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Application of Normal Form Theory to Power Systems: A Proof of Concept of a Novel Structure-Preserving Approach

Bin Wang[†], Xavier Kestelyn[‡], Erfan Kharazian[†], Aurélien Grolet^{*}

Abstract—Differential algebraic equations (DAE) of power systems, classically obtained by aggregating all component equations and network equations, are a standard input of transient stability simulation software. To add a parametric approach to the provided transient stability simulation, the linear modal analysis is a commonly adopted approach because it provides insights about system dynamics, such as eigenvalues and eigenvectors, that can be used for control design. When the power system is more heavily loaded, the linear modal analysis becomes less accurate in terms of capturing system dynamic behaviors when subjected to large disturbances. Normal form analysis, based on a higher-order Taylor expansion of the system model, has been proved to capture the major nonlinear effects occurring in the system. Unfortunately, the normal form method is currently only applicable to ordinary differential equations (ODE) and imposes a recast of the DAE system onto an ODE system, which is feasible only under restrictive assumptions and is usually time-consuming. In this paper, we propose a novel method in order to directly apply the normal form theory to power system DAE without the need to recast DAE into an ODE form. A single-machine-infinite-bus (SMIB) system is adopted to show the feasibility and accuracy of the proposed method.

Index Terms—Power system modeling, differential-algebraic equations, ordinary differential equations, Normal Form theory.

I. INTRODUCTION

POWER system dynamics are dominated by the underlying differential-algebraic equations (DAE) associated with system dynamic components and their controls, as well as the nonlinear algebraic power flow equations. The ever-increasing penetration of renewables further introduces unprecedented complexities to the power system DAE.

Power system DAE can be solved numerically by transient stability simulation software tools, which are a widely adopted approach by the power industry, to obtain system trajectories in pre-specified “what-if” scenarios and the results can be used for dynamic security assessment. In addition to the time-domain simulation, analytical approaches can also be applied to power system DAE to generate useful insights about system dynamics and stability. When only small disturbances are considered, system dynamics can be approximated by a linear

dynamic system and the eigenanalysis can be applied to the associated linearized DAE around an operating point to infer system modal properties. However, when large disturbances are considered, system dynamics can easily become nonlinear due to, e.g., the high loading of the lines, such that the predictions by the linear analysis may become inaccurate.

Compared to relying only on the linearized dynamic system, including higher-order terms has been demonstrated to be more effective in designing damping controllers by the Power System Stability Subcommittee of the Power System Dynamic Performance Committee [1], where Normal Form was adopted to deal with the higher-order terms. Normal form analysis, an extension of the linear modal analysis based on a higher-order Taylor expansion of the system model, has been proven to capture the major nonlinear effects occurring in the system and offers some specific analytic tools for nonlinear characterization and analysis of power systems [1]–[9]. Unfortunately, apart from a sole trial in [10], normal form method is, up to now, only applied to ordinary differential equations (ODE). For DAE systems, a recast onto the associated ODE systems is needed, which is time-consuming and only applicable under restrictive assumptions such as constant impedance loads.

In this paper, we propose a novel method, based on [11], to directly apply normal form to power system DAE without the need to recast DAE onto ODE. The proposed method is applied to a single-machine-infinite-bus (SMIB) system in order to be able to compare it to the conventional Normal Form.

The rest of the paper is organized in the following way. Section II reviews normal form analysis on ODE. Section III presents the proposed application of normal form on a class of DAE systems with second-order nonlinearities. Section IV presents detailed results on an SMIB system and numerical experiments. Section V gives the conclusion and future work.

II. REVIEW OF NORMAL FORM THEORY ON ODE

A. Normal Form Theory on ODE

Consider a nonlinear dynamic system represented by ODE:

$$\dot{\mathbf{x}} = \hat{\mathbf{f}}(\mathbf{x}) \quad (1)$$

where \mathbf{x} is an n -dimensional state vector and $\hat{\mathbf{f}}(\mathbf{x})$ is an autonomous, real, n -dimensional vector field. Let $\mathbf{x} = \mathbf{0}$ be

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a stable operating point¹ such that $\hat{\mathbf{f}}(\mathbf{0}) = \mathbf{0}$ and assume that the vector field $\hat{\mathbf{f}}(\mathbf{x})$ is continuous and k -times differentiable.

First, the k -th order Taylor expansion of (1) at $\mathbf{x} = \mathbf{0}$ is computed:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \sum_{i=2}^k \hat{\mathbf{f}}_i(\mathbf{x}) \quad (2)$$

where $\hat{\mathbf{f}}_i(\mathbf{x})$ is the $n \times 1$ vector composed of homogeneous polynomials of degree i , and \mathbf{A} is the $n \times n$ Jacobian matrix.

Second, (2) is transformed to its Jordan form by applying the similarity transformation $\mathbf{x} = \mathbf{U}\mathbf{y}$, where \mathbf{U} is the matrix consisting of \mathbf{A} 's right eigenvectors. Eq. (2) then becomes:

$$\dot{\mathbf{y}} = \mathbf{\Lambda}\mathbf{y} + \sum_{i=2}^k \hat{\mathbf{F}}_i(\mathbf{y}) \quad (3)$$

where $\mathbf{\Lambda}$ is a diagonal matrix composed of \mathbf{A} 's eigenvalues, and $\hat{\mathbf{F}}_i(\mathbf{y})$ contains homogeneous polynomials of degree i .

The last step is to transform (3) into a simpler form, i.e., its Normal Form, by eliminating nonlinear terms as much as possible. To achieve that, the transformation in (4) is applied, where unknown coefficients are to be determined:

$$\mathbf{y} = \mathbf{z} + \sum_{i=2}^k \hat{\mathbf{h}}_i(\mathbf{z}) \quad (4)$$

$\hat{\mathbf{h}}_i(\mathbf{z})$ being homogeneous polynomials of order i .

To determine unknown coefficients in (4), substitute (4) into (3) and equate right- and left-hand like terms. The unknown coefficients can be computed recursively by eliminating i -th order terms as much as possible, leading to:

$$\dot{\mathbf{z}} = \mathbf{\Lambda}\mathbf{z} + \hat{\mathbf{G}}(\mathbf{z}) + \mathbf{H.O.T}^{k+1} \quad (5)$$

where $\hat{\mathbf{G}}(\mathbf{z})$ is polynomials composed of all monomials that can not be eliminated due to resonance conditions and $\mathbf{H.O.T}^{k+1}$ denotes terms at order $k+1$ and above [12].

B. Normal Form Theory on Power System ODE

Normal Form Theory on Power System ODE can offer the following benefits:

- Making it possible to analyze nonlinear dynamics while keeping the notion of linear modes, a tool that is universally used by the power system community.
- Capturing the non-constant oscillation frequency of nonlinear systems, which is missing in linear analysis [1].
- Correcting the linear participation factors and offering higher-order indices such as nonlinearity index, nonlinear interaction index, and nonlinear modal persistence measures, helping to measure the level and effects of nonlinearities in system response [13].
- Capturing the amplitude-frequency dependency of a nonlinear system, i.e., the backbone curve [14].

¹If not, it can be moved to the origin by a linear coordinate transformation.

- Offering a nonlinear stability index to analytically assess the nonlinear stability of the system [7] [9].

The above benefits come at a price, not surprisingly, in terms of computation and assumption/simplification, as listed below:

- *Computation*: particularly due to the need for a high-order Taylor expansion of the system. This weakness has been particularly addressed in [6].
- *Assumption/simplification*: Putting a power system model into an ODE format is required as the starting point of classical Normal Form analysis. However, this usually involves strong assumptions like constant impedance load and other modeling simplifications on machine models and controls [1]. As a matter of fact, practical power systems are DAE systems, instead of ODE systems.

This paper aims to propose a novel method to overcome the latter shortcoming by extending the application of Normal Form Theory from power system ODE to power system DAE.

III. NORMAL FORM THEORY ON DAE SYSTEMS WITH SECOND-ORDER NONLINEARITIES

This section shows, based on [11], how to apply Normal Form Theory to a class of DAE systems with second-order nonlinearities as:

$$\mathbf{M}\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{F}_2(\mathbf{x}, \mathbf{x}) \quad (6)$$

where \mathbf{x} is the augmented vector of system variables, including both differential or algebraic ones, of size n_{aug} , \mathbf{M} is the augmented mass matrix, which is diagonal consisting of only ones (associated with differential variables) and zeros (associated with algebraic variables), and \mathbf{A} is the augmented linear state matrix, both matrices being of size $n_{\text{aug}} \times n_{\text{aug}}$. Similarly, assume that the origin is a stable operating point of the system¹. Finally note that mass matrix \mathbf{M} can be singular and that \mathbf{F}_2 is a purely quadratic vector function.

Given the DAE system in (6), the Normal Form analysis aims to find a nonlinear change of variables given by (7) that makes possible to get the Normal Form system in (8), where the vector of normal variables is $\mathbf{z} = (z_1, z_2, \dots)^T$. If only a single mode is retained in the Normal Form system, \mathbf{z} contains only two complex conjugate normal variables, i.e., $\mathbf{z} = (z_1, z_2)^T$. Without loss of generality, the following derivations assume only a single mode in the Normal Form system, while the extension to multiple modes is straightforward.

$$\mathbf{x} = \mathbf{h}(\mathbf{z}) \quad (7)$$

$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}) \quad (8)$$

To determine $\mathbf{h}(\mathbf{z})$ and $\mathbf{f}(\mathbf{z})$, we first use multivariate polynomials to rewrite them as:

$$\begin{aligned} \mathbf{h}(\mathbf{z}) = & \mathbf{H}_0 + \mathbf{H}_1 z_1 + \mathbf{H}_2 z_2 + \mathbf{H}_3 z_1^2 + \mathbf{H}_4 z_1 z_2 + \mathbf{H}_5 z_2^2 \\ & + \mathbf{H}_6 z_1^3 + \mathbf{H}_7 z_1^2 z_2 + \mathbf{H}_8 z_1 z_2^2 + \mathbf{H}_9 z_2^3 + \mathbf{H.O.T}^4 \quad (9) \end{aligned}$$

$$\mathbf{f}(\mathbf{z}) = \mathbf{F}_0 + \mathbf{F}_1 z_1 + \mathbf{F}_2 z_2 + \mathbf{F}_3 z_1^2 + \mathbf{F}_4 z_1 z_2 + \mathbf{F}_5 z_2^2 + \mathbf{F}_6 z_1^3 + \mathbf{F}_7 z_1^2 z_2 + \mathbf{F}_8 z_1 z_2^2 + \mathbf{F}_9 z_2^3 + \mathbf{H.O.T}^4 \quad (10)$$

where $\mathbf{H}_0, \mathbf{H}_1, \dots$ are coefficient vectors of size $n_{\text{aug}} \times 1$, and $\mathbf{F}_0, \mathbf{F}_1, \dots$ are coefficient vectors of size 2×1 .

Time derivative of (7) gives (11). Substituting (8) and (11) into (6) gives the following homological equation in (12). Equating like terms in (12) in \mathbf{z} can give the equations for determining the unknown coefficients in $\mathbf{h}(\mathbf{z})$, which can be computed recursively by eliminating i -th order terms as much as possible, i.e., reaching the Normal Form, unless resonance conditions. Note that the solvability of (12) is independent of the mass matrix \mathbf{M} .

$$\dot{\mathbf{x}} = \frac{\partial \mathbf{h}(\mathbf{z})}{\partial \mathbf{z}} \dot{\mathbf{z}} \quad (11)$$

$$\mathbf{M} \frac{\partial \mathbf{h}(\mathbf{z})}{\partial \mathbf{z}} \mathbf{f}(\mathbf{z}) = \mathbf{A} \mathbf{h}(\mathbf{z}) + \mathbf{F}_2(\mathbf{h}(\mathbf{z}), \mathbf{h}(\mathbf{z})) \quad (12)$$

where

$$\frac{\partial \mathbf{h}(\mathbf{z})}{\partial \mathbf{z}} = \left(\frac{\partial \mathbf{h}(\mathbf{z})}{\partial z_1} \quad \frac{\partial \mathbf{h}(\mathbf{z})}{\partial z_2} \right) = \begin{pmatrix} \mathbf{H}_1 + 2\mathbf{H}_3 z_1 + \mathbf{H}_4 z_2 + 3\mathbf{H}_6 z_1^2 + 2\mathbf{H}_7 z_1 z_2 + \mathbf{H}_8 z_2^2 \\ \mathbf{H}_2 + \mathbf{H}_4 z_1 + 2\mathbf{H}_5 z_2 + \mathbf{H}_7 z_1^2 + 2\mathbf{H}_8 z_1 z_2 + 3\mathbf{H}_9 z_2^2 \end{pmatrix}^T \quad (13)$$

IV. CASE STUDY ON SMIB

This section applies the proposed approach to an SMIB power system to show its feasibility. The SMIB system under study is composed of a simple generator model represented by swing equations plus an inductive line connecting it to an infinite bus.

A. Original System Modeling

The differential-algebraic equations of the above SMIB are

$$\frac{d\delta}{dt} = \omega_n (\omega - \omega_s) \quad (14a)$$

$$\frac{d\omega}{dt} = \frac{p_m - p_e - D(\omega - \omega_s)}{2H} \quad (14b)$$

$$0 = i_d (v_d + i_d r_a) - p_e + i_q (v_q + i_q r_a) \quad (14c)$$

$$0 = v_q - e_q + i_q r_a + i_d x_d \quad (14d)$$

$$0 = v_d + i_d r_a - i_q x_d \quad (14e)$$

$$0 = V \sin(\delta - \theta) - v_d \quad (14f)$$

$$0 = V \cos(\delta - \theta) - v_q \quad (14g)$$

$$0 = i_d v_d + i_q v_q - \frac{V V_{\text{inf}} \sin(\theta - \theta_{\text{inf}})}{x_L} \quad (14h)$$

$$0 = i_d v_q - i_q v_d - \frac{V^2}{x_L} + \frac{V V_{\text{inf}} \cos(\theta - \theta_{\text{inf}})}{x_L} \quad (14i)$$

where δ is rotor angle in per unit, ω is rotor angular speed, ω_n is the synchronous angular speed, ω_s is per-unit steady-state

angular speed, p_m and p_e are respectively mechanical power and electrical power, i_d, i_q, v_d , and v_q are respectively d- and q-axes stator currents and voltages, r_a and x_d are machine source impedance, e_q is machine field voltage on q-axis, $V, V_{\text{inf}}, \theta$ and θ_{inf} are respectively voltages and angles of the machine terminal bus and the infinite bus, and x_L is the line reactance between the generator bus and the infinite bus.

B. System Modeling In Expected Form

The exact system (14) is first Taylor expanded up to order three to get the form in (15):

$$\hat{\mathbf{M}} \dot{\hat{\mathbf{x}}} = \hat{\mathbf{A}} \hat{\mathbf{x}} + \mathbf{T2}(\hat{\mathbf{x}}, \hat{\mathbf{x}}) + \mathbf{T3}(\hat{\mathbf{x}}, \hat{\mathbf{x}}, \hat{\mathbf{x}}) \quad (15)$$

where $\hat{\mathbf{x}} = (x_1, x_2, \dots, x_9)^T = (\delta, \omega, v_d, v_q, i_d, i_q, p_e, V, \theta)^T$,

$$\hat{\mathbf{M}} = \begin{pmatrix} \mathbf{I}_{2 \times 2} & \mathbf{0}_{2 \times 7} \\ \mathbf{0}_{7 \times 2} & \mathbf{0}_{7 \times 7} \end{pmatrix} \quad (16)$$

and $\hat{\mathbf{A}}, \mathbf{T2}(\hat{\mathbf{x}}, \hat{\mathbf{x}})$ and $\mathbf{T3}(\hat{\mathbf{x}}, \hat{\mathbf{x}}, \hat{\mathbf{x}})$ are given in Appendix.

By introducing three new dummy variables, i.e., $\Delta = \delta^2$, $\Gamma = \delta\theta$ and $\Theta = \theta^2$, (15) becomes (18) which only contains quadratic terms as required by the proposed method.

$$\mathbf{x} = (x_1, \dots, x_{12})^T = (\delta, \omega, v_d, v_q, i_d, i_q, p_e, V, \theta, \Delta, \Gamma, \Theta)^T \quad (17)$$

$$\mathbf{M} \dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{Q2}(\mathbf{x}, \mathbf{x}) \quad (18)$$

where $\mathbf{Q2}$ is given in Appendix and others are given below.

$$\mathbf{M} = \begin{pmatrix} \hat{\mathbf{M}} & \mathbf{0}_{9 \times 1} & \mathbf{0}_{9 \times 1} & \mathbf{0}_{9 \times 1} \\ \mathbf{0}_{1 \times 9} & 0 & 0 & 0 \\ \mathbf{0}_{1 \times 9} & 0 & 0 & 0 \\ \mathbf{0}_{1 \times 9} & 0 & 0 & 0 \end{pmatrix} \quad (19)$$

$$\mathbf{A} = \begin{pmatrix} \hat{\mathbf{A}} & \mathbf{B} & \mathbf{C} & \mathbf{D} \\ \mathbf{0}_{1 \times 9} & -1 & 0 & 0 \\ \mathbf{0}_{1 \times 9} & 0 & -1 & 0 \\ \mathbf{0}_{1 \times 9} & 0 & 0 & -1 \end{pmatrix} \quad (20)$$

$$\mathbf{B} = \begin{pmatrix} \mathbf{0}_{5 \times 1} \\ (V_s \sin(\delta_s - \theta_s))/2 \\ -(V_s \cos(\delta_s - \theta_s))/2 \\ \mathbf{0}_{2 \times 1} \end{pmatrix} \quad (21)$$

$$\mathbf{C} = \begin{pmatrix} \mathbf{0}_{5 \times 1} \\ -V_s \sin(\delta_s - \theta_s) \\ V_s \cos(\delta_s - \theta_s) \\ \mathbf{0}_{2 \times 1} \end{pmatrix} \quad (22)$$

$$\mathbf{D} = \begin{pmatrix} \mathbf{0}_{5 \times 1} \\ (V_s \sin(\delta_s - \theta_s))/2 \\ -(V_s \cos(\delta_s - \theta_s))/2 \\ -(V_{\text{inf}} V_s \sin(\theta_{\text{inf}} - \theta_s))/(2x_L) \\ -(V_{\text{inf}} V_s \cos(\theta_{\text{inf}} - \theta_s))/(2x_L) \end{pmatrix} \quad (23)$$

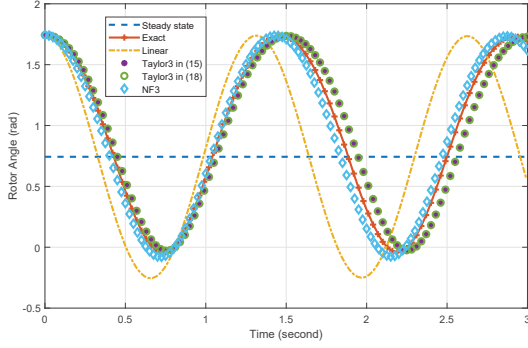


Fig. 1. Responses of different systems: steady state, exact, linear, 3^{rd} Taylor expansion, and Normal Form

C. Numerical Results

Consider the following parameters (all in per unit unless otherwise notified): $\omega_n = 2\pi \times 50$ rad/s, $\omega_s = 1$, $p_m = 1$, $D = 0.1$, $H = 8$, $r_a = 0.1$, $x_d = 0.2$, $e_q = 1$, $V_{inf} = 1$, $\theta_{inf} = 0$ and $x_L = 0.5$, the corresponding operating point is: $\delta_s = 0.7432$, $\omega_s = 1$, $v_{ds} = 0.1766$, $v_{qs} = 0.8532$, $i_{ds} = 0.2338$, $i_{qs} = 1$, $p_{es} = 1$, $V_s = 0.8713$ and $\theta_s = 0.5390$.

A large disturbance is considered by taking the following initial condition: $\delta(0) = 1.7432$ rad and $\omega(0) = 1$ pu.

Figure 1 shows responses of different systems: steady state, exact, linear, 3^{rd} Taylor expansion, and Normal Form. As expected, the linear approach is not able to capture the right amplitude nor the right frequency of the exact system. The Taylor expansions of the system under study, expressed by (15) and (18), give the same response and are very close to the response of the exact system, validating the two ways of expressing the same approximated model of the system. Finally, the Normal Form of the system under study can, contrary to the linear approach, maintain a high accuracy in the amplitude and the frequency compared to the exact system response when subject to a large disturbance.

D. Proposed Normal Form V.S. Classical Normal Form

Figure 2 shows the comparison between the Normal Form model obtained by the classical approach and the proposed Normal Form. It can be concluded that the two approaches give the same responses².

V. CONCLUSION AND FUTURE WORK

This paper shows that applying Normal Form Theory to power system models represented by differential-algebraic equations (DAE) is possible. The obtained Normal Form is strictly the same as the one that can be obtained by using the classical Normal Form, which applies to dynamic systems represented by ordinary differential equations. The proposed Normal Form makes it possible for more realistic power system modeling by DAE, allowing, e.g., the consideration

²Note that $r_a=0$ is used in this comparison due to a limitation in our implementation of the classic Normal Form, which could be avoided.

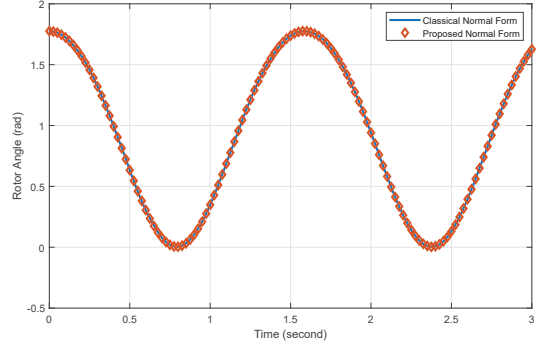


Fig. 2. Comparison between classical and proposed Normal Form

of nonlinear loads and other algebraic constraints, offering the power system community a novel tool for analytically assessing the transient stability of modern power systems.

This paper only tested DAE that is reducible to ODE. Future works include applying more detailed machine, control, and load models, larger test systems that are not reducible to ODE, and integrating a transient stability simulation program.

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APPENDIX

$$\hat{\mathbf{A}} = \begin{pmatrix} 0 & \omega_n & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{D}{2H} & 0 & 0 & 0 & 0 & -\frac{1}{2H} & 0 & 0 \\ 0 & 0 & i_{ds} & i_{qs} & v_{ds} + 2i_{ds}r_a & v_{qs} + 2i_{qs}r_a & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_d & r_a & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & r_a & -x_d & 0 & 0 & 0 \\ V_s \cos(\delta_s - \theta_s) & 0 & -1 & 0 & 0 & 0 & 0 & \sin(\delta_s - \theta_s) & -V_s \cos(\delta_s - \theta_s) \\ -V_s \sin(\delta_s - \theta_s) & 0 & 0 & -1 & 0 & 0 & 0 & \cos(\delta_s - \theta_s) & V_s \sin(\delta_s - \theta_s) \\ 0 & 0 & i_{ds} & i_{qs} & v_{ds} & v_{qs} & 0 & -\frac{V_{inf} \sin(\theta_s - \theta_{inf})}{x_L} & -\frac{V_s V_{inf} \cos(\theta_s - \theta_{inf})}{x_L} \\ 0 & 0 & -i_{qs} & i_{ds} & v_{qs} & -v_{ds} & 0 & \frac{V_{inf} \cos(\theta_s - \theta_{inf})}{x_L} - \frac{2V_s}{x_L} & -\frac{V_s V_{inf} \sin(\theta_s - \theta_{inf})}{x_L} \end{pmatrix}$$

$$\mathbf{T2}(\hat{\mathbf{x}}, \hat{\mathbf{x}}) = \begin{pmatrix} 0 \\ 0 \\ r_a x_6^2 + r_a x_5^2 + x_4 x_6 + x_3 x_5 \\ 0 \\ 0 \\ \frac{(-x_9 + x_1) (\sin(-x_9s + x_1s) x_9 x_{8s} + 2 \cos(-x_9s + x_1s) x_8 - \sin(-x_9s + x_1s) x_1 x_{8s})}{2} \\ -\frac{(-x_9 + x_1) (-\cos(-x_9s + x_1s) x_9 x_{8s} + 2 \sin(-x_9s + x_1s) x_8 + \cos(-x_9s + x_1s) x_1 x_{8s})}{2} \\ -\frac{\sin(\theta_{inf} - x_9s) V_{inf} x_9^2 x_{8s} - 2 \cos(\theta_{inf} - x_9s) V_{inf} x_8 x_9 + 2 x_4 x_6 x_L + 2 x_3 x_5 x_L}{2 x_L} \\ -\frac{\cos(\theta_{inf} - x_9s) V_{inf} x_9^2 x_{8s} - 2 \sin(\theta_{inf} - x_9s) V_{inf} x_8 x_9 + 2 x_8^2 - 2 x_4 x_5 x_L + 2 x_3 x_6 x_L}{2 x_L} \end{pmatrix}$$

$$\mathbf{T3}(\hat{\mathbf{x}}, \hat{\mathbf{x}}, \hat{\mathbf{x}}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{(-x_9 + x_1)^2 (-\cos(-x_9s + x_1s) x_9 x_{8s} + 3 \sin(-x_9s + x_1s) x_8 + \cos(-x_9s + x_1s) x_1 x_{8s})}{6} \\ -\frac{(-x_9 + x_1)^2 (\sin(-x_9s + x_1s) x_9 x_{8s} + 3 \cos(-x_9s + x_1s) x_8 - \sin(-x_9s + x_1s) x_1 x_{8s})}{6} \\ -\frac{V_{inf} x_9^2 (-\cos(\theta_{inf} - x_9s) x_9 x_{8s} + 3 \sin(\theta_{inf} - x_9s) x_8)}{6 x_L} \\ -\frac{V_{inf} x_9^2 (\sin(\theta_{inf} - x_9s) x_9 x_{8s} + 3 \cos(\theta_{inf} - x_9s) x_8)}{6 x_L} \end{pmatrix}$$

$$\mathbf{Q2}(\mathbf{x}, \mathbf{y}) = \begin{pmatrix} 0 \\ 0 \\ r_a x_5 y_5 + x_3 y_5 + r_a x_6 y_6 + x_4 y_6 \\ 0 \\ 0 \\ \frac{(x_1 - x_9) (2 y_8 \cos(\delta_s - \theta_s) - V_s y_1 \sin(\delta_s - \theta_s) + V_s y_9 \sin(\delta_s - \theta_s))}{2} - \frac{x_{10} (3 y_8 \sin(\delta_s - \theta_s) + V_s y_1 \cos(\delta_s - \theta_s) - V_s y_9 \cos(\delta_s - \theta_s))}{6} \\ -\frac{(x_1 - x_9) (2 y_8 \sin(\delta_s - \theta_s) + V_s y_1 \cos(\delta_s - \theta_s) - V_s y_9 \cos(\delta_s - \theta_s))}{2} - \frac{x_{10} (3 y_8 \cos(\delta_s - \theta_s) - V_s y_1 \sin(\delta_s - \theta_s) + V_s y_9 \sin(\delta_s - \theta_s))}{6} \\ \frac{V_s V_{inf} \sin(\theta_s - \theta_{inf}) x_9 y_9 - 2 x_8 V_{inf} \cos(\theta_s - \theta_{inf}) y_9 + 2 x_5 y_3 x_L + 2 x_6 y_4 x_L}{2 x_L} + \frac{V_{inf} x_{11} (3 y_8 \sin(\theta_s - \theta_{inf}) + V_s y_9 \cos(\theta_s - \theta_{inf}))}{6 x_L} \\ -\frac{2 x_8 y_8 + 2 V_{inf} \sin(\theta_s - \theta_{inf}) x_8 y_9 + V_0 V_{inf} \cos(\theta_s - \theta_{inf}) x_9 y_9 + 2 x_6 y_3 x_L - 2 x_5 y_4 x_L}{2 x_L} - \frac{V_{inf} x_{11} (3 y_8 \cos(\theta_s - \theta_{inf}) - V_s y_9 \sin(\theta_s - \theta_{inf}))}{6 x_L} \\ x_1 y_1 \\ x_1 y_9 \\ x_9 y_9 \end{pmatrix}$$