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### ► **To cite this version:**

Andrea Gherzi, Lara Monformoso, Sara Bagossi, Sonia Palha, Ornella Robutti. Use of conceptual blending to investigate students approaching functions in an emerging embodied technology. Proceedings of the Fourteenth Congress of the European Society for Research in Mathematics Education (CERME14), Free University of Bozen-Bolzano; ERME, Feb 2025, Bozen-Bolzano, Italy. <hal-05307171>

**HAL Id: hal-05307171**

**<https://hal.science/hal-05307171v1>**

Submitted on 10 Oct 2025

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# Use of conceptual blending to investigate students approaching functions in an emerging embodied technology

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*When emerging technologies meet embodied activities, new mathematical learning opportunities arise together with new questions about underlying conceptualization processes. In this paper, we present the results of a study involving secondary school students developed upon the use of an emerging embodied technology, GeoGebra plus. To analyse the students' cognitive processes, we used conceptual blending (Fauconnier & Turner) and we built a model to describe the complexity of integrating embodiment, a new technology and mathematics. In this paper we show that GeoGebra plus can support the object-oriented and dynamic view of functions. Moreover, we suggest that such technologies are the engine to move from the paradigm of 'humans-with-media', introduced by Borba and Villarreal, towards a 'humans-in-media' one.*

*Keywords: Conceptual blending, emerging embodied technology, GeoGebra plus, function, secondary education.*

## Introduction

Over the last two decades, mathematics education research has provided evidence for the benefit of using digital technologies in mathematics learning (Clark-Wilson et al., 2020; Drijvers & Sinclair, 2024; Lagrange & Kynigos, 2014). In these digital environments, several embodied approaches have been adopted to foster functional thinking, for instance, bimanual coordination connected to a monitor to engage with proportions (Abrahamson & Sánchez-García, 2016), bimanual coordination through a multi-touch digital interface to promote correspondence and covariation approach (Wei et al., 2023). More recently, even the so-called emerging technologies (Veletsianos, 2010) (e.g., augmented reality, virtual reality, and artificial intelligence) have started to be used in mathematics learning, and some of their affordances exploited to incorporate the use of the body in the digital environments and support the learning process.

In this paper, we will examine a pilot study (consisting of two experiments) designed to support an object-oriented and dynamic view on functions making use of the emerging embodied technology GeoGebra plus (GGB+ from now on). GGB+ is an extension of GeoGebra (GGB) developed by David Hornsby (JKU, Linz, Austria). By adding a pose-estimation model (Posenet) via camera to GeoGebra, GGB+ enables embodied interaction with digital mathematical elements. GGB+ identifies specific points of the user's body (e.g., eyes, shoulders, elbows) and builds an avatar of the user in the GGB graphics view that moves in accordance with the user's movements. In this way, the avatar coexists with representations of mathematical objects (e.g. points, lines, or shapes) interacting with them both statically and dynamically. As a result, when the user moves in front of the screen, their own avatar mirrors the movements, allowing them to interact with mathematical objects and to

receive different visual types of immediate feedback from GGB+ (e.g., the simultaneous movement of the avatar with the user's body, a green Ok! appearing on the screen, or numerical values associated with some coefficients changing with the body's movements). The immediate feedback loop guides the user to align with specific poses or movements and, from now on, we will refer to the process of intentionally adjusting the body posture or pose in response to real-time feedback displayed on a screen with the word *positioning*. This experience goes beyond typical multisensory interactions with technology in mathematics education (Arzarello & Robutti, 2010; Nemirovsky & Borba, 2003), because here the user, through the avatar, co-exists with the representation of mathematical objects.

The pilot study here presented was carried out in the context of the European Widera project TransEET – Transforming Education with Emerging Technologies – and we use the *conceptual blending* model (Fauconnier & Turner, 2002) to describe the students' cognitive processes that emerge in the learning experiences with GGB+. Such a model was chosen for its suitability to describe the complexity of integrating embodiment, a new emerging embodied technology and mathematics.

## **Theoretical framework**

The theory of conceptual blending, developed by Fauconnier and Turner (2002), explains how we unconsciously and instantly integrate inputs from diverse knowledge domains to create new knowledge. This process is represented through a network of mental spaces: *input spaces* (knowledge domains), the *cross-mapping space* (connecting corresponding elements), and the *blended space* (the outcome of merging and elaborating inputs). The same inputs can produce different blends. Fauconnier and Turner also introduced *double-scope blends*, where inputs with distinct organizing frames (e.g., a computer desktop blending a desk and software) combine to form unique structures. These blends support specific actions or meanings, such as “moving a file to the trash bin.”

Earliest studies on multimodality, related to the field of cognitive science on embodied cognition (Wilson, 2002), supported mathematics education research in deepening and outlining the relevance of gestures and body movements in teaching and learning processes (e.g., Arzarello et al., 2015; Ferrari, 2020; Shvarts et al., 2021). Similarly, the construct of *grounded blends* was born (Yoon et al., 2011) within the framework of conceptual blending, with the aim of investigating how gestures can be endowed with and used to communicate mathematical meanings. In this type of conceptual blend network, one of the input spaces is the person's *real space*, including gestures and non-verbal communication that actively contributes to the elaboration of the final blend. Several studies adopting the conceptual blending model have also carefully reflected on the role of technology in conceptualization activities: for instance, Gregorcic and Haglund (2021) analyse a learning activity on celestial motion in which several digital tools are involved, offering various external stimuli, and each one of them (e.g., computer simulation or touchscreen device) is considered as an input space.

In light of the previous considerations, the research question guiding our study is: *What are the blending processes of secondary school students while exploring linear and quadratic functions in an emerging embodied technology such as GGB+?*

## Method

### Participants and research context

In this contribution, we will adopt the blending network introduced and described before to analyse two short pieces of data from the pilot study in the TransEET project.

The first experiment was conducted in Italy with a pair of 10<sup>th</sup>-grade female students from a linguistic secondary school. The experiment took place in the context of a summer maths stage and lasted 75 minutes. The activities were administered by the first and third authors who were present during the whole duration of the experiment, with the role of participant observers. The teacher informed the authors that the students had not studied the Cartesian plane or functions, but they may have encountered them in their previous grades.

The second experiment was conducted in the Netherlands and involved six 10<sup>th</sup>-grade students (four girls and two boys) of a secondary school devoting special attention to dance education. The activity took place during an extracurricular class, lasted 90 minutes, and was structured as a field experiment, with the second and fourth authors present with the role of participant observers. The students had already studied functions (e.g., linear, polynomial, exponential functions), their representations and their general properties. In the data selected for this study, we will present the activity of the two girls.

### Task design

In both experiments, students received tasks to be solved in GGB+. After an initial exploration, aimed at letting students familiarise themselves with the new technology GGB+, three structured tasks followed, each to be solved in a specifically designed applet: (1) *GeoGebra Mirror*, in which students should fit the graph of given linear or quadratic functions on the screen with their body; (2) *Get the coefficient*, in which students' positioning affects linear/quadratic function coefficients, whose values vary in real time (Figure 1, this to enable students connecting how the changing of the coefficients affect the graphical representation; and (3) *Level Up*, in which students should reproduce the graphs of given linear or quadratic functions whose formulas are shown on the screen and so connect two different types of function representation.

In the first experiment, given the little mathematical background of the two students, the task focused only on the case of linear functions, while in the second experiment, the task involved both linear and quadratic functions.

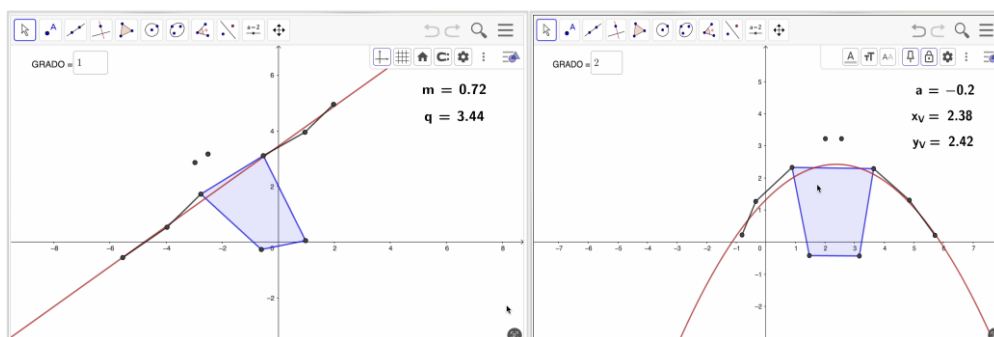


Figure 1: Interface of a GeoGebra+ applet called *Get the coefficient*

In this contribution, we focus on the second task (see Figure 1). In this task, groups of two students were involved in solving the task in GGB+ and filling in a worksheet which asked for a reflection on what they observed. One student at a time moved in front of the screen to position her body in order to: statically represent a function (linear or quadratic) and dynamically vary her positioning. Then both the students observed how the variation of the position of the one in front of the screen determines a change in the values of the function's parameters ( $m$  and  $q$  if  $y=mx+q$ , or  $a$  and vertex coordinates if  $y=ax^2+bx+c$ ), and which of these variations were involved in this change. After this, students filled in their worksheet asking to reflect first on the static representation of the student position in front of the screen and corresponding to a linear or quadratic function, then to the dynamic representation of the variation of such positions corresponding to function transformations (e.g., translation). In this task, by means of the avatar in GGB+, students can link the meaning of the parameters' values to the graphs of the functions, building a mathematical meaning for them.

### **Our adaptation of the conceptual blending model**

In our study, the emerging embodied technology GGB+ is considered a micro-learning environment by itself. The construct that seems most appropriate to frame the richness of GGB+ is that of the *double-scope blend*. Indeed, it can be seen as the result of the blend of input spaces with different organising frames, in our case the mathematical Cartesian system offered in GeoGebra and the virtual avatar controlled by the human body.

The cognitive processes can be represented through a conceptual blending network, adapted from the one by Fauconnier and Turner (2002) in which continuous lines indicate the matching and the cross-space mapping between the inputs and dashed lines indicate the connections between inputs and the blended space (see Figures 2 and 3); we coloured some words and images to highlight the corresponding elements between different spaces. The network consists of three input spaces: (i) the *maths space* referring to the mathematical knowledge that students possess and on which they draw, (ii) the *real space* with the bodily expressions of conceptualization, and (iii) the GGB+ environment intended as both an input and a blended space, that is a double-scope blend (the input spaces contributing to GGB+ as a double-scope blend in Figures 2 and 3 are grey). While we recognize (i) and (ii) the role of input spaces, (iii) is the result of a further blend involving (iii-a) maths in GGB and (iii-b) the user's avatar positioning as input spaces. Therefore (iii) should not be seen as a standalone technology, but as an integration of students' thoughts, words and actions that makes sense only within the GGB+ environment (e.g., a student saying "If I lift the function..."). It also works as an interface providing multiple forms of feedback, triggered by both students' body movements and the GGB tools.

### **Data collection and data analysis**

In both experiments, we video-recorded the students' activity with a camera and the computer screen recording. The videos of students' activity and the screen recording for each experiment were synchronously coupled, to be seen side by side. For the preliminary analysis presented in this contribution, we selected two critical incidents (Tripp, 2011), namely representative episodes of the students' activity, relevant to our research question. Such incidents were selected because they

revealed the blending of the three main conceptual components at play: maths, body, and technology. These two episodes were transcribed and then qualitatively analysed using the blending network.

## Results

In this section, we contextualise each of the two critical incidents (the first focused on linear functions and from the Italian context, the second focused on quadratic functions from the Dutch context), and provide a qualitative analysis both in a descriptive form and using a blending network representation, adapted from Fauconnier and Turner (2002).

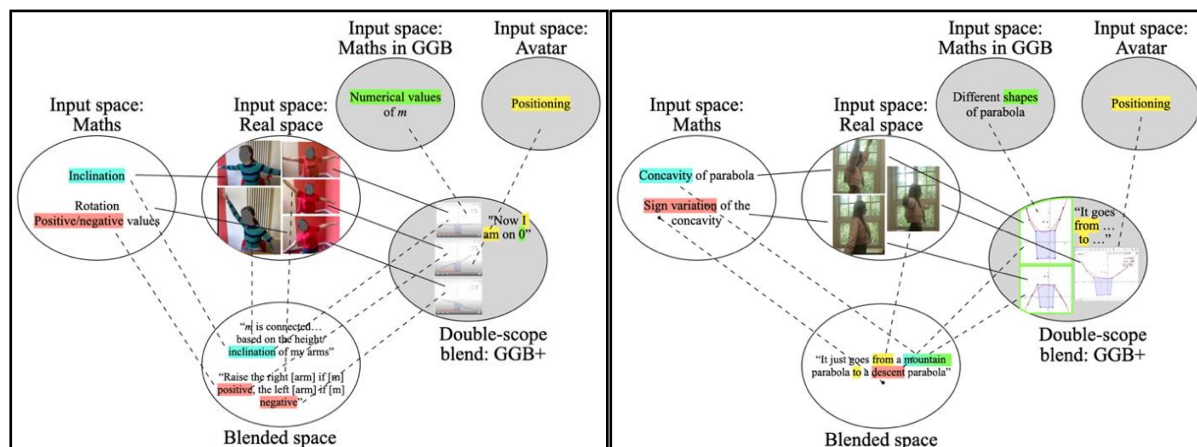
### Critical incident 1 on linear functions

During the task, the researcher asked the two students (S1 and S2) how the number  $m$ , displayed in GGB+ while working with linear functions, was associated with their body position. S2, who was sitting at the table, started moving her open arms in different positions, describing then her variation of position as a “circular movement”. S1, who instead was in front of the screen, positioned her arms horizontally ( $m = 0$ ) and then started raising her shoulders while claiming that it was connected to her arms height, and then to her arms inclination. The researcher then asked “How is this number [ $m$ ] connected to your positioning?”. And S2 claimed: “Now I am on 0”, a sentence making sense only *in* the GGB+ environment where her avatar’s positioning can be connected to a numerical value (0). The horizontal positioning of the arms was simultaneously assumed also by S1. And again, under the researcher’s prompts, S2 inclined her arms differently to have on the screen a representation of  $m$ , first positive and then negative. S1 intervened by adding that she raised “the right [arm], if [ $m$ ] positive, the left, if [ $m$ ] negative”. Thereby the blending network representation (see Figure 2) shows that S1 and S2 linked: (i) their mathematical knowledge about positive/negative values and inclination (*maths space*); (ii) the arm movements done by S2 in front of the computer (*real space*); and (iii) the numerical values of  $m$  provided by GGB+ and controlling the linear function created through her arms (*GGB+ space*). The conceptual blending of these elements generates new knowledge in which the students collectively elaborate on the mathematical meaning of  $m$  by connecting the variations of body position to its corresponding numerical values.

### Critical incident 2 on quadratic functions

In the task students were asked to think about how the shape of the quadratic function had changed when the concavity changed from positive to negative. In this fragment, after having read the task, S3 asked S4, “What changed in the parabola with [...that] movement?”. While speaking, S3 was moving her arms up and down in front of the computer, in order to test and explore how the quadratic function’s graph was changing on the screen. S4 looked at S3, while she was moving, and then at the avatar on the screen, which was embodying a quadratic function fitting to the avatar’s arms points. Hence, S4 replied “It just goes from a mountain parabola to a descent parabola”. S4 elaborated on already known mathematical concepts, i.e. a mathematical property of the quadratic function (mountain and descendent, to refer to concave-down and concave-up quadratic function) conceiving it as a mathematical object. The network representation of the blending process (see Figure 3) shows that S4 linked: (i) her mathematical knowledge of the concavity of a parabola and its sign variation (*maths space*); (ii) the arm movements done by S3 in front of the computer (*real space*); and (iii) the

feedback provided by the avatar controlling a quadratic function through its arms (*GGB+ space*). The conceptual blending of these elements generates new knowledge: S4 elaborated on a mathematical transformation by linking bodily movements to mathematical expressions.



Figures 2 and 3: Network representation for critical incident 1 and 2 on linear and quadratic functions

## Concluding discussion

In this study we introduced an emerging embodied technology, GGB+, developed within the TransEET project, that can support meaningful learning of abstract mathematical concepts by providing students with means to explore and to grasp these concepts in embodied forms that make sense to them. We provided two critical incidents of the use of this technology, and we discussed its potential for learning about functions adopting the analytical lens of conceptual blending.

First, GGB+ allows the students to build their own embodied representation of the mathematical objects, which were in this case linear and quadratic functions, and explore their properties. The technology, together with the proposed tasks, engaged students in making conjectures, testing and adapting them starting from their embodied experience. The novel aspect of GGB+ is that students position themselves as the mathematical object, thereby simulating specific functions with their bodies. For instance, in critical incident 1, students discovered the slope  $m$  by moving their arms and causing a change in the avatar's position, while in critical incident 2 they explored a change of concavity in a quadratic function by means of variation of their own position. The reasoning accompanying these variations is directed to noticing properties rather than calculations helping students to build and develop an object-oriented and dynamic view of the function concept. Wei et al. (2023) pointed out that these approaches to functional thinking are scarcely promoted in learning practices and so our study practically contributes to research on this matter.

Secondly, in this experiment, we elaborated on the model of conceptual blending to include learning situations in which technology allows for the embodiment of mathematical objects. Thanks to affordances of GGB+, which allow the user to "live" within a world of mathematical representations, this model aligns with the perspectives like those of Sinclair et al. (2023), and as earlier proposed by Borba and Villarreal (2005) with the *humans-with-media* paradigm, fostering a more integrated view

of human and technological interaction. For that reason, we frame this new kind of technology as emerging embodied technology, in a paradigm that we want to call *humans-in-media*, to strengthen the one by Borba and Villarreal. This model provided the analytical method to study the critical incidents and to reveal the special role of GGB+ in them. Framing GGB+ as a double-scope blend within our conceptual blending network (see Figures 2 and 3) provides a means to describe the mathematical concepts as understood by the students while keeping track of the elements that contribute to it (the conceptualization process). This was valuable in our experiment because it highlighted a new form of technology which students are working *in* rather than just working *with*. The network revealed how students were interacting and reasoning *in* a technology and the mathematical meanings built through it. Future research is needed to analyse longer-term use and learning with GGB+ and compare it to other humans-in-media technologies. Further deepening of our study should also consider how the cultural background of the involved students determines their processes of building mathematical concepts, the way they approach the tasks and engage within the technology.

## Acknowledgment

This study is funded by the HORIZON-WIDERA project of the European Research Executive Agency (REA) named ‘TransEET’ (Transforming Education with Emerging Technologies - Project No. 101078875). The EU’s support for the production of this contribution does not constitute an endorsement of the content, which reflects the views only of the authors.

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