), while affirming that indigenous languages can function as didactic and cultural resources that expand access to geometry

and foster engagement with both local and global ways of knowing.

## 6 Discussion

This study examined how Master's courses in mathematics education enabled student-researchers to design MIR that integrated mathematical, cultural, and linguistic elements. Atl's case illustrates both the outcomes of MIR design and, more importantly, the institutional and collaborative processes that made such designs possible.

Results show that the co-design process with teachers ensured the cultural and linguistic validity of each MIR. These collaborations adapted task content to pupils' contexts, supported the visibility of indigenous knowledge in formal schooling, and contributed to revitalizing local languages as legitimate media for mathematical reasoning. This echoes findings by Avila (2014), Cortina (2013), and Cortina and Rojas (2016) on linguistic analysis, and then extends them by showing how Master's level preparation explicitly enabled these student-researchers to apply such analyses in collaboration with indigenous teachers.

Each MIR also reveals how multilingual and multicultural dimensions were mobilized in mathematics education. By positioning indigenous languages alongside Spanish as vehicles for mathematical meaning-making, the designs reinforced pupils' metalinguistic awareness and intercultural reflections. These results complement Rosa et al. (2017) and Meaney and Trinick (2020) by moving beyond ethnomathematics approaches to adapt existing tasks in ways that are less time-consuming yet effective for revitalizing indigenous languages in the classroom. Results also show the viability of MIR by integrating the primary indigenous school's pedagogical model of launch, development, and closure.

At the institutional level, the Master's program structured opportunities for students to engage with theory, research tools, and school-based practice in an integrated manner. In this sense, our findings reinforce Barwell et al. (2016) and Halai and Clarkson (2016) for they demonstrate that researcher education can become a site for advancing diversity in mathematics education. The flexibility of the *Questioning the World Paradigm* further encouraged student autonomy, by allowing them to extend the didactic milieu with additional resources, such as new linguistic analyses or theoretical principles.

Atl's reflections illustrate the value of research-based, inquiry-oriented practice, showing how researcher education –when grounded in a robust didactic framework and connected to local realities– can empower future researchers to design MIR that contribute meaningfully to multilingual mathematics education. Our aim was not to assess the full range of research competencies developed, but the study does highlight how MIR design constitutes a first step toward preparing researchers to work at the intersection of linguistic, cultural, and mathematical diversity. This focus on MIR as a first step also clarifies the scope of our work, as we discuss below.

An additional point concerns the scope of our analysis. We chose not to focus on indigenous mathematics itself (e.g., cosmogonies, community-specific practices), as this would require a depth of ethnographic engagement that goes beyond the limits of this study. As Hunter and Hunter (2024) argue, developing indigenous pedagogical models calls for long-term collaboration with communities to avoid imposing external categories. In our case, we considered that revitalization through MIR was a necessary first step because it introduces indigenous languages into classroom mathematics immediately and provides a foundation for later, more ambitious, projects that can analyze and integrate indigenous mathematical practices. This positioning also responds to the risk of subordinating cultural elements to Western frameworks. By foregrounding language revitalization, we sought to respect cultural particularities while still contributing to multilingual and multicultural mathematics education, in line with current approaches to indigenous education in Latin America (Parra et al., 2016).

## 7 Conclusion

This study shows how Master's courses in mathematics education can provide future researchers with opportunities to design Multilingual Instructional Resources (MIR) that integrate mathematical, cultural, and linguistic dimensions. Atl's case illustrates how the courses, grounded in the Anthropological Theory of the Didactic and the *Questioning the World Paradigm*, enabled student-researchers to enrich the didactic milieu through the use of multilingual resources, co-design with indigenous teachers, and the selective extension of didactic milieu, including theoretical and methodological tools.

Our findings highlight three main contributions. First, they demonstrate that research education can become a site for advancing multilingual and multicultural approaches by going beyond contexts where multilingualism is treated as an external or optional dimension. Second, they show the viability of MIR for indigenous schools, where their design –organized around launch, development, and closure– ensured immediate usability and supported the revitalization of indigenous languages as legitimate media for mathematical teaching and learning. Third, the study underscores the importance of institutional preparation combined with co-design processes as, together, they made it possible to integrate cultural references, local practices, and multilingual resources into mathematical tasks.

However, the study has some limitations, mainly its focus on one sole case study and the fact that it does not assess the full range of research competencies developed by the student-researchers. Moreover, it does not provide an in-depth analysis of indigenous mathematical knowledge because that requires long-term ethnographic collaboration. Instead, our work emphasizes the revitalization of indigenous languages through MIR as a necessary step in establishing a foundation for later research that could engage more directly with indigenous mathematical practices.

In closing, future work must address how MIR design can be scaled across diverse schools and languages, and how researcher education programs can consolidate broader research competencies to accompany MIR. The revitalization of indigenous languages is not only essential for their preservation but also for appreciating the cultural richness that multilingual mathematics education brings to teaching and learning. In this sense, mathematics education plays a decisive role in affirming that indigenous languages are powerful means for doing mathematics, both in classrooms and across broader educational contexts.

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## Declarations

**Competing Interests** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## References

- Adler, J. (2002). *Teaching mathematics in multilingual classrooms*. Kluwer Academic Publishers.
- Avila, A. (2014). La etnomatemática en la educación indígena: Así se concibe, así se pone en práctica. *Revista Latinoamericana de Etnomatemática*, 7(1), 19–49. <https://www.revista.etnomatematica.org/index.php/RevLatEm/article/view/104>.
- Barquero, B., & Romo-Vázquez, A. (2022). Study and research for teacher education: Some advances on teacher education in the paradigm of questioning the world. In Y. Chevallard, B. Barquero, M. Bosch, I. Florensa, J. Gascón, P. Nicolás, & N. Ruiz-Munzón (Eds.), *Advances in the anthropological theory of the didactic* (pp. 125–137). Birkhäuser. [https://doi.org/10.1007/978-3-030-76791-4\\_11](https://doi.org/10.1007/978-3-030-76791-4_11).
- Balderas, F. (2023). *La medición con unidades de medida no convencionales en la comunidad de El Ibte. Una propuesta para su recuperación y vinculación en el aula*. Unpublished master's thesis. México: Universidad Pedagógica Nacional.
- Barquera, E. (2003). *Numeración y Medida en la Cultura Otomí*. Unpublished master's thesis. México: Cinvestav.
- Barquero, B., Bosch, M., Candy, J., Florensa, I., Gascón, J., et al. (2022). Points of contact between the ATD and the TDS: Questions raised by the implementation of study and research paths. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the twelfth congress of the European society for research in mathematics education* (pp. 2930–2937). ERME/Free University of Bozen-Bolzano.
- Barton, B. (1996). Making sense of ethnomathematics: Ethnomathematics is making sense. *Educational Studies in Mathematics*, 31(1–2), 201–233. <https://doi.org/10.1007/BF00143932>.
- Barton, B., Fairhall, U., & Trinick, T. (1998). Tikanga reo tātai: Issues in the development of a Māori mathematics register. *For the Learning of Mathematics*, 18(1), 3–9.
- Barwell, R. (2018). From language as a resource to sources of meaning in multilingual mathematics classrooms. *The Journal of Mathematical Behavior*, 50, 155–168. <https://doi.org/10.1016/j.jmathb.2018.02.007>.
- Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J., Planas, N., Setati-Phakeng, M., Valero, P., & Villavicencio Ubilús, M. (Eds.) (2016). *Mathematics education and language diversity: The 21st ICMI study*. Springer. <https://doi.org/10.1007/978-3-319-14511-2>.
- Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Kluwer Academic Publishers.
- Bishop, A. J. (1983). Space and geometry. In R. A. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 173–203). Academic Press.
- Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Kluwer Academic Publishers.
- Blanchet, P., & Kervran, M. (2016). *Langues minoritaires locales et éducation à la diversité: Des dispositifs didactiques à l'épreuve*. L'harmattan.
- Bosch, M., Chevallard, Y., Garcia, F. J., & Monhagan, J. (2020). *Working with the anthropological theory of the didactic. A comprehensive casebook*. Routledge.
- Campbell, E., & Cruz, E. (2009). El sistema numérico del protochato. In *Memorias del Congreso de Idiomas Indígenas de Latinoamérica-IV* (pp. 1–35). University of Texas at Austin.
- Candelier, M., Camilleri-Grima, A., Castellotti, V., de Pietro, J.-F. R., Lőrincz, I., Meißner, F.-J., Noguerol, A., & Schröder-Sura, A. (2012). *A framework of reference for pluralistic approaches to languages and cultures*. Council of Europe.
- Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en Didactique des Mathématiques*, 19(2), 221–266.
- Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *Proceedings of the 12th international congress on mathematical education* (pp. 173–187). Springer. [https://doi.org/10.1007/978-3-319-12688-3\\_13](https://doi.org/10.1007/978-3-319-12688-3_13).
- Cohen, L., Manion, L., & Morrison, K. (2009). *Research methods in education*. Routledge.
- Consejo Nacional de Evaluación de la Política de Desarrollo Social (CONEVAL) (2022). *Educación para la población indígena en México: El derecho a una educación intercultural bilingüe*. [https://www.coneval.org.mx/InformesPublicaciones/Documents/Educacion\\_poblacion\\_indigena.pdf](https://www.coneval.org.mx/InformesPublicaciones/Documents/Educacion_poblacion_indigena.pdf).
- Cortina, J., & Rojas, C. (2016). Didáctica de los sistemas de numeración de las lenguas indígenas: el diseño de una propuesta para escuelas primarias unidocentes. *Revista Latinoamericana de Etnomatemática*, 9(2), 103–126. <https://www.revista.etnomatematica.org/index.php/RevLatEm/article/view/303>.
- Cortina, J. L. (2013). Supporting indigenous students' understanding of the numeration system of their first language. *Mathematics Education Research Journal*, 25(1), 23–42. <https://doi.org/10.1007/s13394-012-0043-6>.
- Coste, D., Moore, D., & Zarate, G. (1997/2009). *Plurilingual and pluricultural competence. Studies towards a common European framework of reference for language learning and teaching*. Council of Europe.
- de Abreu, G. (2014). Cultural diversity in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (1st ed., pp. 125–129). Springer. [https://doi.org/10.1007/978-94-007-4978-8\\_37](https://doi.org/10.1007/978-94-007-4978-8_37).
- D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics*, 5(1), 44–48.
- Erath, K., Ingram, J., Moschkovich, J., & Prediger, S. (2021). Designing and enacting instruction that enhances language for mathematics learning: A review of the state of development and research. *ZDM – Mathematics Education*, 53(2), 245–262. <https://doi.org/10.1007/s11858-020-01213-2>.
- Gal, H., & Linchevski, L. (2010). To see or not to see: Analyzing difficulties in geometry from the perspective of visual perception. *Educational Studies in Mathematics*, 74(2), 163–183. <https://doi.org/10.1007/s10649-010-9232-y>.
- Gates, P. (2020). Equity and access in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (2nd ed., pp. 279–283). [https://doi.org/10.1007/978-3-030-15789-0\\_58](https://doi.org/10.1007/978-3-030-15789-0_58).

- Gelman, R., & Gallistel, C. R. (1978). *The child's concept of number*. Harvard University Press.
- Gilsdorf, T. E. (2009). Mathematics of the Hñähñü: The Otomies. *Journal of Mathematics and Culture*, 4(1), 84–105.
- Gutiérrez, A. (1991). Procesos y habilidades en visualización espacial. In *Memorias del tercer encuentro sobre investigación en Educación Matemática: Geometría* (pp. 44–59). España.
- Halai, A. & Clarkson, P. (Eds.) (2016). *Teaching and learning mathematics in multilingual classrooms*. Sense Publishers.
- Hélot, C. (2008). Awareness raising and multilingualism in primary education. In J. Cenoz & D. Gorter (Eds.), *Encyclopedia of language and education* (pp. 1–14). Springer. [https://doi.org/10.1007/978-3-319-02240-6\\_29](https://doi.org/10.1007/978-3-319-02240-6_29).
- Hershkowitz, R. (2020). Shape and space: Geometry teaching and learning. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 774–779). Springer. [https://doi.org/10.1007/978-3-030-15789-0\\_138](https://doi.org/10.1007/978-3-030-15789-0_138).
- Hunter, J., & Hunter, R. (2024). Weaving together the threads of Indigenous knowledge and mathematics. *Educational Studies in Mathematics*, 116(3), 501–518. <https://doi.org/10.1007/s10649-023-10256-7>.
- Jones, K., & Tzekaki, M. (2016). Research on the teaching and learning of geometry. In A. Gutiérrez, G. Leder, & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education: The journey continues* (pp. 109–149). Sense Publishers. [https://doi.org/10.1007/978-94-6300-561-6\\_4](https://doi.org/10.1007/978-94-6300-561-6_4).
- Kidron, I., Artigue, M., Bosch, M., Dreyfus, T., & Haspekian, M. (2014). Context, milieu, and media-milieus dialectic: A case study on networking of AiC, TDS, and ATD. In *Advances in mathematics education. Networking of theories as a research practice in mathematics education* (pp. 153–177). Springer. [https://doi.org/10.1007/978-3-319-05389-9\\_10](https://doi.org/10.1007/978-3-319-05389-9_10).
- Meaney, T., & Trinick, T. (2020). Indigenous students in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (2nd ed., pp. 369–373). Springer. [https://doi.org/10.1007/978-3-030-15789-0\\_76](https://doi.org/10.1007/978-3-030-15789-0_76).
- Parra, A., Mendes, J. R., Valero, P., & Villavicencio Ubillús, M. (2016). Mathematics education in multilingual contexts for the indigenous population in Latin America. In R. Barwell, et al. (Eds.), *Mathematics education and language diversity* (pp. 47–66). Springer. [https://doi.org/10.1007/978-3-319-14511-2\\_4](https://doi.org/10.1007/978-3-319-14511-2_4).
- Planas, N. (2018). Language as resource: A key notion for understanding the complexity of mathematics learning. *Educational Studies in Mathematics*, 98(3), 215–229. <https://doi.org/10.1007/s10649-018-9810-y>.
- Planas, N., Morgan, C., & Schütte, M. (2018). Mathematics education and language: Lessons and directions from two decades of research. In *Developing research in mathematics education* (pp. 196–210).
- Poisard, C., Kervran, M., Le Pipec, E., Alliot, S., Guedet, G., Hili, H., Jeudy-Karakoç, N., & Larvol, G. (2014). Enseignement et apprentissage des mathématiques à l'école primaire dans un contexte bilingue breton-français. *Spirale - Revue de Recherches en Éducation*, 54, 129–150. [https://www.persee.fr/doc/spira\\_0994-3722\\_2014\\_num\\_54\\_1\\_1040](https://www.persee.fr/doc/spira_0994-3722_2014_num_54_1_1040). Association pour la Recherche en Éducation (ARED).
- Poisard, C., Kervran, M., Surget, E., & Moumin, E. (2018). Étudier des numérations orales en classe: quels savoirs mathématiques et langagiers? *Au Fils des maths*, 528, 38–45.
- Poisard, C. (2022). Mallette de ressources pour la classe CLEAM. Blog Fabric@maths. <https://fabricamaths.hypotheses.org/mallette-cleam>.
- Poisard, C., D'hondt, D., Moumin, E., & Surget, E. (2022). L'éveil aux langues pour travailler sur la construction du nombre à l'école. In C. Hache & C. Mendonça Dias (Eds.), *Plurilinguisme et enseignement des mathématiques*. Éditions Lambert Lucas.
- Rosa, M., Shirley, L., Gavarrete, M. E., & Alanguí, W. V. (Eds.) (2017). *Ethnomathematics and its diverse approaches for mathematics education. ICME-13 monographs*. Springer.
- Sarama, J., & Clements, D. H. (2019). Learning trajectories in early mathematics education. In D. Siemon, T. Barkatsas, & R. Seah (Eds.), *Researching and using progressions (trajectories) in mathematics education* (pp. 32–55). Brill. [https://doi.org/10.1163/9789004396449\\_002](https://doi.org/10.1163/9789004396449_002).
- Sharma, S. (2023). Promoting statistical thinking in year 12 multilingual classrooms: A collaborative study. *Mathematics Education Research Journal*, 35(4), 1–23. <https://doi.org/10.1007/s13394-023-00472-y>.
- Stake, R. E. (1995). *The art of case study research*. Sage.
- Trouche, L., Adler, J., & Remillard, J. T. (2023). Conceptualizing teachers' interactions with resources in crossing languages and cultures. *ZDM – Mathematics Education*, 55(3), 497–519.
- Vázquez-Martínez, N. (2024). *Construcciones multiverbales monoclusales en el zapoteco de San Pedro Mixtepec*. Doctoral dissertation, Centro de Investigaciones y Estudios Superiores en Antropología Social (CIESAS), México. <http://ciesas.repositorioinstitucional.mx/jspui/handle/1015/1719>.

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