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# 3DMP and TPACK in professional development: Combining actions and knowledge

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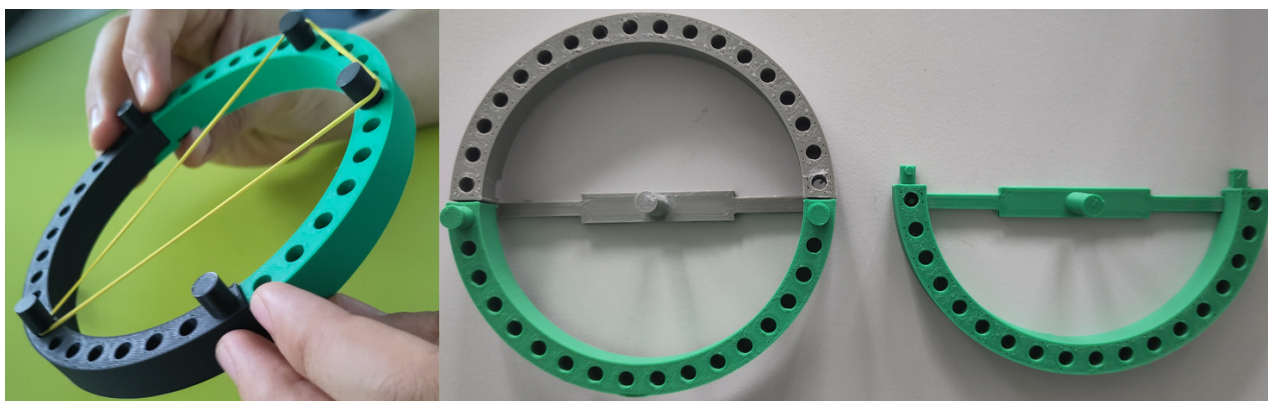
*3D-printing is more present than ever, with new advancements made constantly. Teachers can use 3D-modelling and print tailored learning objects for their students, simultaneously fostering their own TPACK knowledge, making this context very suitable for student teacher education. Still, research on student teachers, 3D-modelling and printing and TPACK is lacking. Therefore, we present our seminar for mathematics teacher education, and how we use our model to describe the 3D-modelling and printing process in combination with TPACK to gain insight into the knowledge needed for the individual steps of the process. This is needed to efficiently develop teacher training and empower teachers with 3D-printing. The qualitative study is presented, and first indications of the combination of 3DMP and TPACK are shown.*

*Keywords: 3D-Printing, TPACK, Manipulatives, Interview, 3DMP*

## 3D-printing and education

Gone may be the days when teachers cannot realize their wish for a tailored manipulative (Stigberg et al., 2022) for their classroom. Through 3D-printing, educators are empowered and do not need to rely on commercial solutions for their learning object wishes, but can instead create their own.

Advances in 3D-printing make the technology viable for private and educational use, yet their implementation in schools is hindered still due to a lack of teacher education among other reasons (Ford & Minshall, 2019; Pearson & Dubé, 2022). Therefore, we have created a seminar for teaching student teachers on how to use 3D-printing to create their own manipulatives (Läufer & Ludwig, 2023b; Stigberg et al., 2022), such as the one in figure 1:



**Figure 1: A 3D-printed manipulative made by a student teacher aimed to show Thales' Theorem and the theorem for the peripheral angle and the central angle using a rubber band. The black pieces where the rubber band sits can be inserted into the holes.**

## The seminar

The target group for our elective seminar is mathematic education students, who already completed the two basic courses, “how to teach algebra” and “how to teach geometry”. We have about 10–12 students every semester, which spans 12–15 weeks. At first, we have an input section (4-5 weeks), outlining the general process, didactics, the tools, and including videos and assignments to learn basic 3D-modelling. Afterward, we have the hands-on section, where the students work on their own projects alone (9-11 weeks). During this phase, we have a first presentation of prototype 1. At the end of the semester, we have presentations of a second prototype (1-2 weeks). The phases are also shown in figure 2. The assignment given to the student teachers is: “Fabricate your own manipulative for a fictional mathematics education lesson, that (1) cannot be bought, (2) is suitable for mathematical education, (3) is complex to some degree, and (4) is 3D-printed.” The example in figure 1 cannot be bought anywhere (criteria 1), but must be fabricated by oneself. It can be used to teach the circumferential angle theorem and more (criteria 2). There is complexity in the design and modelling (criteria 3), and it is 3D-printed (criteria 4). The presentation of the second prototype determines if the students pass or fail. An optional graded 10-page essay can be written for additional credit points.

## Theory: Process and knowledge

Research regarding educators (teachers and student teachers alike) 3D-printing and the needed competencies is lacking. The 3D-modeling and printing process (3DMP, (Läufer & Ludwig, 2023a) can be used to describe the steps taken to fabricate a manipulative for mathematical instruction. This model is split into phases (concept, model, prepare, print), tool usage and interim results.

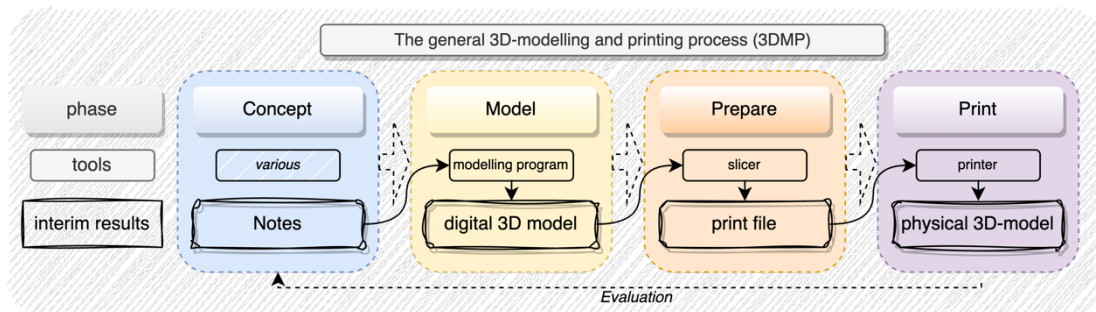


Figure 2: The 3DMP-Process (Läufer & Ludwig, 2023b)

At the beginning, the students have an idea which they want to realize. For that, they need to come up with a **concept** of how they intend to achieve it, answering questions like “What should it look like?”, “What can the students learn?”, “What are the dimensions?” and “What do the students do with it?”. Afterward, they begin **modelling** their concept in a 3D-modelling program. Our student teachers use a programming environment (OpenSCAD) to explicitly require pre-set notes, calculations and thoughts because, in our experience, programming makes it harder to “discover” a manipulative, skipping the concept phase, although this has not been researched yet.

Once the 3D-model is to our expectations, we need to **prepare** the model for printing. This is done in a so-called slicing program, which divides our 3D-model into layers (“slices”; therefore the program is called a “slicer”) and precise directions for our 3D-printer. In the **printing**-phase, the students prepare the printer, the print bed, the filament, and then print their manipulative themselves.

Due to the origin of the process model being a seminar for student teachers, which can pass or fail the seminar, the **evaluation**, which happens anywhere in the process, is only added for completeness but not considered in this paper. Students' evaluation of their manipulatives in a fail/pass seminar does not resemble in-service teacher behavior. Therefore, we focus on the concept, modelling, preparation and printing.

While knowing what the teacher's actions are and how to relate them to each other is needed to describe and conceptualize professional development for mathematics (student) teachers, the competencies and knowledge needed during these actions taken is not yet researched. To accommodate for this, we use the TPACK-Framework (Mishra & Koehler, 2006) in this explorative study, which concerns itself with the qualitative description of teacher knowledge and has been used to investigate student teacher tool use (Akkoç & Hangül, 2023; Engelhardt et al., 2023; Rocha, 2023).

## **Research Question**

Teachers want to have good learning materials for their students, while teacher *educators* aim to provide teachers with education resulting in the best possible competencies needed for teaching. Currently, when teaching 3D modelling and printing, we don't know which actions or phase corresponds to which area of TPACK. To fill this gap, especially to accommodate the growing need for teacher education, studies are needed that investigate the connection between these two models. Therefore, we combine the two mentioned theories into the following research question:

### **What phase of the 3DMP process corresponds with which area of TPACK?**

Due to the technical nature of the seminar and the used tools, as well as the mathematical contents that are to be used in and with the manipulative, we expect to see the Modelling, Slicing and Printing phases as mainly technological, and that especially the Concept phase is mathematical and pedagogical.

In the winter semester 2024/2025 this study will be repeated to provide additional data.

## **Method**

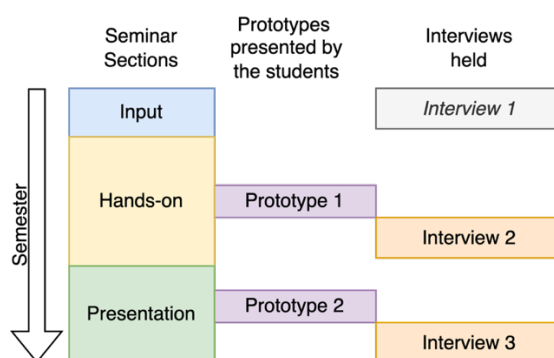
To answer this question, we are performing a qualitative study with guided interviews. The first is at the very beginning of the seminar to gain insight on the motivation of the students and if they arrive at the seminar with a goal in mind, which is not included in this paper. We then hold interviews during the semester every time they are to present a prototype (see figure 4). After every prototype, the students have gone through the 3DMP process at least once per prototype. The interview sections are

- (a) "Topic and Introduction", which serves as an introduction to the interview
- (b) "Concept", where we ask questions on what the students did before 3D-Modelling
- (c) "Modelling", where questions regarding the use of the modelling program are asked
- (d) "Slicing", where questions regarding the use of the slicing program are asked
- (e) "Printing", where questions regarding the printing process are asked
- (f) "Feedback", with which we want to see how much influence the instructor and the assistants have on the way the students work on their own manipulatives, and

- (g) “Self-locating on TPACK”, in where we intend to gain additional insights on how the students view their own work and where they locate each phase of 3DMP. The students have not been taught TPACK but were only shown a very reduced figure of a typical TPACK Venn-diagram.

For this paper, we focus only on (a) – (e). The interviews are not held by the authors (who also grade the students), but by an assistant that has no influence on the grade of the students. The study was piloted and the guided interview, as well as the following methods, were tested (Läufer & Ludwig, 2023b). The interviews are on a voluntary basis.

Overall, they present two prototypes, which do not have to concern the same topic:



**Figure 4: The phases of the seminar, the prototype presentations, and the interviews being held.**

After the semester, once the students’ grades have been entered, the authors get access to the transcribed, anonymized interviews, ranging from 45 min to 1h 20 min. The answers of the students are then numbered and, if they are very long, split into semantic parts, which are commonly between 1 and 6 sentences. These parts are then rated by 2 raters according to a piloted rating manual for TPACK and 3DMP respectively (see Läufer & Ludwig, 2023b). Due to the *overlap* of the 3DMP ratings and the TPACK ratings, we can then draw conclusions of the connections between the two theoretical models.

First they were rated into the base categories of TPACK (CK, TK, PK) whenever they showed mathematical (CK), pedagogical (PK), or technological (TK) knowledge. We are using the base categories of TPACK to not get lost in details and to avoid an overly complex construct of the 3DMP phases with TPACK, which happened during the piloting phase. In this study, CK is rated when the student uses or mentions explicit mathematical content (i.e. rotation, angles, formulas, ...). We rate PK whenever a student references school, education, pedagogy or didactics. TK is rated whenever technology is referenced, like the programs used, research done with computers, and other digital resources.

Afterward, they were rated into the phases of 3DMP every time the actions that the students describe can be attributed to a phase of 3DMP, especially if they reference a tool or an interim result. Every semantic part can be attributed to all phases, if the students talk about their actions in this phase. An example of the rating is shown here (translated from German):

Interviewer: “What notes or preparation did you make during conceptualization?”  
 Student: “(...) I would use the edge of a semicircle and its diameter, and I would simply construct a semicircle, have it printed, and then I imagined using triangles. I thought

of printing the triangles separately—some right-angled, some acute-angled, and some obtuse-angled. My first idea was that students, like very young children, would insert the triangles into the semicircle and see which ones fit. (...)”

This is one semantic part that has been agreed on by both raters to be **CK**, **PK** and **Concept**, due to the mathematical shapes (CK), the direct reference to students (PK), and the actions described that can be attributed to the concept phase.

To have methodological solidness, we calculated the inter-rater-reliabilities (IRR) not only in the form of Cohen’s Kappa for every category (TPACK) and phase (3DMP), but also calculated Moon’s Kappa, which is an abstraction of Fleiss’ Kappa (Moons, 2023). Through this, we gain insight not just into the general agreements (Moon’s Kappa), but also which agreement is difficult (Cohen’s Kappa). The “Self-locating on TPACK” section is only rated during the TPACK rating because the questions are not focused on the *actions* that the students have taken, but with their own *views* on 3D printing. Therefore, we also don’t include it in this paper, and it is not included in table 1.

## Results

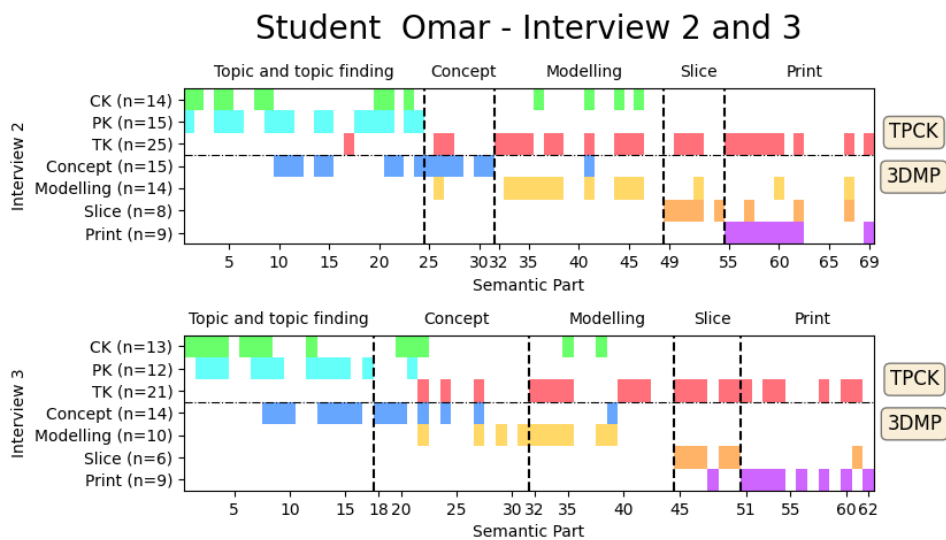
Every students’ 2<sup>nd</sup> and 3<sup>rd</sup> interview with the respective IRRs in form of Cohen’s and Moon’s Kappa values (Moons, 2023) is listed in the following table. The value ranges are 0.2-0.4 (slight), 0.41-0.6 (moderate), 0.61-0.8 (substantial) and 0.81-1 (almost perfect) according to Landis & Koch (1977 cited by Moons, 2023).

**Table 1: Descriptive Data on the interviews and the agreements of both raters in TPACK and 3DMP. Bold IRR values are  $\geq 0.6$ . All names are anonymized. *n* refers to the amount of overall ratings over both interviews. The numbers behind the names describe the sum of all semantic parts in interview sections (a)-(e).**

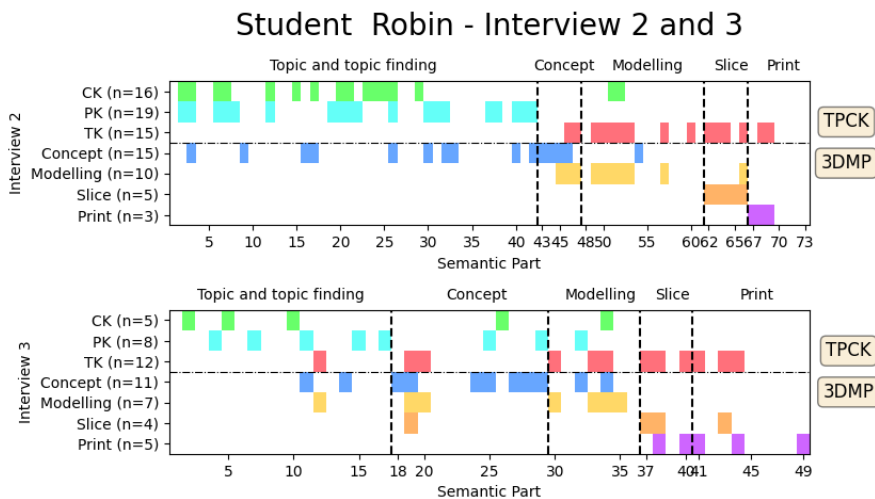
Student (# of semantic parts over both interviews without (f) and (g))	IRR TPACK				IRR 3DMP				
	Cohen’s Kappa			Moon’s Kappa	Cohen’s Kappa				Moon’s Kappa
	CK (n)	PK (n)	TK (n)		Conc. (n)	Mod. (n)	Slice (n)	Print (n)	
Tamino (111)	0.35 (15)	<b>0.67</b> (12)	<b>0.64</b> (34)	<b>0.67</b>	<b>0.65</b> (20)	0.51 (12)	<b>0.66</b> (10)	0.58 (14)	<b>0.79</b>
Sare (85)	0.38 (10)	0.44 (9)	0.57 (20)	<b>0.60</b>	0.45 (13)	0.45 (6)	<b>0.82</b> (5)	<b>0.70</b> (13)	<b>0.79</b>
Jonathan (99)	0.49 (25)	0.52 (15)	<b>0.72</b> (35)	<b>0.63</b>	0.54 (19)	<b>0.62</b> (13)	<b>0.76</b> (8)	0.46 (14)	<b>0.72</b>
Omar (131)	<b>0.75</b> (27)	<b>0.81</b> (27)	<b>0.70</b> (46)	<b>0.75</b>	<b>0.75</b> (29)	<b>0.65</b> (24)	<b>0.84</b> (14)	<b>0.69</b> (18)	<b>0.84</b>
Jürgen (110)	0.51 (18)	<b>0.68</b> (18)	0.54 (29)	<b>0.71</b>	<b>0.60</b> (17)	<b>0.70</b> (16)	<b>0.8</b> (9)	0.49 (9)	<b>0.82</b>

Robin (122)	0.31 (21)	<b>0.60</b> (27)	0.55 (27)	0.51	0.45 (24)	<b>0.69</b> (17)	<b>0.66</b> (9)	0.53 (8)	<b>0.77</b>
Emily (93)	0.48 (14)	0.53 (10)	<b>0.65</b> (23)	<b>0.70</b>	0.57 (19)	<b>0.77</b> (12)	<b>0.86</b> (9)	0.49 (10)	<b>0.80</b>

For TPACK, the interview with the highest agreement is Omar, while the one with the lowest is Robin. For 3DMP, it is Omar (highest) and Jonathan (lowest). For 6 out of 7 interviews, CK (mathematical knowledge) was the lowest, indicating that this was the least clear to identify. Looking at 3DMP, no such negative trend is visible, but in 6 out of 7 cases, **Slicing** was the one with the highest agreement. For every student, we also generated activity diagrams (Albarracin et al., 2019), as shown here for the students Omar and Robin:



**Figure 5: The Activity Diagram of Omar interview 2 and 3, split into TPACK and 3DMP coding. The top sections (vertical splits) divide the interviews into the parts (a)-(e). The x-axis shows the semantic parts (like the one in the interview above, it can span multiple sentences), while the y-axis shows the agreement of both raters for one semantic part. Each row has its own color to help distinguish itself.**



**Figure 6: The Activity Diagram of Robin interview 2 and 3, split into TPACK and 3DMP coding.**

The diagrams show, that the sections of the interview and the answers of the students are fitting with each other in the sense, that i.e. the student's answers are rated for Modelling inside the Modelling section. Still, the sections are heterogeneous, meaning that the students also mention other phases of 3DMP than the one that's currently the topic of the interview.

In the diagrams (including the 5 not shown in this paper), we can see clear trends that begin to answer the research question:

- 1) The Concept-phase is not easy to locate in TPACK, since the start of the interview "Topic and topic finding", where the students describe what their manipulative is about, already brings out a lot regarding the concept phase. A small link between CK, PK and TK to the Concept Phase can be seen.
- 2) The Modelling-phase seems to be linked to CK and TK.
- 3) Additionally, in all interviews, no semantic part is attributed to Modelling and PK, indicating that the modelling phase may have no pedagogical content.
- 4) Slicing and Printing are mainly connected to technological knowledge. There is no semantic part that is rated as Printing and CK/PK, while Slicing has a small connection to CK. This is mostly when the students rotate the objects in the slicer or change the size.

## **Limitations and Discussion**

In this paper, we have presented our study on 3D-Modelling and Printing and how we combine our Model describing this process with TPACK to investigate mathematical student teachers' actions through retrospective interviews. Due to the small number of students and the limited space, only basic trends can be surmised, as the detailed investigations of the student's actions are yet to be done.

While the agreement rates, especially with Moon's Kappa, are well and acceptable, the individual categories seem difficult to rate consistently for some students. This can be explained due to the retrospective interviews. The 3DMP model is not yet proven or investigated fully. Still, shows promise in being able to describe the student's actions appropriately for qualitative investigations. This is also since the phases directly relate to the tools and interim results, and the interview was sectioned into (among others) the usage of the tools. The research question is not yet fully answered, as additional interviews should still (and will) be held, and an analysis of additional data taken during the seminar is yet open.

## **Acknowledgements**

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