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Using Gaussian Mixture Model to predict bead geometry of WAAM-CMT beads

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Abstract

Mastering the prediction of geometry of a part is vital during designing fabrication protocols for a manufacturing process. But for Additive Manufacturing (AM) techniques like Wire Arc Additive Manufacturing (WAAM), the complex nature of the process produces parts with irregular surfaces. This adversely affects the design of experiments and makes the development of bead geometry prediction models more complex. The existing models often uses several hypothesis which simplifies the geometry making predictions which are different from that of reality. Therefore, in the present study several single beads are fabricated using different input parameters and their surface is measured comprising of irregularities. This data is then used to develop a geometry prediction model using Gaussian Mixture Model (GMM) with curve-fitted polynomial functions as input. GMM is then used to generate random variables using which beads are simulated using Monte-Carlo Simulations. The model is then validated by comparing the simulated beads with the fabricated beads. The proposed model is found to be capable of simulating beads with surface irregularities similar to that of experiments. This methodology can now serve as base to simulate more complex geometries of parts fabricated using WAAM.

Keywords : WAAM-CMT, Gaussian Mixture Model, Geometry Prediction, Uncertainty

Key Message : The data and the python scripts used to develop the model can be found at https://git.utt.fr/pulickan/robust-am_wire-arc-additive-manufacturing_waam

1 Introduction

WAAM is a leading Additive Manufacturing process widely used to fabricate parts of high dimensions. But the inherent complex physics of the process requiring complex control of the process parameter makes the geometry of the fabricated parts irregular. WAAM which uses the principles of welding, functions by short-circuiting a metallic electrode as it comes into contact with a metallic substrate. Traditionally the process is controlled by the manipulation of Current and Voltage among the important process control parameters. But recent advancements in the process has led to metal transfer which are regulated by microcontrollers in real-time, such as the Cold Metal Transfer (CMT) which is used in the current study. With WAAM-CMT, the most important user controllable parameters which determine

the bead geometry are the Wire Feed Rate (V_{feed}), Travel Speed (V_T), Contact-tip-to-Work Distance (CTWD), Torch Angle (T_A), and Wire Diameter (d_W). Even though the process physics is much more complicated than what can be described using a handful of parameters, it is identified that the other parameters and the variation in the geometry are in one way or the other depended on the variations in these five parameters

As the process physics and bead geometry relationship is complicated to analytically establish different authors have developed different methods to predict bead geometry based on input parameters. In [1] Karmuhilan and Sood develops a Artificial Neural Network to predict the bead height and width of WAAM beads as a function of Voltage, Wire Feed Rate and Travel Speed. In [2] Surface Waviness Index is developed as a function of maximum width observed, minimum width observed and bead height so as to aid machining after fabrication. In [3] the bead height, width and penetration is modelled using regression as a function of V_{feed} , V_T and CTWD. In [4] Wang, develops an analytical model for the bead height and width as a function of wire diameter, V_{feed} , V_T and T_A .

The present study is done with the objective of making predictions of the geometry of a part with irregularities due to the uncertainties included in the model rather than neglecting or simplifying the irregularities. Therefore, the modelling process relies on an observation-based approach, where a statistically significant number of single beads are fabricated per set of input parameters chosen. These beads are then scanned using a laser scanner, whose data is analysed to quantify the surface irregularities which serves as the input for the model. Thus, this approach relates the input parameters used for fabrication with the geometric irregularities of the respective beads. This enables the manufacturer to make the plan of experiments more suited for the part to be fabricated as the variations to be expected are quantified and integrated into the model.

Prediction models developed are simplified and far from reality, as they neglect the surface irregularities of the bead and generalise the bead profiles irrespective of their actual shape [5]. While handling uncertainties, for a parameter under consideration, it can be a range of values of different probabilities, which makes the parameter a random variable [6]. Therefore these are defined using probability distributions. The geometrical accuracy of the WAAM-CMT is low, even though the process is a significant advancement compared to its predecessors [7]. The uncertainties in the process is being widely researched, nevertheless, most of the research explains uncertainties qualitatively.

The model development is executed using statistical tools as these are efficient while handling data with variations which have to be considered into the model. These distributions of data are analysed using statistical tools which could enable the simulation of bead's geometry numerically. Among the different tools available, considering the complexity of the distributions, while keeping the model easy to manipulate a Gaussian Mixture Model (GMM) is used to develop the prediction model. GMM gives promising results with the simulation of beads as discussed in Section 3 while making negligible compromise for precision. The advantage with GMM is that the original data does not have to be stored once the parameters are determined making it less easy to handle. GMM uses weighted sum of Gaussian distributions of clusters to model a given parent distribution [8].

The GMM based simulated beads are finally compared with the experimental beads for validating the model. Thus, this study proposes a methodology which can be used to develop prediction models when the process physics involved is complex and the variations in the geometry plays an important role in the design of experiments .

2 Experiments

Fabrication of parts using WAAM-CMT is controlled by different parameters. Two parameters which are important and are manipulated from a fabrication standpoint under the current study are the Wire Feed Rate (V_{feed}) and Travel Speed (V_T). In the scope of current experiments four different V_{feed} & V_T are chosen such that beads of four different geometric characteristics can be fabricated and analysed. This assures that the model does not generalise the process with only one set of parameters and rather respects the possible variations which can occur due to the interplay of different set of input parameters used for fabrication.

Therefore, 140 single beads are fabricated using four different set of parameters (35 beads per set of parameters) [Table 1]. This enables the study of variations in beads with the same set of parameters, as well as variations with different set of parameters.

TABLE 1 – Input set of parameters used for fabrication

Parameter Set	Wire Feed Rate V_{feed} m/min	Travel Speed V_T mm/s
P1	9.0	13.3
P2	7.5	18.3
P3	10	8.3
P4	10	18.3

The beads are then digitalised using a 3D laser scanner, which outputs point clouds of data, with around 2000 sections for a single bead over the length of 100 mm. This data is used in the present study to develop prediction model.

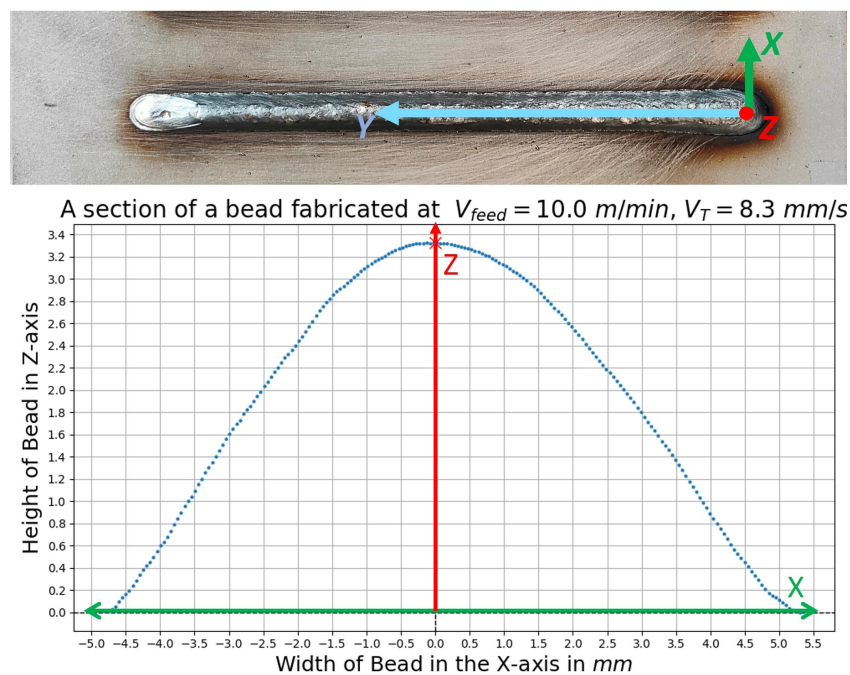


FIGURE 1 – Fabricated bead (on top) and the reconstructed scan data showing a section of a bead fabricated with P3 set of parameters.

3 Development of Numerical Model

The first step employed in the present study to develop a numerical model is the transformation the point cloud of data to mathematical functions. This helps in a simpler representation of data and thereby the beads themselves, using functions which can then be easily used while developing models.

Different authors have used different approaches such as - parabola, arc and sinusoidal functions [9], parabola and cosine functions [10], parabola, sine function, Gaussian and logistic functions [11].

In the current study the point clouds of data obtained from the scan are curve fitted using a 4th degree polynomial function ($R^2 = 0.99$) of the form $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$ for each bead sections. These polynomial functions then replace the point cloud and the variation in the polynomial coefficients is studied using their distributions which shows the extend of variations in the shape of the bead sections [Fig. 2]. This approach using the distributions make sure that the variations due to the several uncertainties in the process are considered while modelling.

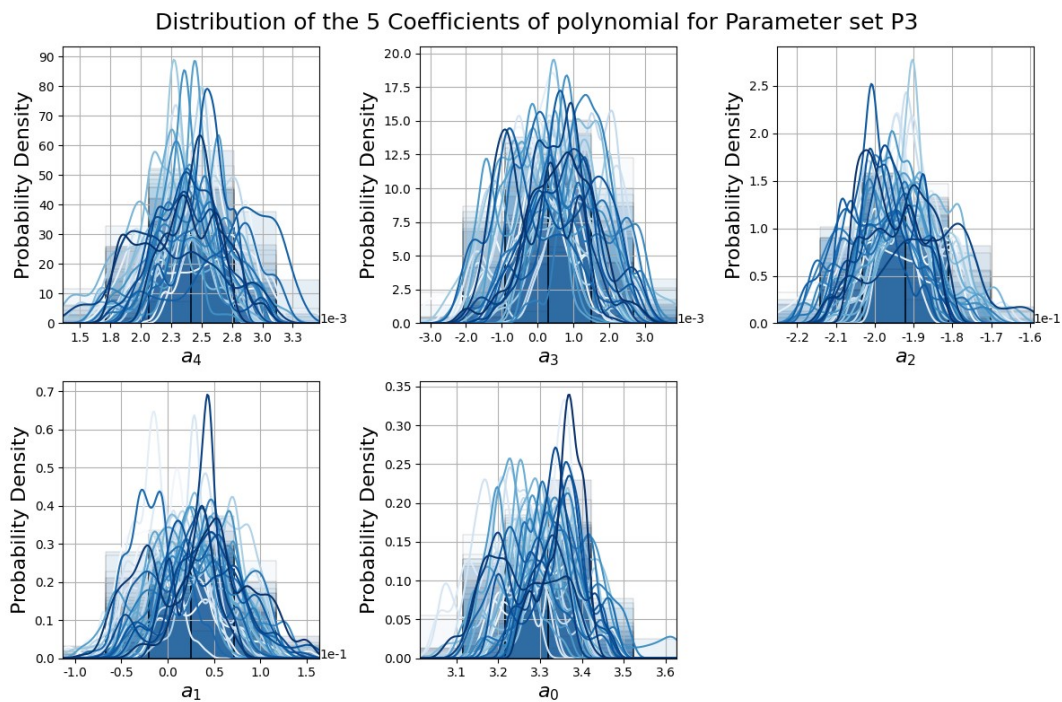


FIGURE 2 – Distributions of the five coefficients of the polynomial functions for P3 set of parameters.

Each curve corresponds to the distribution of a coefficient along the length of a bead among the 35 beads fabricated using the P3 set of parameters.

But as the distributions of the coefficients cannot be represented using a classical Probability Distribution Function (PDF), the distributions are modeled using GMM. GMM segregates a distribution into several weighted clusters each of which are represented by a Gaussian PDF. The weighted sum of these distributions forms the distribution of the original population. Thus, GMM enables to approximate any kind of distribution using a weighted sum of Gaussian distributions [12].

Thus, a GMM can be reconstructed using the mean, variance and the weight (contribution of a cluster towards the GMM). For the data obtained under the current study the number of clusters (K) is estimated using the Bayesian Information Criterion (BIC) which finds a healthy compromise between precision

of the GMM with experimental data and computational cost, or in other words it avoids under-fitting or over-fitting of the GMM [13]. For the current experiments the number of clusters is estimated to be 8 [Fig. 3].

Nevertheless, a strong correlation is found between the polynomial coefficients meaning that instead of using variance for individual distributions, covariance matrix has to be used such that the relationship between the coefficients are also respected while modelling. Avoiding the use of covariance can result in bead sections with geometries which does not physically exist.

Therefore a multivariate GM Model which takes into account the covariance of the polynomial coefficients are used which is given by [14] :

$$p(x) = \sum_{k=1}^K \phi_k \mathcal{N}(x|\mu_k, \Sigma_k) \quad (1)$$

where,

- Σ_k is the covariance matrix of k -th cluster, where k varies from 1 to 8 as estimated using BIC.
- K is the number of components/sub-populations in the mixture model.
- ϕ_k is the mixing coefficient/weight for the k -th Gaussian component with the constraint that the sum of $\phi_k = 1$.

$$\sum_{k=1}^K \phi_k = 1 \quad (2)$$

The multivariate Normal Density Function of d -dimension, and covariance Σ is given by [15] :

$$\mathcal{N}(x|\mu_k, \Sigma_k) = \frac{1}{\sqrt{(2\pi)^d |\Sigma_k|}} \exp\left(-\frac{1}{2}(x - \mu_k)^T \Sigma_k^{-1} (x - \mu_k)\right) \quad (3)$$

Being able to represent the distributions of the coefficients of polynomial functions using GMM means that point cloud of data obtained after the laser scan and the polynomial coefficients obtained after the curve fitting can now be reproduced by just using the parameters of the GMM.

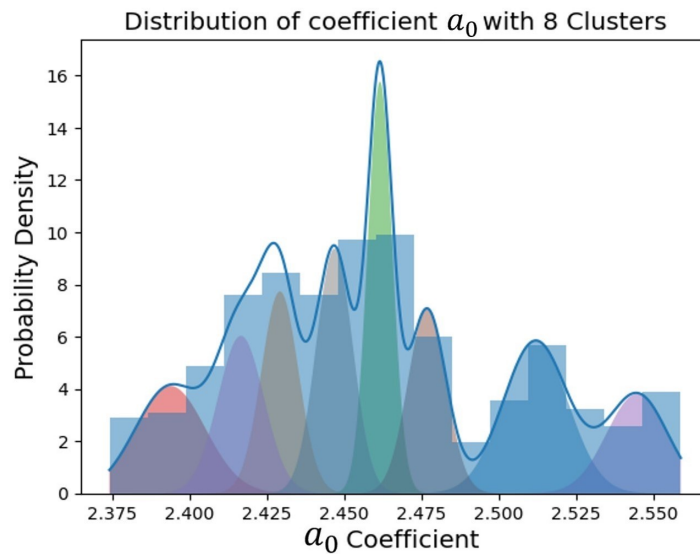


FIGURE 3 – Distribution of coefficient a_0 of a bead and its GMM using 8 clusters

The geometry of the beads thus generated is compared with the experimental beads for validation. The methodology of the modelling approach from experiments to simulation is illustrated in Figure 4.

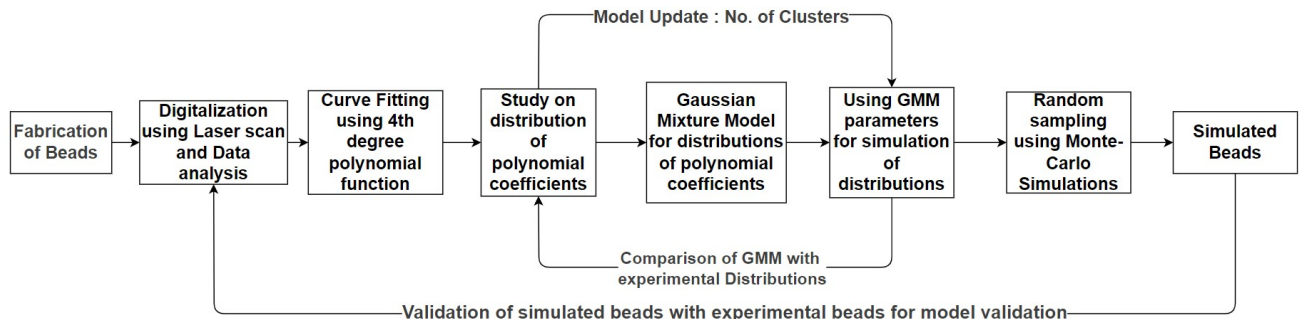


FIGURE 4 – Flow diagram illustrating the steps in the development of model for the simulation of beads

Using the thus parametrized distributions, random variables of polynomial coefficients can be generated. Monte-Carlo Simulations are done on these random variables to sample polynomial coefficients which can then be used to simulate beads [Fig. 5].

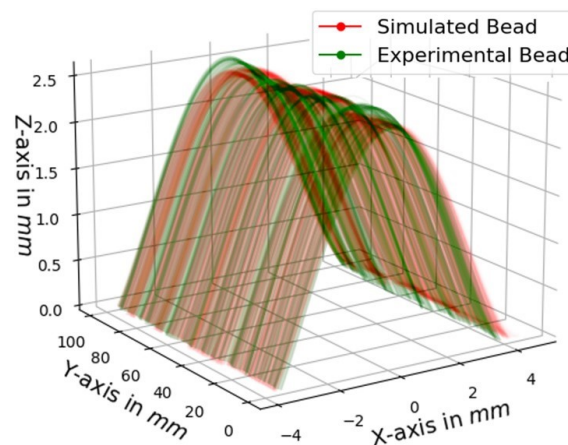


FIGURE 5 – Simulated and experimental beads

4 Results

The simulations of the beads using GMM returned promising results, as the distributions of the polynomial coefficients, bead width and bead height were found to be similar to that of the experimentally obtained distributions [Fig. 6].

For instance, the mean of the bead height and bead width found experimentally are compared with that of simulated beads in Table 4. It can be seen that the maximum percentage error found is less than 3%.

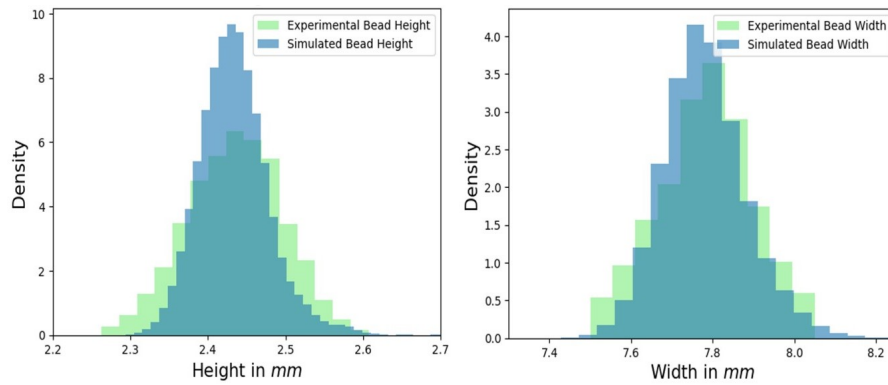


FIGURE 6 – Comparison between the distributions of bead height and width of beads for parameter P1

Parameter Set	Bead Width		Bead Width	
	μ (mm)	Error (%)	μ (mm)	Error (%)
P1	7.75	2.17	2.42	0.81
P2	6.74	1.26	1.91	2.15
P3	9.95	1.15	3.29	2.80
P4	6.99	0.21	2.24	0.75

Thereby, it can be seen that from a statistical point of view the simulation of WAAM-CMT beads can be successfully done using GMM, as the distributions of the two of the most important characteristics - bead height and bead width are found to be coherent with that of the experimental beads.

5 Conclusions

For fabrication techniques like WAAM-CMT where the surface irregularities can be profound, an observation based statistical approach can be used to develop prediction models as proposed in this study.

The use of GMM can be considered as a viable method for modelling as it is proved to be able to successfully simulate the experimentally obtained beads. It can be seen that the numerically simulated bead is similar to that of the corresponding fabricated bead.

The variation in the bead form found at some locations could be because of the random sampling approach using the GMM based distributions which neglects the continuous variation of the bead surface.

Thus, a methodology to simulate WAAM-CMT beads is developed and the beads thus simulated are found to be coherent with the bead geometry of the fabricated beads. As this numerical approach can simulate WAAM-CMT beads successfully, these can be later used to simulate overlapping beads and layers of the process. The proposed methodology can be used not only for WAAM-CMT beads but also other processes for which the observed parameters under consideration exhibit significant variations and does not follow a classical probability Density Function.

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