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Two-way Coupling of Fluid–Structure Interaction for Elastic Magneto-Swimmers: A Finite Element ALE Approach

Christophe Prud'homme* Vincent Chabannes* Laetitia Giraldi†

Agathe Chouippe‡ Céline Van Landeghem*

Abstract

Artificial micro-swimmers actuated by external magnetic fields hold significant promise for targeted biomedical applications, including drug delivery and micro-robot-assisted therapy. However, their dynamics remain challenging to control due to the complex nonlinear coupling between magnetic actuation, elastic deformations, and fluid interactions in confined biological environments. Numerical modeling is therefore essential to better understand, predict, and optimize their behavior for practical applications. In this work, we present a comprehensive finite element framework based on the Arbitrary Lagrangian–Eulerian formulation to simulate deformable elastic micro-swimmers in confined fluid domains. The method employs a full-order model that resolves the complete fluid dynamics while simultaneously tracking swimmer deformation and global displacement on conforming meshes. Numerical experiments are performed with the open-source finite element library `Feel++`, demonstrating excellent agreement with experimental data from the literature. The validation benchmarks in both two and three dimensions confirm the accuracy, robustness, and computational efficiency of the proposed framework, representing a foundational step toward developing digital twins of magneto-swimmers for biomedical applications.

Keywords: Fluid-structure interaction; Finite element method; Arbitrary Lagrangian–Eulerian framework; Two-way coupling; Elasto-magneto swimmer; `Feel++`.

1 Introduction

In recent years, significant attention has been devoted to the development of artificial micro-swimmers for biomedical applications, such as targeted drug delivery or minimally invasive diagnostics [10]. One promising approach involves actuating these micro-swimmers using external magnetic fields, often generated by MRI technology [17], to enable wireless control inside the human body. However, controlling flagellated swimmers in such conditions remains a challenging task. These systems are typically non-homologous, and their dynamics are often poorly controllable [19, 2] due to the complex coupling between magnetic actuation, elastic deformations, and hydrodynamic interactions at low Reynolds number. This complexity motivates the development of accurate numerical models capable of capturing the coupled fluid-elasto-magneto interactions governing their dynamics.

Modeling the motion of micro-swimmers in fluids involves the coupling of fluid dynamics with rigid and, eventually, elastic body mechanics, as well as complex fluid–structure interactions.

*IRMA, University of Strasbourg & CEMOSIS, France.

†Inria Sophia Antipolis Méditerranée, Calisto Team, France.

‡Université de Strasbourg, CNRS ICUBE UMR 7357, France.

Various numerical approaches have been developed to address this problem, each offering a trade-off between computational efficiency and modeling accuracy.

At low Reynolds numbers, where inertial effects are negligible, simplified models, such as the *Resistive Force Theory* [20, 15, 23] and the *Slender Body Theory* [14, 4, 26, 24], provide computationally efficient approximations. These methods are particularly useful for slender filaments, but they rely on strong geometric assumptions and do not fully capture near-field hydrodynamic interactions or swimmer–boundary effects.

To improve accuracy, *Boundary Integral Methods* [39, 42] have been introduced, offering a more refined description of fluid-structure coupling without discretization of the entire fluid domain [38]. These methods rely on Green’s functions of the Stokes equations [9], which are available in closed form and allow the velocity of the fluid to be expressed as an integral over the swimmer’s surface. However, this approach is restricted to the Stokes regime and cannot be generalized to inertial flows. In addition, the singular behavior of the Green functions near the boundary requires regularization techniques to ensure numerical stability and precision [30, 22, 34].

When more complex geometries, large deformations, or interactions with boundaries are involved, full discretization of both fluid and swimmer domains becomes necessary. In such cases, the *Finite Element Method* provides a versatile framework, particularly when combined with techniques like the *Immersed Boundary Method* [5, 33, 27, 6, 7] or *CutFEM* [32, 21, 11]. These approaches allow the simulation of moving rigid swimmers immersed in a fluid, but they introduce additional challenges, such as force interpolation errors and numerical instabilities near the fluid–structure interface. Overall, while existing models provide valuable insight into micro-swimmer dynamics, many approaches that include both fluid- and elastic-solid interactions are limited to configurations where the solid undergoes only passive deformations, without rigid-body motion. As a result, these models do not capture the full dynamics of a self-propelled swimmer, particularly in scenarios where elastic deformation allows the swimmer to push the fluid, enabling its displacement.

In this work, we introduce a finite element method based on the *Arbitrary Lagrangian–Eulerian* framework to compute the displacement of deformable swimmers immersed in viscous flows within geometrically complex domains [13, 37, 8, 40, 41]. This approach resolves the full fluid dynamics while tracking both swimmer deformation and global motion on a conforming mesh. Compared to immersed boundary methods, the explicit tracking of the interface during the computation allows for precise evaluation of all physical quantities at the fluid–structure boundary. Thus, it provides an accurate description of the fluid–structure interface and can be extended to complex fluids relevant to biological applications.

The paper is organized as follows. In Section 2 we introduce the mathematical models, starting with the fluid equations, then describing the swimmer through rigid-body motion and elasticity, and finally presenting the coupled fluid–structure interaction problem. Section 3 details the spatial and temporal discretization of all these equations. Section 4 focuses on computational strategies, including the remeshing procedure, and the preconditioning technique. Section 5 presents the numerical results, consisting of validation tests, all performed using the open-source finite element library *Feel++* [16]. Finally, section 6 provides conclusions and perspectives.

2 Mathematical modeling

This section introduces the mathematical model for the fluid–magneto-swimmer interaction problem. We first describe the hydrodynamics of incompressible Newtonian fluids and the dynamics of elastic bodies, governed by hyper-elasticity equations. We then present the coupling between these two components, taking into account the rigid motion of the swimmer.

2.1 Fluid model

Let $\mathcal{F}^t \subset \mathbb{R}^d$, with $d = 2, 3$ denoting the spatial dimension, be the region occupied by the fluid at time $t \in]0, T]$, where $T > 0$ is the final time. In what follows, we use the notation with superscript t to emphasize that the corresponding quantity is time-dependent. The physical properties of the incompressible Newtonian fluid are its dynamic viscosity $\mu_{\mathcal{F}} \in \mathbb{R}^+$ and its density $\rho_{\mathcal{F}} \in \mathbb{R}^+$. Their hydrodynamics are described by the non-linear Navier-Stokes equations. These equations are formulated in the Eulerian framework and describe the time evolution of the velocity field $u^t :]0, T] \times \mathcal{F}^t \rightarrow \mathbb{R}^d$, and the pressure field $p^t :]0, T] \times \mathcal{F}^t \rightarrow \mathbb{R}$ of the fluid.

The Navier-Stokes equations are given at time $t > 0$ by

$$\begin{cases} \rho_{\mathcal{F}} \left(\partial_t u^t + (u^t \cdot \nabla) u^t \right) - \nabla \cdot \sigma(u^t, p^t) = 0_{\mathbb{R}^d} & \text{in } \mathcal{F}^t, \\ \nabla \cdot u^t = 0 & \text{in } \mathcal{F}^t, \\ u^t = 0_{\mathbb{R}^d} & \text{on } \partial\mathcal{F}_D^t, \\ \sigma(u^t, p^t) n_{\mathcal{F}}^t = 0_{\mathbb{R}^d} & \text{on } \partial\mathcal{F}_N^t. \end{cases}$$

The boundary conditions are either homogeneous Dirichlet boundary conditions, imposed on $\partial\mathcal{F}_D^t$, or homogeneous Neumann boundary conditions, imposed on $\partial\mathcal{F}_N^t$, such that $\partial\mathcal{F}^t = \partial\mathcal{F}_D^t \cup \partial\mathcal{F}_N^t$. Here, $n_{\mathcal{F}}^t$ is the outward unit normal vector to $\partial\mathcal{F}_N^t$. The stress tensor is defined as

$$\sigma(u^t, p^t) = -p^t \mathbb{I}_d + \mu_{\mathcal{F}} (\nabla u^t + (\nabla u^t)^T),$$

where \mathbb{I}_d is the identity matrix of size d .

2.2 Elasto-magneto-swimmer model

The swimmer \mathcal{S} consists of its magnetic head $\mathcal{S}_{\text{head}}$ and its elastic tail $\mathcal{S}_{\text{tail}}$, such that $\mathcal{S} = \mathcal{S}_{\text{head}} \cup \mathcal{S}_{\text{tail}}$. We introduce the following notation: \mathcal{S}^* denotes the reference domain of the swimmer, i.e., the initial domain at each remeshing step, while \mathcal{S}^t denotes the domain at the current time. The same notation is used for the fluid. The notations are shown in Fig. 1. The swimmer is described by the position of its center of mass x_{cm}^t and the configuration of its structure, i.e., its deformation. These two quantities are determined by its time-dependent translational velocity U^t , angular velocity ω^t , and the elastic displacement η^t of the flagella. This elastic displacement is actuated by an external magnetic field $B^t :]0, T] \rightarrow \mathbb{R}^d$.

The linear velocity $U^t :]0, T] \rightarrow \mathbb{R}^d$ is described by the Newton's second law

$$m^t \text{d}_t U^t = f_H^t, \quad (1)$$

and the angular velocity $\omega^t :]0, T] \rightarrow \mathbb{R}^{d^*}$, with $d^* = 1$ in two dimensions and $d^* = 3$ in three dimensions, is described by the Euler's equation

$$\text{d}_t [R(\theta^t) J^t R(\theta^t)^T \omega^t] = T_H^t + T_m^t. \quad (2)$$

In (1), $m^t \in \mathbb{R}^+$ describes the mass of the swimmer, which is computed from its density $\rho_{\mathcal{S}} \in \mathbb{R}^+$. In (2), $J^t \in \mathcal{S}_{++}^{d^*}$ represents its inertia tensor, R the rotation matrix, and $\theta^t :]0, T] \rightarrow \Theta$ the rotation angle of the swimmer, which is derived from

$$\text{d}_t \theta^t = \omega^t.$$

In two dimensions, the interval $\Theta = [-\pi, \pi]$, whereas in three dimensions it is given by $\Theta = [-\pi, \pi] \times [0, \pi] \times [0, \pi/2]$. In (1) and (2), $f_H^t :]0, T] \times \partial\mathcal{S}^t \rightarrow \mathbb{R}^d$ and $T_H^t :]0, T] \times \partial\mathcal{S}^t \rightarrow \mathbb{R}^{d^*}$ describe the hydrodynamical forces and torques

$$f_H^t = \int_{\partial\mathcal{S}^t} \sigma(u^t, p^t) n_{\mathcal{S}}^t, \quad T_H^t = \int_{\partial\mathcal{S}^t} (x^t - x_{cm}^t) \times (\sigma(u^t, p^t) n_{\mathcal{S}}^t), \quad (3)$$

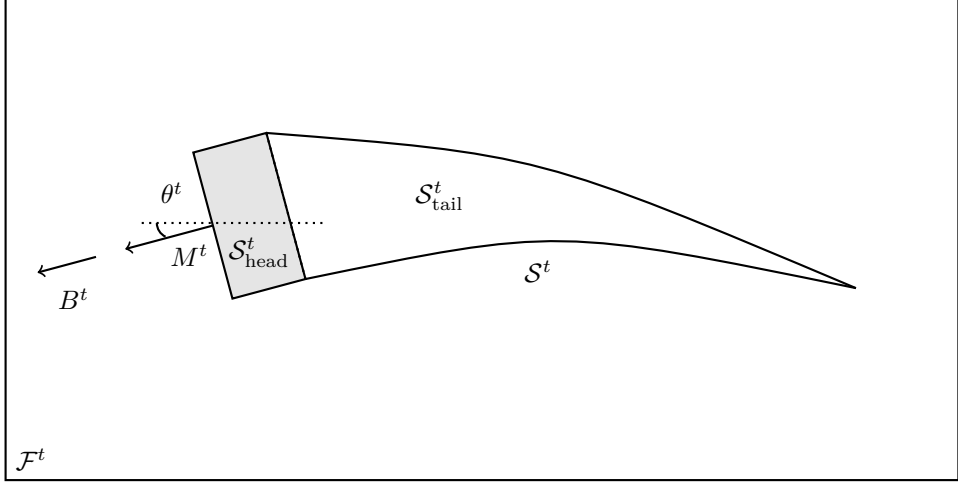


Figure 1: Configuration of the magneto-swimmer at time t inside a fluid domain \mathcal{F}^t . The magnetic moment M^t tends to align with the external magnetic field B^t , and θ^t represents the swimmer's head orientation.

where $n_{\mathcal{S}}^t$ is the current outward unit normal vector to $\partial\mathcal{S}^t$. The swimmer is also subjected to the magnetic torque $T_m^t :]0, T] \times \mathcal{S}_{\text{head}}^* \rightarrow \mathbb{R}^{d^*}$ defined as

$$T_m^t = \mathbf{m}R(\theta^t)M^t \times B^t,$$

where $M^t :]0, T] \rightarrow \mathbb{R}^d$ is its magnetic moment and $\mathbf{m} \in \mathbb{R}^+$ its magnetization.

Moreover, the elastic displacement $\eta^t :]0, T] \times \mathcal{S}^* \rightarrow \mathbb{R}^d$ of the flagella is governed by the hyper-elasticity equations

$$\begin{cases} \rho_S \partial_{tt}\eta^t - \nabla \cdot (F(\eta^t) \Sigma(\eta^t)) = 0_{\mathbb{R}^d} & \text{in } \mathcal{S}^*, \\ \eta^t = R(\theta^t)(x^* - x_{cm}^*) - (x^* - x_{cm}^*) & \text{on } \mathcal{S}_{\text{head}}^*, \\ F(\eta^t) \Sigma(\eta^t) n_{\mathcal{S}}^* = 0_{\mathbb{R}^d} & \text{on } \partial\mathcal{S}_{\text{tail}}^*, \end{cases} \quad (4)$$

here x_{cm}^* is the center of the swimmer's head $\mathcal{S}_{\text{head}}^*$. To describe the deformation of the hyper-elastic flagella, we consider the Saint-Venant-Kirchhoff model. In this framework, the inertial and surface loads are defined in terms of the deformation gradient

$$F(\eta^t) = \mathbb{I}_d + \nabla\eta^t,$$

and the second Piola-Kirchhoff tensor

$$\Sigma(\eta^t) = \lambda \text{Tr}(\epsilon(\eta^t)) \mathbb{I}_d + 2\mu \epsilon(\eta^t),$$

with λ, μ the Lamé coefficients and $\epsilon(\eta^t)$ the Green-Lagrange tensor.

2.3 Arbitrary Lagrangian-Eulerian framework

The Navier-Stokes equations are formulated in an Eulerian reference frame, where they describe the evolution of hydrodynamics within a fixed computational domain. However, the dynamics of the swimmers are described in a Lagrangian reference frame. To account for the geometric coupling condition, we adopt the Arbitrary Lagrangian-Eulerian framework [12]. This approach allows the fluid domain to follow the motion of the fluid-structure interaction interface. Specifically, the reference frame remains Lagrangian near the swimmers but stays Eulerian farther from the fluid-structure interaction interface. Both the swimmer and the fluid are discretized, and the discretized fluid domain follows the motion of the swimmer.

We define the ALE map $\mathcal{A}_{\mathcal{F}^*}^t :]0, T] \times \mathcal{F}^* \rightarrow \mathcal{F}^t$ as a continuous and bijective function that gives the position of a particle in the current fluid domain \mathcal{F}^t based on its position in the reference domain \mathcal{F}^*

$$\mathcal{A}_{\mathcal{F}^*}^t(x^*) = x^t.$$

To construct this ALE map, we introduce the displacement of the discretized fluid domain over time, denoted as $\eta_{\mathcal{F}}^t :]0, T] \times \mathcal{F}^* \rightarrow \mathcal{F}^t$. A possible definition of the ALE map is then:

$$\mathcal{A}_{\mathcal{F}^*}^t(x^*) = x^* + \eta_{\mathcal{F}}^t(x^*).$$

The fluid domain displacement $\eta_{\mathcal{F}}^t$ can be obtained, for instance, by a harmonic extension of the swimmer's displacement at time t , defined on $\partial\mathcal{S}^*$, to the interior of the fluid domain. This extension is determined by solving the following Laplace smoothing equation

$$\begin{cases} \nabla \cdot ((1 + \tau)\nabla\eta_{\mathcal{F}}^t) = 0_{\mathbb{R}^d} & \text{in } \mathcal{F}^*, \\ \eta_{\mathcal{F}}^t = \eta^t & \text{on } \partial\mathcal{S}_i^*, \end{cases} \quad (5)$$

where η^t is the solution of (4), and τ acts as a space-dependent diffusion coefficient. It is a piecewise constant coefficient, defined on each element e of the domain's discretization as

$$\tau|_e = \frac{1 - V_{\min}/V_{\max}}{V_e/V_{\max}},$$

where V_{\max} , V_{\min} , and V_e are the volumes of the largest, smallest, and current elements of the domain discretization, respectively [25]. This coefficient τ ensures that mesh deformation is applied primarily to elements with larger volumes.

When transitioning from the fluid Eulerian frame to an ALE frame, an additional term appears in the Navier-Stokes equations to account for the velocity of the moving domain, denoted by $u_{\mathcal{A}}^t$. The full system in the ALE reference frame is divided into two problems. First, the *fluid-rigid* problem, solving the hydrodynamics and the swimmer rigid motion, is governed by the following equations

$$\left\{ \begin{array}{ll} \rho_{\mathcal{F}} \left(\partial_t u^t + ((u^t - u_{\mathcal{A}}^t) \cdot \nabla) u^t \right) - \nabla \cdot \sigma(u^t, p^t) = 0_{\mathbb{R}^d} & \text{in } \mathcal{F}^t, \\ \nabla \cdot u^t = 0 & \text{in } \mathcal{F}^t, \\ u^t = 0_{\mathbb{R}^d} & \text{on } \partial\mathcal{F}_D^t, \\ \sigma(u^t, p^t) n_{\mathcal{F}}^t = 0_{\mathbb{R}^d} & \text{on } \partial\mathcal{F}_N^t, \\ u^t = \tilde{u}^t & \text{on } \partial\mathcal{S}^t, \\ m^t \, \text{d}_t U^t = f_H^t, \\ \text{d}_t \left[R(\theta^t) J^t R(\theta^t)^T \omega^t \right] = T_H^t + T_m^t, \end{array} \right. \quad (6)$$

where $\tilde{u}^t = U^t + \omega^t \times (x^t - x_{cm}^t) + \partial\eta^t \circ (\mathcal{A}_{\mathcal{F}^*}^t)^{-1}$ the velocity of the swimmer. Then, the *fluid-elastic* problem computes the elastic deformations and introduces the continuity of displacements and stresses. This problem is described by

$$\left\{ \begin{array}{ll} \rho_{\mathcal{S}} \partial_{tt} \eta^t - \nabla \cdot (F(\eta^t) \Sigma(\eta^t)) = 0_{\mathbb{R}^d} & \text{in } \mathcal{S}^*, \\ \eta^t = \eta_T^t + \eta_R^t & \text{on } \mathcal{S}_{\text{head}}^*, \\ F(\eta^t) \Sigma(\eta^t) n_{\mathcal{S}}^* = -(\sigma(u^t, p^t) n_{\mathcal{F}}^t) \circ \mathcal{A}_{\mathcal{F}^*}^t & \text{on } \partial\mathcal{S}^*, \end{array} \right. \quad (7)$$

where the translational and rotational displacements are respectively given by

$$\text{d}_t \eta_T^t = U^t, \quad \eta_R^t = R(\theta^t)(x^* - x_{cm}^*) - (x^* - x_{cm}^*).$$

3 Numerical discretization

For the spatial discretization of the discretized domain $\Omega_h^t = \mathcal{F}_h^t \cup \mathcal{S}_h^t$, we use standard Lagrange finite elements. The associated discrete approximation spaces are constructed from piecewise polynomial functions of degree N , denoted $P_c^N(\Omega_h^t)$

$$P_c^N(\Omega_h^t) = \left\{ v \in C^0(\Omega_h^t) \mid v \circ \varphi_{K_e}^{\text{geo}} \in \mathbb{P}^N(\widehat{K}), \quad \forall K_e \right\},$$

where $\mathbb{P}^N(\widehat{K})$ denotes the space of scalar polynomials of total degree less than or equal to N defined on the reference element \widehat{K} , and $\varphi_{K_e}^{\text{geo}} : \widehat{K} \rightarrow K_e$ the geometric transformation allowing to obtain each element K_e from the reference element \widehat{K} . Vector-valued finite element spaces are constructed as Cartesian products of the scalar-valued spaces, denoted by $[P_c^N(\Omega_h^t)]^d$. We also use the notation $H^m(\Omega_h^t)$ to denote the Sobolev space of order m , and with boundary condition g on a subset $\Gamma \subset \partial\Omega_h^t$ it is given by

$$H_{(g,\Gamma)}^m(\Omega_h^t) = \{v \in H^m(\Omega_h^t) \mid v|_{\Gamma} = g\}.$$

The vector-valued versions are respectively given by $[H^m(\Omega_h^t)]^d$ and $[H_{(g,\Gamma)}^m(\Omega_h^t)]^d$.

Regarding temporal discretization, the time interval $]t_0 = 0, T = t_{N_t}]$, with $T > 0$ denoting the final time, is divided into N_t time steps of size δt . The discrete time instances are denoted by t_n for $n = 1, \dots, N_t$. Finally, a mesh at time t_{n+1} is denoted by Ω_h^{n+1} , and a discrete solution at this time is written as u_h^{n+1} .

We will first present the spatial and temporal discretization of the ALE map before introducing that of the *fluid-rigid* and *fluid-elastic* problems.

3.1 Discretization of the ALE map

The approximation spaces at time t_{n+1} of the trial and test functions are respectively given by:

$$\begin{aligned} V_{\mathcal{A},h}^* &= \{\mathbf{v}, \mathbf{v} \in [H_{(0_{\mathbb{R}^d}, \partial\mathcal{F}_h^*)}^1(\mathcal{F}_h^*)]^d \cap [P_c^1(\mathcal{F}_h^*)]^d, \mathbf{v} = \eta_{\mathcal{S}_h^*}^{n+1} \text{ on } \partial\mathcal{S}_h^*\}, \\ W_{\mathcal{A},h}^* &= \{\mathbf{v}, \mathbf{v} \in [H_{(0_{\mathbb{R}^d}, \partial\mathcal{F}_h^*)}^1(\mathcal{F}_h^*)]^d \cap [P_c^1(\mathcal{F}_h^*)]^d, \mathbf{v} = 0_{\mathbb{R}^d} \text{ on } \partial\mathcal{S}_h^*\}. \end{aligned}$$

Thus, the fluid domain displacement is discretized using continuous affine finite elements. The space- and time-discretized version of (5) at time t_{n+1} gets

Find $\eta_{\mathcal{F},h}^{n+1} \in V_{\mathcal{A},h}^*$ such that

$$\underbrace{\int_{\mathcal{F}_h^*} (1 + \tau) \nabla \eta_{\mathcal{F},h}^{n+1} : \nabla \mathbf{v}}_{:= a_{\mathcal{A}^*}(\eta_{\mathcal{F},h}^{n+1}, \mathbf{v})} = 0 \quad \text{for all } \mathbf{v} \in W_{\mathcal{A},h}^*. \quad (8)$$

The discrete ALE map is then defined as

$$\mathcal{A}_{\mathcal{F}^*,h}^{n+1}(x^*) = x^* + \eta_{\mathcal{F},h}^{n+1}(x^*).$$

Large deformations may lead to element inversions, resulting in invalid triangulation. To avoid this, mesh quality metrics [18] are used. If the quality remains above a fixed threshold, the domain is updated using the ALE map. Otherwise, remeshing is performed before applying it. Details of these procedures are given in [41].

For the numerical resolution, we define the algebraic system associated with (8) as follows

$$A\boldsymbol{\eta}_{\mathcal{F}} = 0_{\mathbb{R}^{N_{\boldsymbol{\eta}_{\mathcal{F}}}}},$$

where we denote by

$$A = (a_{\mathcal{A}^*}(\xi_i, \xi_j))_{i,j} \in \mathbb{R}^{N_{\boldsymbol{\eta}_{\mathcal{F}}} \times N_{\boldsymbol{\eta}_{\mathcal{F}}}}, \quad \text{for } 1 \leq i, j \leq N_{\boldsymbol{\eta}_{\mathcal{F}}},$$

and $(\xi_i)_i$, $i = 1, \dots, N_{\boldsymbol{\eta}_{\mathcal{F}}}$ is the basis of the space $V_{\mathcal{A},h}^*$. The solver and preconditioner applied to this algebraic system are detailed in 4.

3.2 Discretization of the fluid-rigid problem

The Dirichlet boundary condition, enforcing the coupling of velocities at the interface of the swimmer is incorporated both in the function space associated with the fluid velocity and in the space of test functions. The discrete spaces for the fluid velocity and pressure at time t_{n+1} are then respectively defined as

$$\begin{aligned} V_{\mathcal{F},h}^{n+1} &= \left\{ \mathbf{v} : \mathcal{F}_h^{n+1} \rightarrow \mathbb{R}^d, \mathbf{v} = \tilde{\mathbf{v}} \circ (\mathcal{A}_{\mathcal{F}^*,h}^{n+1})^{-1}, \tilde{\mathbf{v}} \in [H_{(0,\mathbb{R}^d),\mathcal{F}_{D,h}^*}^1(\mathcal{F}_h^*)]^d \cap [P_c^M(\mathcal{F}_h^*)]^d, \right. \\ &\quad \left. \tilde{\mathbf{v}} = \tilde{\mathbf{u}} \text{ on } \partial\mathcal{S}_h^{n+1}, \tilde{\mathbf{u}} \in \mathbb{R}^d \right\}, \\ Q_{\mathcal{F},h}^{n+1} &= \left\{ \mathbf{q} : \mathcal{F}_h^{n+1} \rightarrow \mathbb{R}, \mathbf{q} = \tilde{\mathbf{q}} \circ (\mathcal{A}_{\mathcal{F}^*,h}^{n+1})^{-1}, \tilde{\mathbf{q}} \in P_c^N(\mathcal{F}_h^*) \right\}. \end{aligned}$$

We choose the Taylor-Hood inf-sup finite elements to satisfy the compatibility condition. The polynomial degrees are related by $N = M - 1$, and typically we set $M = 2$ and $N = 1$. Thus, the velocity and pressure are respectively discretized using continuous piecewise quadratic, and affine finite elements. Following the ideas of [31], the discretized formulation of the *fluid-rigid* problem (6) deriving from the variational form is stated as follows

Find the fluid velocity and pressure $(u_h^{n+1}, p_h^{n+1}) \in V_{\mathcal{F},h}^{n+1} \times Q_{\mathcal{F},h}^{n+1}$, and the swimmer's linear and angular velocities $U^{n+1}, \omega^{n+1} \in \mathbb{R}^d \times \mathbb{R}^{d^*}$, such that for all $(\mathbf{v}, \mathbf{q}) \in V_{\mathcal{F},h}^{n+1} \times Q_{\mathcal{F},h}^{n+1}$, and $\mathbf{U}, \boldsymbol{\omega} \in \mathbb{R}^d \times \mathbb{R}^{d^*}$, one has

$$\begin{aligned} a_{\mathcal{F}^t}(u_h^{n+1}, \mathbf{v}) - b_{\mathcal{F}^t}(\mathbf{v}, p_h^{n+1}) + m^{n+1} \text{d}_t U^{n+1} \cdot \mathbf{U} \\ + \text{d}_t \left[R(\theta^{n+1}) J^{n+1} R(\theta^{n+1})^T \omega^{n+1} \right] \cdot \boldsymbol{\omega} = l_{\mathcal{F}}(\mathbf{v}, \mathbf{U}, \boldsymbol{\omega}), \end{aligned} \quad (9)$$

$$b_{\mathcal{F}^t}(u_h^{n+1}, \mathbf{q}) = 0.$$

with the bilinear and linear forms

$$\begin{aligned} a_{\mathcal{F}^t}(u_h^{n+1}, \mathbf{v}) &= \rho_{\mathcal{F}} \int_{\mathcal{F}_h^{n+1}} \partial_t u_h^{n+1} \cdot \mathbf{v} + \rho_{\mathcal{F}} \int_{\mathcal{F}_h^{n+1}} \left((u_h^{n+1} - u_{\mathcal{A},h}^{n+1}) \cdot \nabla \right) u_h^{n+1} \cdot \mathbf{v} \\ &\quad + \mu_{\mathcal{F}} \int_{\mathcal{F}_h^{n+1}} \nabla u_h^{n+1} : \nabla \mathbf{v}, \\ l_{\mathcal{F}^t}(\mathbf{v}) &= f_H^{n+1} \cdot \mathbf{U} + T_H^{n+1} \cdot \boldsymbol{\omega} + T_m^{n+1} \cdot \boldsymbol{\omega}, \\ b_{\mathcal{F}^t}(u_h^{n+1}, \mathbf{q}) &= - \int_{\mathcal{F}_h^{n+1}} \mathbf{q} \cdot \nabla \cdot u_h^{n+1}. \end{aligned}$$

The time derivative of u_h^{n+1} as well as the ALE velocity $u_{\mathcal{A},h}^{n+1}$ are time-discretized using the second-order backward differentiation formula, BDF2. For the numerical resolution, the degrees of freedom of the fluid velocity on the interface of the swimmer are treated differently since they depend only on the linear and angular velocities.

We denote the vectors $\mathbf{u} = (\mathbf{u}_1, \dots, \mathbf{u}_{N_u}) = (\mathbf{u}_I, \mathbf{u}_\Gamma)$ for the velocity, where the subscript Γ represent the degrees of freedom associated with the interface, $\mathbf{p} = (\mathbf{p}_1, \dots, \mathbf{p}_{N_p})$ for the pressure, and $\mathbf{U} \in \mathbb{R}^d$ and $\boldsymbol{\omega} \in \mathbb{R}^{d^*}$ represent the linear and angular velocities, respectively. The resulting algebraic system of (9) is then given by

$$\begin{bmatrix} A_{II} & A_{I\Gamma} & 0 & 0 & B_I^T \\ A_{\Gamma I} & A_{\Gamma\Gamma} & 0 & 0 & B_\Gamma^T \\ 0 & 0 & m^{n+1} \mathbb{I}_d & 0 & 0 \\ 0 & 0 & 0 & R(\theta^{n+1}) J^{n+1} R(\theta^{n+1})^T & 0 \\ B_I & B_\Gamma & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{u}_I \\ \mathbf{u}_\Gamma \\ \mathbf{U} \\ \boldsymbol{\omega} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_U \\ l_\omega \\ 0 \end{bmatrix}, \quad (10)$$

where

$$\begin{aligned} A_{JK} &= (a_{\mathcal{F}t}(\phi_{J_i}, \phi_{K_j}))_{i,j} \in \mathbb{R}^{N_{\mathbf{u}_J} \times N_{\mathbf{u}_K}}, \text{ for } J, K \in \{I, \Gamma\}, \\ B_J &= (b_{\mathcal{F}t}(\phi_{J_i}, \psi_j))_{i,j} \in \mathbb{R}^{N_{\mathbf{u}_J} \times N_{\mathbf{p}}}, \text{ for } J \in \{I, \Gamma\}, \\ l_U &= f_H^{n+1} \cdot \mathbf{U}, \\ l_\omega &= T_H^{n+1} \cdot \boldsymbol{\omega} + T_m^{n+1} \cdot \boldsymbol{\omega}, \end{aligned}$$

and $(\phi_i)_i, i = 1, \dots, N_{\mathbf{u}}$, respectively $(\psi_i)_i, i = 1, \dots, N_{\mathbf{p}}$, the basis of the spaces $V_{\mathcal{F},h}^{n+1}$ and $Q_{\mathcal{F},h}^{n+1}$.

Using the no-slip boundary conditions $\mathbf{u} = \tilde{\mathbf{u}} = \mathbf{U} + \boldsymbol{\omega} \times (x^{n+1} - x_{cm}^{n+1})$ on $\partial\mathcal{S}_h^{n+1}$, we introduce the operator \mathcal{P} such as

$$(\mathbf{u}_I, \mathbf{u}_\Gamma, \mathbf{U}, \boldsymbol{\omega}, \mathbf{p})^T = \mathcal{P}(\mathbf{u}_I, \mathbf{U}, \boldsymbol{\omega}, \mathbf{p}), \quad (11)$$

with

$$\mathcal{P} = \begin{bmatrix} \mathbb{I}_d & 0 & 0 & 0 \\ 0 & \tilde{P}_U & \tilde{P}_\omega & 0 \\ 0 & \mathbb{I}_d & 0 & 0 \\ 0 & 0 & \mathbb{I}_d & 0 \\ 0 & 0 & 0 & \mathbb{I}_d \end{bmatrix}.$$

Finally, by using (11), and by multiplying by \mathcal{P} , one obtains the algebraic system of the fluid-swimmer problem

$$\mathcal{P}^T \begin{bmatrix} A_{II} & A_{I\Gamma} & 0 & 0 & B_I^T \\ A_{\Gamma I} & A_{\Gamma\Gamma} & 0 & 0 & B_\Gamma^T \\ 0 & 0 & m^{n+1}\mathbb{I}_d & 0 & 0 \\ 0 & 0 & 0 & R(\theta^{n+1})J^{n+1}R(\theta^{n+1})^T & 0 \\ B_I & B_\Gamma & 0 & 0 & 0 \end{bmatrix} \mathcal{P} \begin{bmatrix} \mathbf{u}_I \\ \mathbf{U} \\ \boldsymbol{\omega} \\ \mathbf{p} \end{bmatrix} = \mathcal{P}^T \begin{bmatrix} 0 \\ 0 \\ l_U \\ l_\omega \\ 0 \end{bmatrix}.$$

3.3 Discretization of the fluid–elastic problem

The approximation spaces of admissible elastic displacements, and of the tests functions, at time t_{n+1} , are respectively defined by

$$\begin{aligned} V_{\mathcal{S},h}^* &= \{\mathbf{v}, \mathbf{v} \in [H_{(\eta_T^{n+1} + \eta_R^{n+1}, \partial\mathcal{S}_{\text{head},h}^*)}^1(\mathcal{S}_h^*)]^d \cap [P_c^1(\mathcal{S}_h^*)]^d\}, \\ W_{\mathcal{S},h}^* &= \{\mathbf{v}, \mathbf{v} \in [H_{(0_{\mathbb{R}^d}, \partial\mathcal{S}_{\text{head},h}^*)}^1(\mathcal{S}_h^*)]^d \cap [P_c^1(\mathcal{S}_h^*)]^d\}. \end{aligned}$$

The fully discretized *fluid–elastic* problem is stated as follows

Find the elastic displacement $\eta_h^{n+1} \in V_{\mathcal{S},h}^*$ such that, for all $\mathbf{v} \in W_{\mathcal{S},h}^*$, one has

$$a_{\mathcal{S}^*}(\eta_h^{n+1}, \mathbf{v}) = - \int_{\partial\mathcal{S}_h^{n+1}} |\det F(\eta_h^{n+1})| \|(F(\eta_h^{n+1}))^{-T}\| (\sigma(u_h^{n+1}, p_h^{n+1}) n_{\mathcal{F}}^{n+1}) \cdot \mathbf{v},$$

where the boundary condition is expressed on the current interface $\partial\mathcal{S}_h^{n+1}$, since the hydrodynamic forces and torques are resolved onto the current domain, see (3). The bilinear form is defined as

$$a_{\mathcal{S}^*}(\eta_h^{n+1}, \mathbf{v}) = \rho_{\mathcal{S}} \int_{\mathcal{S}_h^*} \partial_t \eta_h^{n+1} \cdot \mathbf{v} + \int_{\mathcal{S}_h^*} F(\eta_h^{n+1}) \Sigma(\eta_h^{n+1}) : \nabla \mathbf{v}.$$

Once again, we consider the BDF2 scheme to discretize the time derivative. The corresponding algebraic system is written as

$$A\boldsymbol{\eta}_{\mathcal{S}} = G, \quad (12)$$

where the stiffness matrix $A \in \mathbb{R}^{N_{\boldsymbol{\eta}_{\mathcal{S}}} \times N_{\boldsymbol{\eta}_{\mathcal{S}}}}$ is expressed in the basis $(\varphi_i)_i, i = 1, \dots, N_{\boldsymbol{\eta}_{\mathcal{S}}}$ of the discrete space $V_{\mathcal{S},h}^*$ as

$$A = (a_{\mathcal{S}^*}(\varphi_i, \varphi_j))_{i,j},$$

and the right-hand side vector $G \in \mathbb{R}^{N_{\boldsymbol{\eta}_{\mathcal{S}}}}$ includes the boundary conditions.

4 Resolution strategies

4.1 Full algorithm

The complete coupled algorithm for the elasto-magneto-swimmer is given in [Algorithm 1](#). At each iteration, the swimmer's displacement is updated using a relaxation method, where the relaxation parameter t^{k+1} is computed using the Aitken method, as described in [\[28\]](#).

Algorithm 1 Fixed-Point algorithm for the magneto-swimmer

- 1: **Input:** Solid displacements at previous time steps η_h^{n-1} and η_h^n , tolerance tol , maximum iterations k_{\max} .
- 2: **Output:** New solid displacement η_h^{n+1} .
- 3: Initialize iteration counter: $k = 0$.
- 4: Predict initial displacement using BDF2-consistent extrapolation

$$\eta_h^{n+1,0} = 2\eta_h^n - \eta_h^{n-1}.$$

- 5: **while** $\epsilon > \text{tol}$ and $k < k_{\max}$ **do**
- 6: Compute ALE map $\mathcal{A}_{\mathcal{F}^*,h}^{n+1,k+1}$ from $\eta_h^{n+1,k}$.
- 7: Solve *fluid-rigid* problem on $\tilde{\mathcal{A}}_{\mathcal{F}^*,h}^{n+1,k+1}(\mathcal{F}_h^*)$ to obtain $\eta_T^{n+1,k+1} + \eta_R^{n+1,k+1}$.
- 8: Update total solid displacement on the reference interface

$$\tilde{\eta}_h^{n+1,k+1} = \eta_h^{n+1,k} + \eta_T^{n+1,k+1} + \eta_R^{n+1,k+1}.$$

- 9: Compute ALE map $\mathcal{A}_{\mathcal{F}^*,h}^{n+1,k+1}$ from $\tilde{\eta}_h^{n+1,k+1}$.
- 10: Solve *fluid-elastic* problem on \mathcal{S}_h^* to obtain total displacement $\tilde{\eta}_h^{n+1,k+1}$.
- 11: Compute error

$$\epsilon = \|\tilde{\eta}_h^{n+1,k+1} - \eta_h^{n+1,k}\|_{L^2(\mathcal{S}_h^*)}.$$

- 12: **if** $\epsilon > \text{tol}$ **then**
- 13: Update swimmer displacement

$$\eta_h^{n+1,k+1} = t^{k+1}\tilde{\eta}_h^{n+1,k+1} + (1 - t^{k+1})\eta_h^{n+1,k}.$$

- 14: Subtract rigid motion

$$\eta_h^{n+1,k+1} = \eta_h^{n+1,k+1} - (\eta_T^{n+1,k+1} + \eta_R^{n+1,k+1}).$$

- 15: Increment iteration counter $k = k + 1$.
 - 16: **else**
 - 17: Set $\eta_h^{n+1} = \tilde{\eta}_h^{n+1,k+1}$ and exit loop.
-

4.2 Remeshing strategy

The mesh deformation described by the ALE map may lead to significant element distortion during long-term simulations. To preserve mesh quality, a remeshing procedure is performed at a prescribed frequency, treated as a user-defined parameter depending on the simulated trajectories. At each remeshing step, the reference domain is updated accordingly, and all fields are projected onto the new mesh. The remeshing operations are carried out using the **MMG** tool in sequential mode, or **ParMMG** in parallel [\[3\]](#). The latter libraries take as input the current domain, which could be of bad quality, and a scalar function defined as the remeshing metric. This metric specifies the desired characteristic size of the elements in the resulting mesh. To accurately capture the fluid-swimmer interaction, we adopt a graded remeshing strategy, meaning that the mesh size is adjusted according to the distance to the swimmer's boundary in the current fluid domain.

4.3 Solver and preconditioner

The numerical scheme of the magneto-swimmer model is based on solving a fixed-point algorithm which iterates between the ALE (5), *fluid-rigid* (6), and *fluid-elastic* (7) problems corresponding to the step 6, 7 and 10 of the full algorithm. Each sub-problem is solved using appropriate solvers and preconditioners, which are summarized for both two- and three-dimensional cases in Table 1. The `Feel++` library interfaces with PETSc [29] for the efficient solution of large-scale linear and nonlinear systems.

Sub-problem	Two Dimensions	Three Dimensions
ALE map	CG + GAMG	CG + GAMG
<i>Fluid-rigid</i>	Direct solver + LU	GMRES + Block preconditioner
<i>Fluid-elastic</i>	Direct solver + LU	GMRES + GASM

Table 1: Solvers and preconditioners for the magneto-swimmer model.

5 Numerical results

We perform different numerical tests to validate and illustrate the proposed magneto-elastic swimmer model. First, we analyze the relationship between the swimmer’s net displacement and the area enclosed by its stroke in the configuration space. Second, we investigate the net displacement of the swimmer as a function of the external magnetic field frequency in two dimensions. Finally, we extend the validation to three dimensions by simulating the full 3D magneto-swimmer and comparing the results with the 2D case and available literature. The main physical parameters used in the simulations are summarized in Table 2, these values are chosen according to [36]. We prescribe the external magnetic field and the swimmer’s magnetic moment based on experimental data from [35, 36],

$$B^n = (b_x, b_y \sin(2\pi f t_n))^T, \quad (13)$$

with $b_x = b_y = 0.005\text{T}$. Under the action of this oscillating magnetic field, the swimmer follows a straight horizontal path.

Physical parameter	Value
Young modulus of the head	41GPa
Poisson coefficient of the head	0.281
Magnetization of the head	10^5A/m
Density of the head	7000kg/m^3
Height of the head	0.5mm
Young modulus of the tail	0.1MPa
Poisson coefficient of the tail	0.4
Density of the tail	1300kg/m^3
Length of the tail	7.5mm
Maximal diameter of the tail	1.5mm
Magnetic field intensity b_x, b_y	5mT

Table 2: Physical properties of the magneto-swimmer taken from references [35, 36].

Net displacement vs swimmer's strokes We introduce the magneto-head and magneto-tail angles θ_{head}^n and θ_{tail}^n relative to the horizontal axis as illustrated in Fig. 2. We build their phase portrait based on their time evolution over one period as depicted in Fig. 3 to represent the swimmer's stroke in the configuration space. The obtained curve approximates an ellipse, and according to [1], when the deformation is small, the mean x -displacement per period is directly correlated to the area of this ellipse. This result confirms the theoretical prediction that the swimmer will achieve an x -net displacement after the stroke cycle.

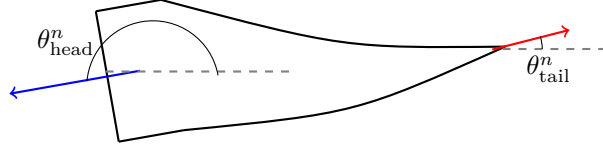


Figure 2: The illustration of the two angles used for the phase configuration space.

Net displacement vs frequency In this case, we work with the two-dimensional model of the magneto-elastic swimmer. In Fig. 4, we vary the frequency f of the magnetic field defined in equation (13) within the range $f \in [0 \text{ Hz}, 3 \text{ Hz}]$, while measuring the net displacement Δx of the swimmer over one oscillation period of duration $\frac{1}{f}$. This analysis is performed for three different values of the tail's Young modulus: $E_S = 5 \times 10^4 \text{ Pa}$, $E_S = 8 \times 10^4 \text{ Pa}$, and $E_S = 2 \times 10^5 \text{ Pa}$. The corresponding results are shown in Fig. 4, which plots the net displacement obtained during the third oscillation cycle, during the time interval $[\frac{2}{f} \text{ s}, \frac{3}{f} \text{ s}]$.

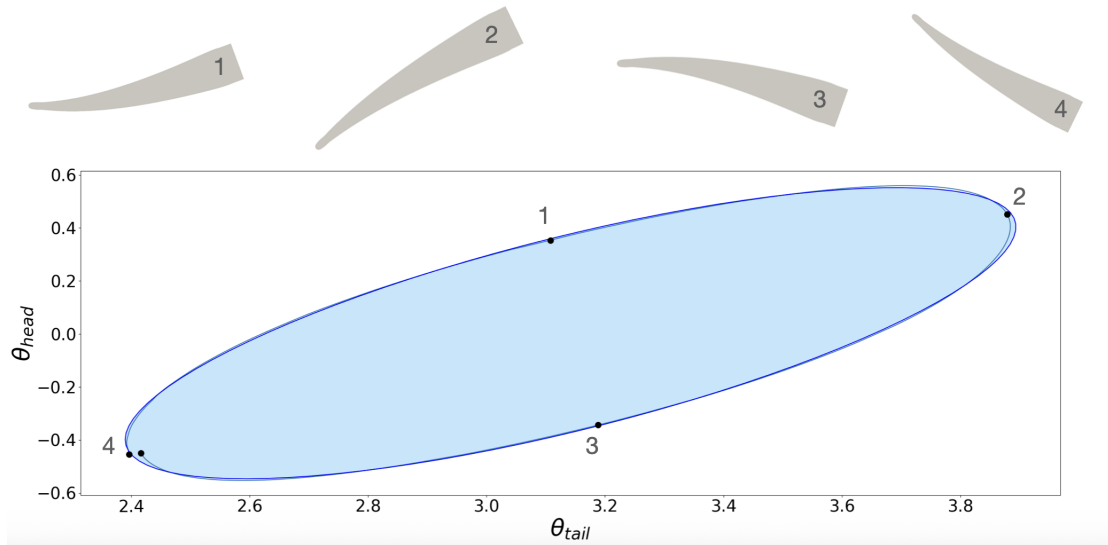


Figure 3: At the top, the deformations of the swimmer with respect to the stroke cycle plotted at the bottom in the configuration space.

We observe that the displacement per cycle increases with frequency, reaching a maximum around $f \approx 0.8 \text{ Hz}$, before decreasing as the frequency continues to rise. This non-monotonic trend is consistent with experimental and theoretical findings reported in the literature [1, 35], where an optimal frequency was also identified, corresponding to a resonance between the magnetic actuation and the elastic response of the swimmer's tail. At low frequencies, the magnetic actuation is too slow to generate significant tail deformation, resulting in limited propulsion. Conversely, at high frequencies, the head cannot follow the rapid field oscillations, leading to attenuated elastic waves along the tail, reducing the net displacement.

Furthermore, we note that a smaller Young modulus leads to larger net displacements per cycle. This is attributed to the increased flexibility of the tail, which allows for greater deformation

and thus more efficient conversion of oscillatory motion into forward propulsion. This observation also agrees with the results of [35], where flexible magnetic swimmers were shown to outperform stiffer ones under similar low-Reynolds-number conditions.

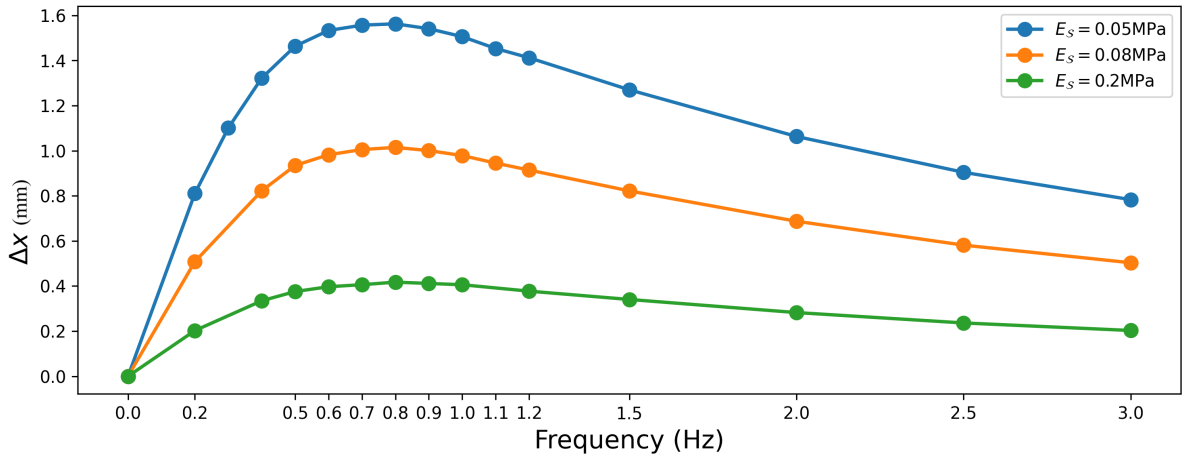


Figure 4: Displacement of the magneto-swimmer over one period as a function of the frequency of the magnetic field.

3D displacement of the elasto-magneto-swimmer We now consider the same test case using the three-dimensional model of the elasto-magneto-swimmer. The geometry of the swimmer is shown in Fig. 5. Its head is cylindrical, with a diameter of 1.5 mm and a height of 0.5 mm, and is attached to a flexible tail of length 7.5 mm and a minimum diameter of 0.2 mm. The density and Poisson’s ratio of both the head and the tail are set to typical values used in the literature for soft magnetic swimmers.

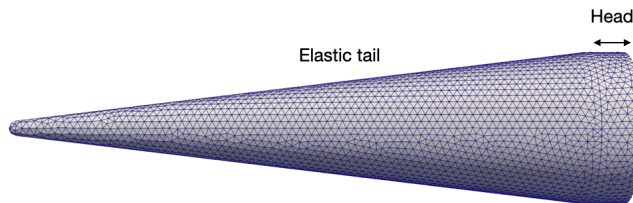


Figure 5: Discretized geometry of the three-dimensional magneto-swimmer.

The Young’s modulus is fixed to $E_S = 2 \times 10^5 \text{ Pa}$, while the frequency of the external magnetic field is varied within the range $[0 \text{ Hz}, 3 \text{ Hz}]$. The external magnetic field in three dimensions is generalized by applying the same oscillatory pattern in the y -direction but keeping the z -component zero, see equations (13). The intensity of the magnetic field is maintained at $b_x = b_y = 5 \text{ mT}$. The magnetization of the magneto-head remains fixed at $m = 10^5 \text{ A/m}$ as in the two-dimensional case and following experimental data from [35, 36].

The Fig. 7 shows the net displacement Δx of the three-dimensional magneto-swimmer over one period as a function of the frequency f of the external magnetic field. It leads to results similar to those in the two-dimensional case: the displacement increases with frequency, reaching a maximum at $f = 0.8 \text{ Hz}$, and then decreases. The behavior is consistent with the two-dimensional simulations and experimental observations reported in the literature [35]. Moreover, Fig. 6 illustrates the deformation and motion of the micro-swimmer at different times during

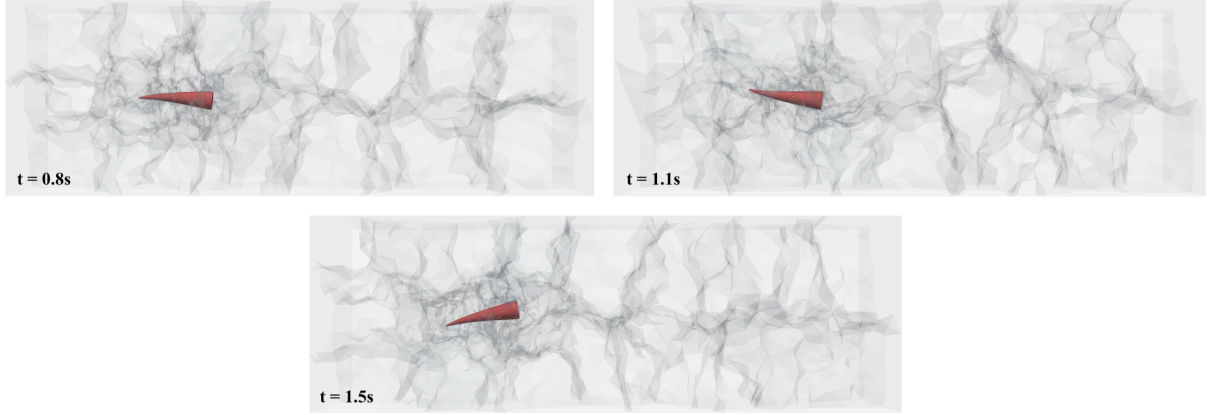


Figure 6: Visualization of the three-dimensional magneto-swimmer at different time instants. Transparent gray surfaces represent the boundaries between domains used by the processors.

a stroke cycle. The transparent gray surfaces represent the boundaries between domains used by the processors in the parallel computation. However, the three-dimensional simulations are computationally expensive, requiring long execution times. One perspective of this work is to improve the solver and preconditioner strategies, as well as to verify the parallel scalability of the model.

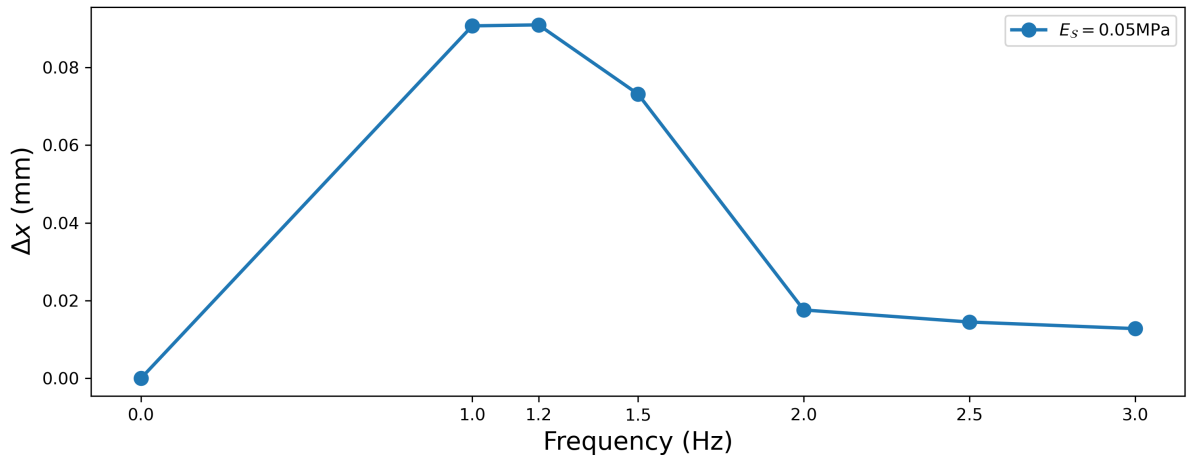


Figure 7: Net displacement of the three-dimensional magneto-swimmer over one period as a function of the frequency.

6 Perspectives and Conclusion

This work presents a comprehensive numerical framework for simulating fluid-structure interactions of elastic magneto-swimmers in confined domains, representing a significant advance toward developing digital twins for biomedical micro-robotics applications. The developed platform combines realistic modeling, numerical robustness, and high-performance computing capabilities to enable accurate simulation of magneto-swimmers.

The proposed full-order approach based on the Arbitrary Lagrangian-Eulerian formulation provides a major advancement in the physical understanding of active swimmer problems. The 2D and 3D validation benchmarks demonstrate the maturity and accuracy of the method, showing excellent agreement with experimental data from the literature. The framework successfully

captures the complex nonlinear coupling between magnetic actuation, elastic deformations, and fluid interactions that characterize these systems.

Key contributions of this work include: (i) a robust finite element discretization strategy that handles large swimmer deformations through adaptive remeshing procedures, (ii) advanced coupling schemes that ensure stability and convergence of the fluid-structure interaction, and (iii) a parallel implementation within the open-source Feel++ library that leverages high-performance computing resources.

The developed platform opens numerous perspectives for biomedical applications. It incorporates advanced path-planning algorithms that allow autonomous navigation of magneto-swimmers through complex biological environments. These algorithms are essential for practical biomedical applications, where precise control and collision avoidance are critical for successful targeted delivery missions. The framework also provides a solid foundation for in silico calibration using experimental data, optimal control strategies for targeted navigation, and the development of model reduction techniques. Its capabilities in handling realistic biological environments make it particularly well-suited for applications in drug delivery and micro-robot-assisted therapy.

Future developments will focus on extending the framework to multi-swimmer configurations to study collective behavior, implementing advanced control algorithms based on machine learning techniques, and optimizing the computational strategies for exascale computing. The integration with experimental validation campaigns will further enhance the platform's predictive capabilities, ultimately enabling the transition from laboratory prototypes to clinical applications in targeted biomedical interventions.

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