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# Empirical Estimation of the Effective Sample Size Based on the Central Limit Theorem in Reverberation Chambers

Anyela Aquino-Velasquez<sup>#§1</sup>, Philippe Besnier<sup>§2</sup>, Priscila Fernandez-Lopez<sup>#3</sup>, Mohsen Koohestani<sup>§4</sup>

<sup>#</sup>Valeo, Créteil, France

<sup>§</sup> Université de Rennes, INSA Rennes, ESEO Angers, IETR UMR 6164 CNRS, CentraleSupélec, Nantes Université, F 35000 Rennes France

{<sup>1</sup>anyela.aquino-velasquez, <sup>3</sup>priscila.fernandez-lopez}@valeo.com, <sup>2</sup>philippe.besnier@insa-rennes.fr, <sup>4</sup>mohsen.koohestani@eseo.fr

**Abstract**—Mode stirrers are typically used to change the boundary conditions when performing measurements in reverberation chambers (RCs). A major concern arises in determining the number of independent realizations or, equivalently, the effective sample size ( $N_{eff}$ ) obtained during stirrer rotations. A significant effort has been put into determining the effective sample size in the literature, leading to the proposal of numerous methods. However, those methods yield different estimations for  $N_{eff}$ , which does not allow a clear diagnosis. This paper aims to provide an empirical estimation based on the average of the standard deviation over the mean estimation in relation to the central limit theorem (CLT). It is confronted with different theoretical models in the literature. Finally, it is confirmed that the spatial correlation of data series provides a better estimation than all other methods.

**Keywords**—Reverberation chamber, effective sample size, central limit theorem, standard deviation, spatial correlation.

## I. INTRODUCTION

Reverberation chambers (RCs) have traditionally been used for electromagnetic compatibility (EMC) tests [1]-[3]. RC has the capability to generate high field strength with moderate input power. Industries such as aviation, automotive, and defense are leveraging this capability to enhance their testing processes. For automotive industry, the ISO 11452-11 [4] is the international standard to test the road vehicle components against radiated disturbances in a RC. However, there is no equivalent test applicable on integrated circuits (ICs). The frequency band extension of EMC test makes the ICs be the weakest part in an electronic product. It is therefore necessary to assess the radiated immunity of ICs in order to design robust products. To do so, the RC is a very interesting candidate especially at frequencies above 4 GHz, but first uncertainties must be estimated. Overall, whatever the application, calibration of RC plays a key role and specifically the estimation of the effective sample size which drives statistical measurement uncertainties.

Since RCs consist of a diffuse field environment with complex test conditions, such as inhomogeneous objects and variations in boundary conditions due to stirrers, RC measurements are studied from a statistical perspective [5]-[6].

Therefore, electromagnetic field calibration relies on statistical field uniformity, which is strongly related to the number of independent realizations. It is important to determine the number of independent realizations provided by the stirring process to ensure accurate statistical analysis of the measured data. The number of independent realizations or  $N_{eff}$  is the equivalent number of states provided by the stirrer that provides uncorrelated fields in the chamber, and thereby uncorrelated power levels. One of the important questions arising from using RC is the determination of the effective sample size. The IEC 61000-4-21 standard [7] highlights that the number of independent realizations must be determined to apply statistical analysis in a RC. This depends on the stirrer's ability to sufficiently alter boundary conditions, ensuring statistically significant changes in the field pattern [8].

For systematic measurements in RC, only the uncorrelated measurements are useful since these may provide new information. Given as series of  $N$  correlated measurements corresponding to  $N$  different boundary conditions (e.g. stirrer positions), the effective sample size is defined as the equivalent to the information that would be provided by a random series, i.e., consisting only of independent events, of length  $N_{eff}$ , such that  $N_{eff} < N$ . The number  $N_{eff}$  is what we call here the effective sample size of the series of length  $N$ .

In the last two decades different methods have been provide in the literature in order to determine the effective sample size  $N_{eff}$  [9]-[10]. Though non correlation do not imply independence, correlation operators are commonly used. The most widely used approach is the autocorrelation function (ACF) method based on a correlation threshold, as discussed in [11]-[13]. However, it is well known that the ACF method provides only rough estimations [14]. To address this limitation, modified versions of the ACF method have been introduced taking a slightly different approach [12]-[14]. In [15], the autoregressive model method is applied by the authors as an alternative way to characterize the correlation and this was used for uncertainty analysis. Likewise, in [16], [17] it was used to compute an ideal stirrer step size. In [18], alternative models are proposed based on the Q-factor. In a multivariate approach, in [19]-[21], an alternative way of evaluating the

effective sample size based on the calculation of the correlation matrix is proposed.

This work aims to provide a simple and efficient empirical estimation based on the average of the standard deviation over the mean estimation in relation to the CLT. The proposed method is easy to implement since it only requires twice the number of the antenna/probe positions with regards to the standard specifications. Indeed, the IEC 61000-4-21 standard requires 8 probe /antenna positions for calibration. Results are consistent with the spatial correlation method in [16] which confirmed that the spatial correlation of data series provide a better estimation than all other methods.

The paper is structured as follows. Section II is devoted to present some of the widely used methods to compute the effective sample size. Section III demonstrates the analytical formulation of our method based on the central limit theorem. In Section IV, experimental results are discussed and comparisons among the presented methods are analyzed. Finally, Section V concludes the contributions of the study.

## II. METHODS FOR ESTIMATING THE EFFECTIVE SAMPLE SIZE

There are several methods for estimating the  $N_{eff}$  and many of these approaches rely on the autocorrelation function and the autocorrelation matrix as fundamental tools. This section presents five of these methods: the standardized method, the first-order correlation function based estimation, the correlation matrix method, and the information theory-based method.

### A. Standardized Method

As the name suggests, the standardized method estimates the effective sample size based on established standard information. It consists of calculating the correlation coefficient between successive stirrer positions. According to [7], the correlation coefficient  $\rho$  can then be calculated using the ACF (1).

$$\rho = \frac{\frac{1}{N-1} \sum_i (x_i - u_x)(y_i - u_y)}{\sqrt{\left(\frac{\sum_i (x_i - u_x)^2}{N-1}\right) \left(\frac{\sum_i (y_i - u_y)^2}{N-1}\right)}} \quad (1)$$

in which  $y_i$  is the same distribution as  $x_i$ , but shifted by one realization (i.e., by one position of the stirrer so that  $y_i = x_{i+1}$ ) and  $N$  is the number of realizations taken over a complete rotation of the stirrer. The terms  $u_x$  and  $u_y$  are averages of the data collected at the different positions of the stirrer.

By convention of the standard EN 61000-4-21 [7], section A.3, the data are regarded as uncorrelated when the value of the amplitude of the correlation coefficient is less than the threshold value  $\rho < \rho_{stand}$  [22]. This threshold value is obtained from the fit to the numerical data for  $N$  not less than 100 (2).

$$\rho_{stand} = \frac{1}{e} \times \left(1 - \frac{7.22}{N^{0.64}}\right) \quad (2)$$

For an infinite number  $N$ ,  $\rho_{stand} = \frac{1}{e}$ . Note that according to this standardized approach, the correlation coefficient is examined at a spatial point or at several points successively.

### B. First Order Correlation Function (FOCF)-based Estimation

This method provides an estimate of the effective sample size by considering the autocorrelation present in the data. The correlation coefficient  $\rho$  is quantified using the first-order autocorrelation function (FOCF) which corresponds to the lag-1 value of the Autocorrelation Function (ACF). The ACF at lag  $k$ , denoted as  $R(k)$ , is defined as (3).

$$R(k) = \frac{1}{N-k} \sum_{i=1}^{N-k} (X_i - \bar{X})(X_{i+k} - \bar{X}) \quad (3)$$

The FOCF refers to the autocorrelation function at lag-1 ( $k = 1$ ) defined in (1). To estimate the effective sample size, it relies on the fact that the FOCF quantifies the dependence between consecutive samples, allowing for a correction in the sample size estimation. The higher the correlation, the lower the effective sample size (4).

$$N_{eff} = N \times \left(\frac{1 - |\rho|}{1 + |\rho|}\right) \quad (4)$$

where  $N$  is the number of realizations taken over a complete rotation of the stirrer. The method proposed in [15] provides an alternative approach to characterize the autocorrelation in RC measurements by using the autoregressive (AR) models. In this sense, the expression in (4) is an approximation of an AR method of order 1 described in [15].

### C. Correlation Matrix Method

The correlation matrix method is a technique used to analyze the intercorrelation coefficients between two different states of the RC. In this approach, the focus is no longer on the autocorrelation between consecutively measured states. Instead, the correlation is analyzed across pairs of stirrer positions, considering measurements taken at different spatial locations across  $P$  positions. This means that, rather than analyzing the temporal correlation between measurements at consecutive points, this method examines how measurements relate to one another across various spatial locations, based on data collected from different stirrer positions within the system.

For an  $N$ -by- $N$  correlation matrix,  $R$ , computed from a  $P$ -by- $N$  submatrix of the original data matrix,  $H$ , the matrix element,  $r_{ij}$ , describing the correlation between the  $i^{th}$  and  $j^{th}$  stirrer positions and occupying the  $i^{th}$  row and  $j^{th}$  column of  $R$  is given by (5) [19].

$$r_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}\sigma_{jj}}} \quad (5)$$

In (5),  $\sigma_{ij}$  denotes the elements of the corresponding covariance matrix,  $\Sigma$  as given by (6) [20].

$$\sigma_{ij} = \frac{1}{P-1} \sum_{k=i}^P \{(h_i(p_k) - \langle h_i(p_k) \rangle_p) \times (h_j(p_k) - \langle h_j(p_k) \rangle_p)^*\} \quad (6)$$

Once the correlation coefficient between states has been obtained, the effective sample size  $N_{eff}$  can be evaluated based on the article by G. Gradoni *et al.* [19], Eq.(11); where the  $N_{eff}$  is computed by dividing the number of elements of the  $N$  correlation matrix by the number of elements greater than the threshold value as (7)  $\rho_{Gradoni}$ , with  $\rho_{Gradoni} = \frac{1}{e}$ .

$$N_{eff} = \frac{N^2}{N_{gradoni}} \quad (7)$$

where  $N_{gradoni}$  is defined as (8).

$$N_{gradoni} = \sum_{i,j} I(|\rho_{ij}| \leq \rho_{gradoni}) \quad (8)$$

where  $I(\cdot)$  is the indicator function, which is 1 if the condition  $|\rho_{ij}| \leq \rho_{gradoni}$  is true and 0 otherwise.

#### D. Information Theory-Based Method

This method was applied in the article by R. J. Pirkl *et al.* [18] based on the calculation of the eigenvalues of the correlation matrix (9).

$$N_{eff} = \frac{\left(\sum_{i=1}^N \lambda_i\right)^2}{\sum_{i=1}^N \lambda_i^2} \quad (9)$$

with  $\lambda_n$  being the eigenvalues of the correlation matrix. In the sense of information theory, if the eigenvalues of the correlation matrix (covariance) are all significant (non-zero) and identical then there is no redundancy of information and  $N_{eff} = N$ . On the contrary, if we find for example a single non-zero eigenvalue, then the information provided by all the states is redundant and  $N_{eff} = 1$ .

The effective number of uncorrelated measurements may alternatively be expressed as (10) [9].

$$N_{eff} = \frac{N^2}{\sum_{i,j=1}^N |r_{ij}|^2} \quad (10)$$

where  $r_{ij}$  are again the elements of the correlation matrix,  $R$ . In practice, (10) tends to be more convenient, because it does not require the calculation of the correlation matrix's eigenvalues.

### III. EFFECTIVE SAMPLE SIZE ESTIMATION USING THE STANDARD DEVIATION METRIC

The proposed method for estimating the effective sample size relies on an empirical estimation based on the average of the standard deviation over the mean estimation in relation to the CLT.

Let  $X_1, X_2, \dots, X_n$  be a sequence of random values of the random variable  $X$  defined on the same probability space,

following the same independent distribution  $\mathcal{L}$ . Suppose that the expectation (or mean)  $\mu_x$  and standard deviation  $\sigma_x$  of the distribution  $\mathcal{L}$  (first and second moment of the distribution) exist and are finite ( $\sigma \neq 0$ ).

Let us consider the sum  $S_n = X_1 + X_2 + \dots + X_n$ . The CLT states the following result: the distribution of  $S_n$  tends to the normal distribution  $\mathcal{N}(n\mu, \sigma\sqrt{n})$  as  $n$  approaches infinity. The expectation of  $S_n$  is given by  $n\mu$  and its standard deviation is  $\sigma\sqrt{n}$ .

The interest is not in the sum  $S_n$ , but rather in the mean  $\langle X_n \rangle$  defined as (11).

$$\langle X_n \rangle = \frac{S_n}{n} = \frac{X_1 + X_2 + \dots + X_n}{n} \quad (11)$$

Then it is also a random variable whose distribution tends to the normal distribution  $\mathcal{N}(\mu, \sigma/\sqrt{n})$  as  $n$  approaches infinity. This result is very important because it allows us to control the uncertainty in estimating a mean quantity, by knowing the distribution and the associated standard deviation. Therefore, when  $N$  is sufficiently large, we can consider that the estimator  $\hat{\mu}$  of the mean value is distributed according to a normal law with standard deviation (12).

$$\sigma_{\mu_x} = \frac{\sigma_x}{\sqrt{N}} \quad (12)$$

We propose an approach based on the average of the standard deviation over the mean estimation in relation to the CLT to arrive at the estimation of the effective sample size of the series.

Consider a population of  $P$  samples of size  $N$  with the presence of a non-zero correlation between successive values. Let  $\hat{\mu}$  represent the estimated mean. In this case, the estimator of the mean would have the same standard deviation to mean ratio as the sample  $X_1, X_2, \dots, X_n$ . This translates into the following relationship (13).

$$\frac{\sigma_{\mu_x}}{\mu_x} = \frac{\hat{\sigma}_{\mu}}{\hat{\mu}} \quad (13)$$

Using (12) in the first member of (17), the effective sample size sought is a solution to the following equation (18).

$$\frac{\sigma_x}{\mu_x} \times \frac{1}{\sqrt{N_{eff}}} = \frac{\hat{\sigma}_{\mu}}{\hat{\mu}} \quad (14)$$

In (14), the relationship  $\frac{\sigma_x}{\mu_x}$  is unknown at low frequencies. Therefore, we compute the empirical estimation based on spatial sampling performed by moving the Rx antenna at  $P$  different positions and  $N$  stirrer positions (15).

Given  $P$  data sets of size  $N$ :

$$\left(\frac{\hat{\sigma}_x}{\hat{\mu}_x}\right) = \frac{1}{P} \sum_{i=1}^P \left(\frac{\sigma_x}{\mu_x}\right)_i \quad (15)$$

The terms  $\hat{\sigma}_{\mu}$  and  $\hat{\mu}$  are estimated over  $P$  spatial samples as follows in (16) and (17), respectively.

$$\hat{\sigma}_{\mu} = \sqrt{\frac{1}{P-1} \sum_{i=1}^P (\mu_i - \hat{\mu})^2} \quad (16)$$



Fig. 1. IETR stirrer-mode reverberation chamber with the Tx and Rx antennas used for measurements.

$$\hat{\mu} = \frac{1}{P} \sum_{j=1}^P \mu_j \quad (17)$$

Therefore, solving (14) for  $N_{eff}$ , the effective sample size is computed as follows (18).

$$N_{eff} = \left( \frac{\left( \frac{\hat{\sigma}_x}{\hat{\mu}_x} \right)^2}{\frac{\hat{\sigma}_\mu}{\hat{\mu}}} \right)^2 \quad (18)$$

Hence, the estimation is obtained based on the measurement of  $P$  antenna positions with  $N$  stirrer positions for each measurement.

#### IV. EXPERIMENTAL VALIDATION

In order to perform a comparison of the values obtained for the  $N_{eff}$  among our proposed method and the ones listed in Section II, a large set of measurements were carried out. This section presents the measurement protocol to have into account during the data collection, and the calculation of the  $N_{eff}$  under two different selections of the random variable (RV): the stirrer position and the frequency.

##### A. Measurement Protocol

All experiments were carried out in the IETR reverberation chamber, shown in Fig.1., whose dimensions are 2.9 m  $\times$  3.7 m  $\times$  8.7 m. The lowest usable frequency (LUF) is established around 200 MHz. The measurements were performed from 200 MHz to 1000 MHz. At each frequency point, we use 16 independent positions of the receiving antenna ( $P$ ). The Tx and Rx antennas (Fig. 1) are log-periodic antennas (ETS-Lindgren Model 3148, 6.4 cm high, 85.6 cm wide, and 73.7 cm deep (length)). The Tx antenna is placed in a corner of the RC, outside the test volume, and directed towards the corner of the

chamber in order to avoid the presence of a direct path between the transmission and the reception. A minimum distance of  $\lambda/2$  at the LUF is respected between each position of the Rx antenna, so as to ensure the absence of correlation between these positions [23]. Hence, we have considered the first eight different Rx antenna positions, which are horizontally polarized and located 10 cm above the top of the antenna mast. The second set of eight different Rx antenna positions are vertically polarized and positioned 45 cm above the top of the antenna mast. For each position of the receiving antenna, we use 60 stirrer positions ( $N$ ), equally distributed over a full rotation. This means that an angular step of  $\Delta\theta = 6^\circ$  separates two successive positions of the stirrer. Furthermore, the test volume is far enough from walls and the stirrer by considering a distance of at least  $\lambda/2$  at the LUF. The antennas were connected to a vector network analyzer (VNA), which was calibrated at the antenna ports. For each of the  $N$  different stirrer positions, the VNA was used to record the complex  $S_{21}$  with a finite frequency at 8001 equispaced frequencies from 200 to 1000 MHz.

##### B. Stirrer Position as Random Variable

A first approach of the calculation of the effective sample size is carried out by considering the stirrer position as an RV. As it was mentioned, a VNA was used to record the complex  $S_{21}$ . From this data all the calculations and analysis were carried out using the magnitude of  $S_{21}$ . The resulting dataset was compiled into a  $P$ -by- $N$  data matrix,  $\mathbf{H}$ , given by (19).

$$\mathbf{H} \equiv \begin{bmatrix} h_1^{(1)} & h_1^{(2)} & \dots & h_1^{(N)} \\ h_2^{(1)} & h_2^{(2)} & \dots & h_2^{(N)} \\ \vdots & \vdots & \ddots & \vdots \\ h_P^{(1)} & h_P^{(2)} & \dots & h_P^{(N)} \end{bmatrix} \quad (19)$$

where  $N$  is the stirrer positions,  $P$  is the antenna positions, and the matrix element  $h_p^{(n)}$  denotes the magnitude of  $S_{21}$  measurement corresponding to the  $p_{th}$  antenna position  $P$  and  $n_{th}$  stirrer position. At each frequency point, a data set was obtained (8001 matrices), organized as shown in the matrix  $\mathbf{H}$ , which would allow for the calculation of  $N_{eff}$  for each frequency value.

For the computation of the  $N_{eff}$  based on Section II (A, and B), the correlation coefficient  $\rho$  was averaged over all the receiving antenna positions  $\langle \rho_p^{(n)} \rangle_p$  to finally obtain a value at each frequency point  $\langle \rho \rangle$ . For the computation of  $N_{eff}$  based on Section II (C and D), the covariance and correlation matrices are computed directly from the matrix  $\mathbf{H}$ . For the application of our proposed method, the standard deviation and the expectation are averaged over all the antenna positions positions  $\langle \sigma_p^{(n)} \rangle_p$  and  $\langle \mu_p^{(n)} \rangle_p$ , respectively. Then, the computation of the  $N_{eff}$  follows as described in Section III.

Fig. 2 (a),(b) present the calculated results of the  $N_{eff}$  obtained for the methods listed in Section II and our method in Section III against the frequency in raw data and fitted data, respectively. The curves shown in Fig. 2 (b) represent

fitted data obtained using polynomial interpolation for each estimation method. Fig. 2 (a) demonstrates that a single frequency measurement is not sufficient to reliably estimate  $N_{eff}$  instead, it should be determined over a large number of frequencies. It is worth mentioning that the value was also calculated using the autoregressive model method described in [15], and the results were similar to those obtained with the FOCF method. Therefore, the AR method results were not included in the figure to provide a clearer view of the other methods. It is observed estimations ranging from 10 to 20 between the correlation matrix method and the information theory based method, and from 10 to 60 when considering the correlation matrix method and the standardized method. This is due to the fact that the correlation matrix method is more restrictive than the others since in this method the correlation is analyzed across pairs of stirrer positions considering measurements taken at different spatial locations. It is noteworthy that the results from our proposed method closely align with those obtained using the correlation matrix method. The key aspect of our method is the calculation of the  $N_{eff}$  by considering measurements taken across a wide range of spatial locations, denoted as  $P$ . This allows for a robust estimation by ensuring that the interdependencies between spatial measurements are appropriately accounted for, resulting in consistent and reliable estimates of the effective sample size.

### C. Frequency as Random Variable

A second approach of the calculation of the effective sample size is carried out by considering the frequency as a RV [21]. Under this approach, the calculation of the  $N_{eff}$  is performed over a 6 MHz bandwidth consisting of 60 frequency points. Hence, the resulting data set was grouped into a  $P$ -by- $M$  data matrix,  $\mathbf{G}$ , given by (20).

$$\mathbf{G} \equiv \begin{bmatrix} g_1^{(f_1)} & g_1^{(f_2)} & \dots & g_1^{(f_M)} \\ g_2^{(f_1)} & g_2^{(f_2)} & \dots & g_2^{(f_M)} \\ \vdots & \vdots & \ddots & \vdots \\ g_P^{(f_1)} & g_P^{(f_2)} & \dots & g_P^{(f_M)} \end{bmatrix} \quad (20)$$

where  $M$  is the frequency points of the 6 MHz bandwidth,  $P$  is the antenna positions, and the matrix element  $g_p^{(f_m)}$  denotes the magnitude of  $S_{21}$  measurement corresponding to the  $p_{th}$  antenna position  $P$  and  $m_{th}$  frequency point  $f_M$ . At each stirrer position, a data set  $\mathbf{G}$  was obtained (8001 / 60  $\approx$  133 matrices), which would allow for the calculation of  $N_{eff}$  at each 6 MHz bandwidth. For the computation of the  $N_{eff}$ , the correlation coefficient  $\rho$  was averaged over all stirrer positions  $N \langle \rho_p^{(f_m)} \rangle_n$  to finally obtain an average value for each 6 MHz bandwidth. For the computation of  $N_{eff}$  based on Section II (C and D), the covariance and correlation matrices are computed.

From Fig. 2 and Fig. 3 it is noted that when frequency is used as the RV,  $N_{eff}$  is higher, around 30, due to lower correlation between frequency points, whereas using stirrer position as the RV results in a lower  $N_{eff}$  due to stronger correlation between consecutive positions. Moreover, from Fig.

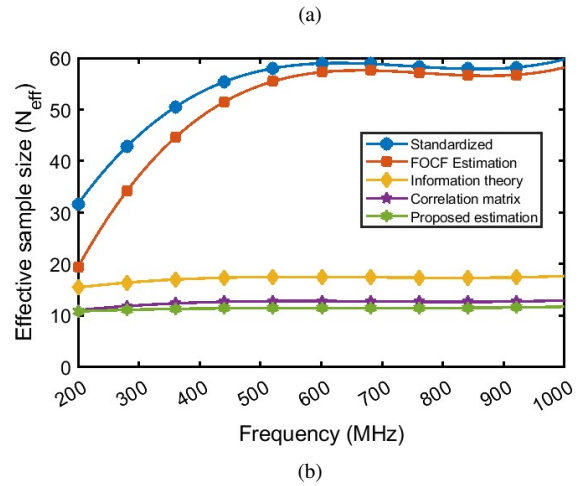
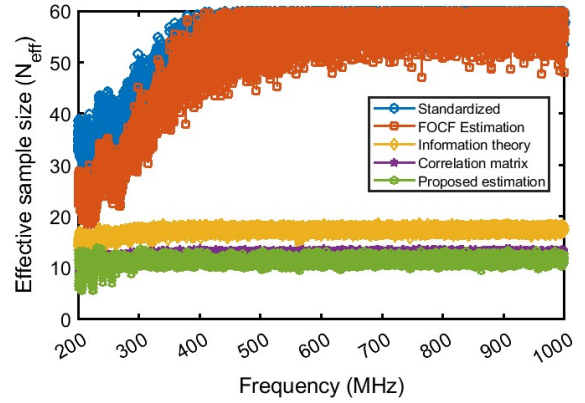


Fig. 2. Comparison of the methods described in Section II and our proposed estimation method to compute the  $N_{eff}$  by considering the stirrer position as RV. (a) Raw data. (b) Fitted data.

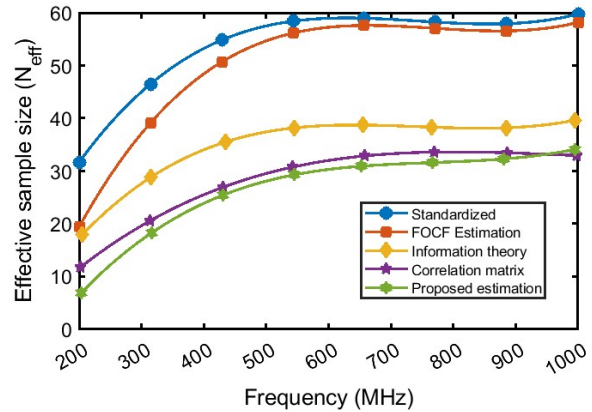


Fig. 3. (Fitted data) Comparison of the methods described in Section II and our proposed estimation method to compute the  $N_{eff}$  by considering the frequency as RV.

3, it is noted the estimations ranging from 20 to 40 between the correlation matrix method and the information theory based method.

## V. CONCLUSION

As described in our work, various methods existing in the literature provide different estimations of the effective

sample size. This paper proposes an empirical estimation based on the average of the standard deviation over the mean estimation in relation to the central limit theorem. The experimental validation of the proposed method, is carried out by analyzing two approaches with regard to two random variables. A first approach estimates the effective sample size by treating the stirrer position as RV, while a second approach considers the frequency as RV. When frequency is treated as the RV, the effective sample size tends to be higher due to the lower correlation between measurements taken at different frequency points because frequency variations significantly alter the electromagnetic modal structure of the RC, leading to more independent realizations. From the results obtained we can draw two main conclusions: First, among the methods available in the literature, the correlation matrix method is one of the most reliable. Additionally, the information theory-based method can also be used, as its results closely align with those of the correlation matrix method. However, the FOCF and standardized methods are less accurate, as their results deviate significantly. Second, the empirical estimation, we propose, provides a good estimation. This implies that even if trusting other methods and performing calculations is challenging due to their complexity, our proposed approach remains a reliable and easily applicable alternative. Overall, the results obtained using our proposed method align with those from the correlation matrix method, confirming that the spatial correlation of data series provide a better estimation than all other methods.

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