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# The implementation of mathematics education programmes: fidelity to design and fidelity to theory

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*Considering fidelity and adaptation are important to the implementation of mathematics education programmes for designers, evaluators, and users. Researchers sometimes view fidelity and adaptation as opposites, where fidelity refers to faithfully implementing processes, and adaptation deviates from this. However, for some mathematics education programmes this is a false dichotomy. An expanded understanding of fidelity can address this in cases where fidelity to programme theory or causal assumptions is important rather than fidelity to planned implementation or design. The differences between these two forms of fidelity and the value of considering both are illustrated by considering a set of four programmes, each with multiple components, that aim to improve outcomes and progression for disadvantaged pupils in England.*

*Keywords: Fidelity, adaptation, implementation.*

## Introduction

The concept of fidelity is central to the study and evaluation of implementation in education generally and important in mathematics education (Munter et al., 2016). In the study of implementation in education, fidelity carries both technical and common language meanings and is also used in various ways in mathematics education (Jankvist et al., 2022). Adaptation is also important to innovation in mathematics education for designers, evaluators, and users, including when implementing research findings. Fidelity and adaptation are sometimes viewed as opposites. However, clarifying different meanings of fidelity dispels this false dichotomy. In this paper, I distinguish between fidelity to design and fidelity to theory.

Fidelity to design is the common meaning used in the study of implementation and means adherence to a programme's design, structure, and processes: doing what was intended and planned for (O'Donnell, 2008). Attention to fidelity to design allows for the identification of essential features of innovations and ensures these are implemented when programmes are replicated. Where the programme is not implemented as planned, attending to fidelity brings a focus to key moderating influences and contextual factors. However, where innovations are being developed, adaptation will be inevitable and desirable to develop an effective innovation. This will often be the case in early applications of research findings or in design research phases in many mathematics education innovations, but also when replicating or scaling to understand contextual influences (Cai et al., 2020). Further, there are features of mathematics education curricula, pedagogy, and professional development where flexibility, agency, and an adaptive orientation are essential to intended practice. In such cases, what is important is fidelity to the underpinning theory of how activity will lead to outcome, the core ideas of the innovation (Cai et al., 2020; Jankvist et al., 2022), and echoes arguments for flexibility within fidelity (Kendall & Frank, 2018).

These two meanings may not cover all cases. In some cases, where very high levels of adaptation may be desirable or acceptable, for example, in pilot or exploratory stages of innovations, a further form of fidelity may be relevant. This is fidelity to programme goals. McNeill et al. (2018) report an example in science education where they contrast fidelity to procedure with fidelity to the goal—in the case they consider the goal of developing scientific argumentation. It is not clear in such cases whether it is then meaningful to consider activity as implementation, and so I note this third possible meaning of fidelity but do not consider it further in this paper.

In the next two sections, I develop the argument for distinguishing the two forms of fidelity. I consider fidelity in implementation science in general and then briefly summarise discussion of fidelity in mathematics education, drawing on examples from mathematics education innovations. Following this, I illustrate the application of the two forms of fidelity through an example of four programmes that aim to improve outcomes and progression for disadvantaged pupils in England.

## **Fidelity and implementation**

As stated, fidelity is a key concept in implementation science, and proponents of its importance have developed various models of different aspects of fidelity and how frameworks to guide its measurement are based to varying extents on conceptual theorising or reviews of how fidelity is used and applied by researchers and others (see Gage et al., 2020 for examples). These frameworks are concerned with the common meaning of fidelity as implementation of the designed programme. For the purposes of distinguishing between the implementation of design and the implementation of theory, and given the limits of this paper, I illustrate with components of an early influential model of fidelity (Dane & Schneider, 1998). In summary in this model, fidelity consists of a combination of:

- adherence of delivery to the prescribed programme
- exposure of recipients to the programme activities
- quality of delivery
- responsiveness of participants
- differentiation of participant experience by planned activities only.

Later developments of models may have different or additional components, but in common with the above features, the focus is on the relationship between the planned activities in a programme and implemented activities.

In the study of implementation, there are different views of the value of fidelity, with two positions that can broadly be summarised as pro-fidelity and pro-adaptation perspectives (Century & Cassata, 2016). A pro-fidelity perspective considers it important that effective practices should be implemented without deviation and that any changes should be made in a design phase. In this view, users should be encouraged and supported to implement with fidelity. A pro-adaptation view considers that high fidelity of implementation may be hard to achieve in practice and innovation users will respond to context, and that those implementing innovations are able to add effective strategies or improve the innovation. Such adaptations can mean innovations can be more widely applied beyond any one specific context, and this can support sustainability (Century & Cassata, 2016). For

new research-informed innovations, adaptation in use aligns with a different model of the relationship between researchers and designers and practitioners in which there is collaboration and co-design. Practitioners are positioned as agentic rather than end users.

Underlying the pro-adaptation perspective is an emphasis on the underpinning causal assumptions for a programme or its component parts or the programme theory (Ahl et al., 2022; Munter et al., 2016). ‘Theory’ is used in multiple ways with nuanced meanings, including when considering programme implementation (Helenius et al., 2023). It is often used as part of a compound phrase, ‘Theory of Change’, but this term too has different interpretations, often being used to mean the visual representation of programmes casual structure (Boylan, 2023; Chen, 2012). Other terms are used instead of or alongside ‘theory’ to refer to the same or similar phenomena, such as determinants (Chen, 2012; Helenius et al., 2023). though often used with reference to implementation and contextual determinants (McLoughlin et al., 2021). Others use ‘mechanism’ to identify causal relationships (Lewis et al., 2018), often drawing on realist social theory (Coldwell & Maxwell, 2018). Conversely, statisticians may more commonly consider ‘assumptions’ in inferential models (Colnet et al., 2024). However, regardless of language used, these different terms draw attention to how designed programme processes and structure may cause intended outcomes. Thus, fidelity to the casual relationships of a programme or a component is fidelity to programme theory.

From a pro-adaptation perspective, considering fidelity to theory as a separate construct is helpful in addressing what adaptations are acceptable or desirable for each specific innovation or component. Acceptable adaptations support the enactment of the programme theory, and desirable adaptations are those that help to amplify the programme theory mechanisms. In cases where programmes are in a mature stage of development, including when core components and essential fidelity are established, then fidelity to design will often include fidelity to theory, as the theory is embedded in the programme design, process, and structure.

## **Mathematics education and fidelity**

Turning to mathematics education specifically, it is important to consider implementation in connection with the specifics of the discipline and the mathematics education ecology (Koichu et al., 2021). In mathematics education, specifically, adaptation is likely and may be desirable and intended (see, Ahl et al., 2022, for an example). In mathematics education, a central concern is for the interplay between mathematical content, teacher activity, the learner, and the learning environment. While the conceptualisation of this interaction varies in different paradigms, it is significant in arguably all traditions in mathematics education and underscores the significance of responsive adaptation of practice. Adaptation is particularly important in pedagogical approaches that stress the importance of both teacher and learner agency in the learning process. Beyond individual classrooms, similarly adaptive practice is important to the implementation of mathematical education programmes.

For some mathematics education practices, fidelity and adaptation are a false dichotomy. This is because fidelity to the underlying principles of the programme or activity may require adaptation of materials or practice. I take as an example the implementation and evaluation of a professional and curriculum development programme in England for lower secondary pupils—Realistic Maths Education (RME) (Demack et al., 2022). In the RME programme, one aim was for teachers to move

towards greater use of models and representations in teaching than their previous practice. Teachers received curriculum and teaching resources, along with professional development, to encourage the use of bar models, ratio tables, and double number lines. It was thought that if teachers could choose and use different models in different ways, it would first help students learn more and then help students learn how to choose the right models to help them solve problems. Given this, although teacher use of programme materials was important (fidelity to design), the fact that these were used flexibly and with adaptation was also important (fidelity to theory).

A typology of three types of professional learning frequently found in innovations provides a heuristic to identify whether fidelity to design or fidelity to theory, or both, are important specifically for mathematics innovations that include professional development (Boylan & Demack, 2018). Technical professional learning aims for teachers to use new techniques or use new materials in specific ways. Here fidelity to design is most relevant. Pedagogical professional learning focuses on growth and change as a professional through reflective and agentic practice. Here fidelity to theory is most relevant. Curriculum professional development combines professional learning with the use of research-informed curriculum materials. Here, as in the RME programme, it is likely that both forms of fidelity should be considered.

Regardless of the forms of professional learning or the value in the typology, for any given programme or component of a programme, three possibilities are possible in terms of the importance of the two different forms of fidelity (for instances where fidelity is important at all):

- fidelity to design
- fidelity to theory
- fidelity to design and fidelity to theory

### **The MX programmes as an illustration**

I now consider an example to illustrate the application of the different forms of fidelity in practice. This example is a set of four programmes – the MX programmes - funded by the Maths Excellence Fund (n.d.) in England, informed by research on relative attainment of students across their education experience in relation to their demographic characteristics (Noyes et al., 2023). The programmes aim to support disadvantaged students to attain highly at 16 and for more to progress to A-level and university mathematics, with 46 secondary schools participating. The National Foundation for Education Research is evaluating the four programmes, and I advise on the application of Theory of Change to the evaluation. The four programmes are led by several types of organisations.

Each programme is comprised of multiple components (see Table 1), with some types of components common across programmes and some specific to individual programmes. The MX programmes' overall meta-theory posits that addressing systemic and long-standing inequalities in mathematical outcomes requires multiple programme components that can have an additive impact.

Some components involve activities similar to activities often reported on in mathematics education research: parental engagement, teacher training and development, classroom technology use, tutoring, and mentoring. Others are more specific to the programme or to the context in England:

- tracking and monitoring target groups of pupils and collecting data on outcomes

- In-school champions who coordinate and encourage participation
- An extended, more challenging mathematics curriculum for 11- to 14-year-olds
- Additional supplementary maths qualification for 16-year-olds
- Y6-Y7 transition to support pupils' mathematical learning as they move from primary (elementary) to secondary (junior high school) phases
- Maths enrichment through after-school maths clubs or mathematics events at universities
- Interschool competitions between local schools inspired by maths Olympiad competitions.

### **Fidelity of design and theory in implementation and evaluation**

Table 1 summarises the different components for the four programmes and, for each programme, posits whether fidelity to design (d), fidelity to theory (t), or fidelity to design and theory (d & t) is most relevant. Empty cells show that the component is not part of a programme. These ascriptions are an interpretation informed by my analysis of programme implementation plans and logic models and then checked with the team responsible for oversight of the four programmes for the funder. For some components, expected outputs from activities are identified, for example, the number of students participating in additional coaching sessions. For others, the programme design indicates that activities will be developed and modified on specific sites.

We can attribute fidelity to the design, if there are specific indicators of adherence to the design, such as the number of students participating in specific activities with defined processes and structures. In the MX programmes, pupil tracking systems most fully exemplify this. Components that demonstrate fidelity to theory are those that allow for significant flexibility in implementation. So, for example, in three programmes, an in-school champion will be identified, but what they will do is not predetermined. In one programme, the in-school champion will be a new member of staff, working closely with the target groups of students undertaking activities that are specified in other programme components. Two types of components are considered to have both fidelity to design and relevance to theory. Some activities have plans for the number of students participating or the number of sessions offered, but programme teams will develop or tailor the details in practice to suit identified needs. Alternatively, the components feature both pre-designed subcomponents and ones that will be developed.

**Table 1: Types of fidelity relevant to selected programme components**

<b>Programme components</b>	<b>United learning</b>	<b>East Maths Community</b>	<b>MEI</b>	<b>Co-op Academies</b>
Tracking and monitoring	d	d	d	d
In school champion	t	t	t	d & t
Parental engagement	t	d		t
Teacher training and development	d		d & t	d & t

Programme components	United learning	East Maths Community	MEI	Co-op Academies
Community learning hubs		t		
Classroom technology use				t
Extended KS3 maths curriculum	d		t	
Additional maths qualifications	d	d	d & t	d
Tutoring	d	d & t	d & t	d & t
Mentoring			d	d & t
Y6-Y7 transition	t			
Maths enrichment	d & t	d & t	t	d & t
Inter-school competitions		t		

Identifying forms of fidelity that are most relevant to components supports Theory of Change development and evaluation activity. Where fidelity to design is relevant, it is important to consider appropriate fidelity measures (O'Donnell, 2008). When fidelity to theory is considered important, fidelity measures might not be suitable. Instead, when developing Theory of Change models and evaluation activity, the underpinning programme theory relevant to the component under consideration should be examined.

## Conclusion

I have considered two forms of fidelity: fidelity to design and fidelity to theory. Fidelity to design corresponds to common meanings of implementation fidelity. Fidelity to theory is important, particularly for programmes in early stages of development where essential components are not yet identified.

It is also important for programmes in which adaptation, agency, and flexibility of mathematics teachers are central to programme aims and important to intended outcomes. Considering fidelity to programme theory. provides a means to assess whether adaptations are potentially useful additions to programmes or should be viewed as a failure of implementation and supports the articulation of causal theory. This may be particularly important when replicating or scaling innovations (Aguilar et al., 2023; Cai et al., 2020). The distinction between these two types of fidelity potentially has wider applicability beyond mathematics education.

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